Mathematik zum Mitmachen und Nachlesen

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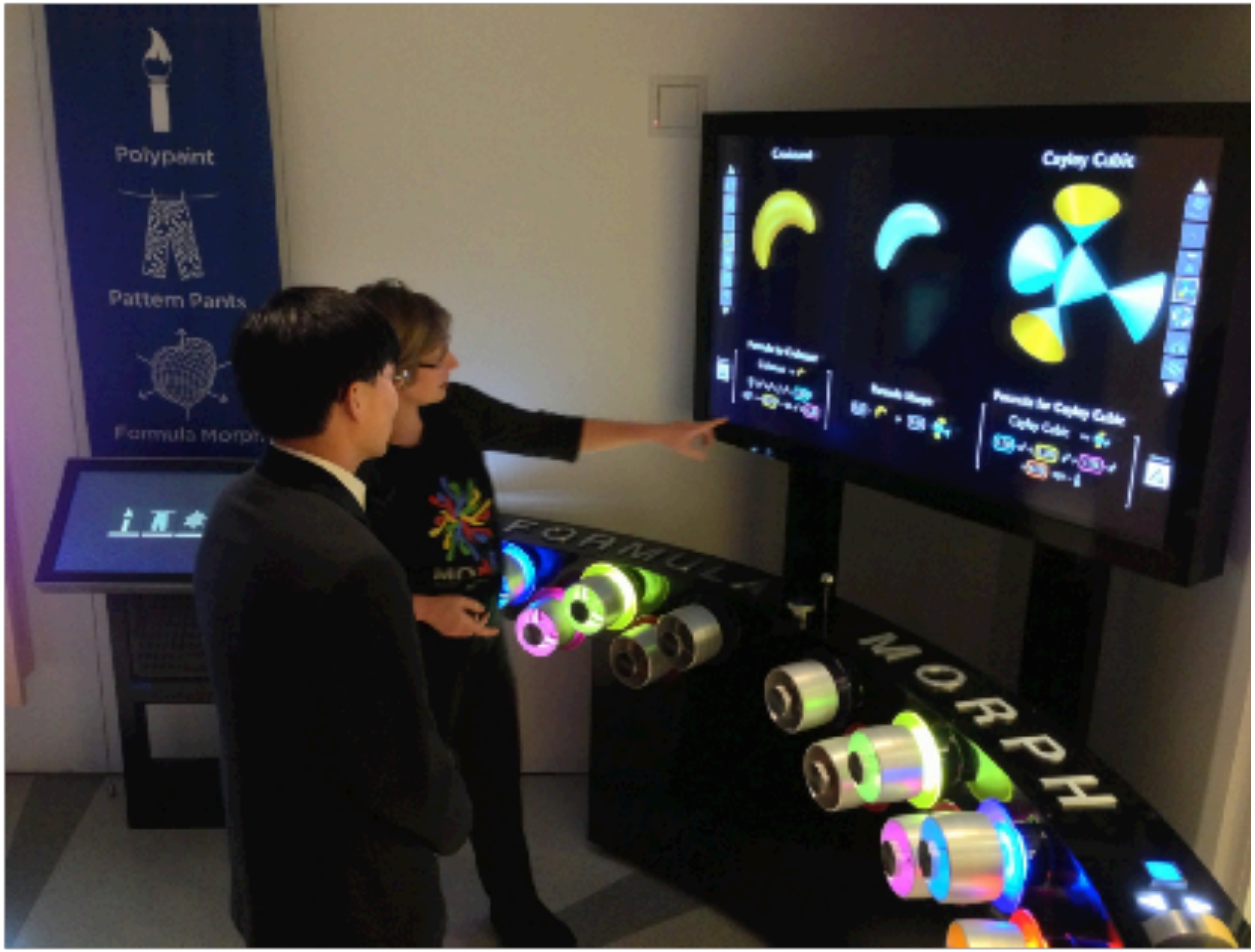
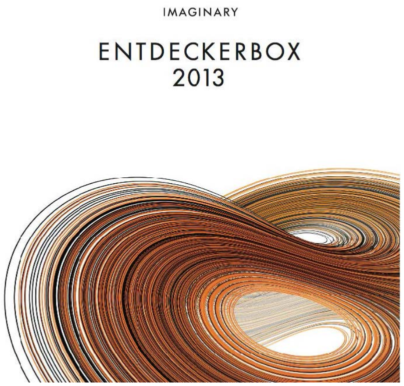
FB Mathematik/
Chefeditorin Schnappschüsse

Mathematik zum Mitmachen
und Nachlesen

Mathematik zum Mitmachen am Touchscreen...



Mathematik zum Mitmachen...



Touchscreen mit Geometrieprogrammen:

- SURFER – Form und Formel
- Morenaments – Muster und Methode
- Karten der Erde

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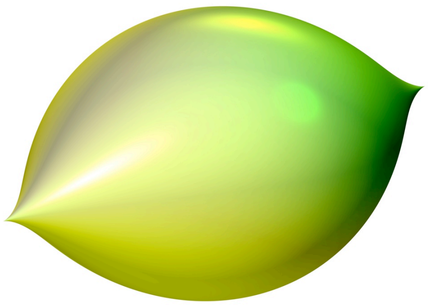
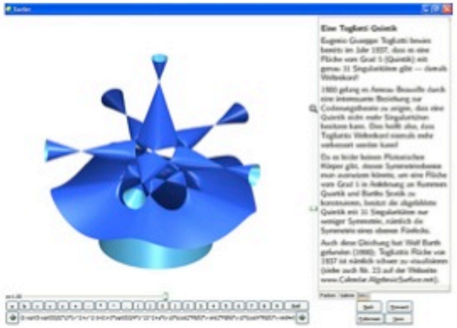


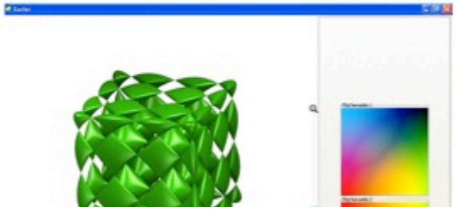
Zitrus $x^2+z^2= y^3(1-y)^3$

SURFER – Form und Formel

surfer

visualization of algebraic surfaces





Programm SURFER

- Visualisierung algebraischer Flächen in Echtzeit
- Erstellen von Bildern und Videos

- Intuitive Bedienung
- Benutzergalerien mit Hintergrundinformationen

- Einsatz im Unterricht

- Seit 2014 Teil der **Entdeckerbox**

http://www.imaginary.org/entdeckerbox

Motivation: Welche Fläche gehört zu dieser Gleichung?

$$((3x^2+(y-1.9)^2+4z^2-1)^2+0.2z)\cdot(((((0.8z+1.2)^3+5y-6)^2+16x^2-0.5)$$
$$\cdot(x^2+(y+6)^2+(z-2.8)^2-0.3)\cdot(x^2+(y-1)^2+(z+3.3)^2-0.03)+290)$$
$$\cdot(9x^2+(y-0.1\cdot z+2.5)^2+(4z-5+y)^2-1)-400)-99=0$$

Finde alle (x,y,z) im Raum, die die Gleichung erfüllen!

SURFER – Form und Formel

Die Lösung:

$$((3x^2+(y-1.9)^2+4z^2-1)^2+0.2z)\cdot(((((0.8z+1.2)^3+5y-6)^2+16x^2-0.5)$$
$$\cdot(x^2+(y+6)^2+(z-2.8)^2-0.3)\cdot(x^2+(y-1)^2+(z+3.3)^2-0.03)+290)$$
$$\cdot(9x^2+(y-0.1\cdot z+2.5)^2+(4z-5+y)^2-1)-400)-99=0$$

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... betreibt Mathematik-Kommunikationsplattform mit vielen weiteren Inhalten

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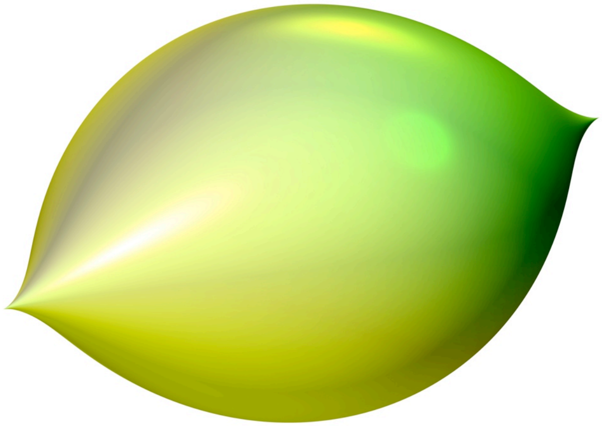

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Mathematik zum Mitmachen und **Nachlesen**

Mathematik zum Nachlesen:
Schnappschüsse moderner Mathematik aus Oberwolfach

Kurze Texte über aktuelle mathematische Forschung:

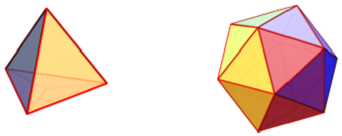

Figure 1: The Tetrahedron $\mathcal{T}$ (left) and the Icosahedron $\mathcal{I}$ (right).

mirror plane, i.e., the plane where any point is reflected on.[1] Only considering rotations, we obtain for the tetrahedron 12 rotational symmetries in total:

For rotations fixing the tetrahedron one has two different kinds of rotation axes: on the one hand, going perpendicular through the center of a face and meeting the opposite vertex (there are four of them, one for each face; see Fig. 2) and on the other hand axes connecting the midpoints of two vis-á-vis edges (three of them). For the "face-vertex" axes one can rotate two times by 120 degree about the axes and for the "vis-á-vis-edges" axes once about 180 degrees before reaching the initial position. So one gets $2 \cdot 4 + 1 \cdot 3 + 1 \cdot 1 = 12$ different rotations, where $1 \cdot 1$ comes from the identity rotation that does nothing.

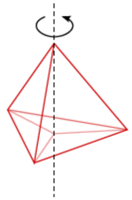

Figure 2: An example for a rotation axis going through one vertex and meeting the opposite face perpendicularly

[1] Note that the rotation axis is exactly the set of points, which is not moved when performing the rotation. Similarly, the mirror plane is the set of fixed points of a reflection. In general, we can write each rotation as the composition of two reflections. Also, by a theorem of Leonard Euler (1707-1783), a composition of two rotations is again a rotation.

Snapshots of modern mathematics from Oberwolfach   № 6/2014

Dirichlet Series

John E. McCarthy [1]

Mathematicians are very interested in prime numbers. In this snapshot, we will discuss some problems concerning the distribution of primes and introduce some special infinite series in order to study them.

1 Convergent Series

A *series* is a sequence of terms added together, *e.g.*

$$1 + 2 + 3 + \ldots + 100,$$

a series with 100 terms.[2] Series may have a finite number of terms, in which case one wants to find a formula that gives the sum, either exactly or approximately; or they may be what we call an infinite series, which means they have an infinite number of terms. We use the notation a_n for the n^{th} term in a series, so we shall be considering series of the form

$$a_1 + a_2 + a_3 + \ldots . \tag{1}$$

How can one make sense of (1)? How can one sum an infinite number of terms? Consider the example $a_n = \frac{1}{2^n}$. This gives the series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots . \tag{2}$$

[1] Partially supported by National Science Foundation Grant DMS 1300280

[2] This particular sum was supposedly given to Carl Friedrich Gauß (1777–1855) as a schoolboy to keep him occupied; but he found a clever way to find the sum quickly. Can you see how to find it without too much work?

$SO_3(\mathbb{R})$, one can relate a complex subgroup of $SU_2(\mathbb{C})$, which acts[7] on complex polynomials in two variables.
In the case of the tetrahedron, one looks at so-called invariant polynomials of the complex group $\tilde{\mathcal{T}}$ in $SU_2(\mathbb{C})$ corresponding to $\mathcal{T}$. These are all polynomials which are not changed under the group action, meaning that

$$f(x_1, x_2) = f(\varphi^{-1}(x_1, x_2))$$

for all φ in $\tilde{\mathcal{T}}$). Finally, one finds 3 polynomials u, v, w in 2 variables with complex coefficients such that any other invariant polynomial is a polynomial in u, v, w. These three invariants satisfy the equation $u^2 + v^3 + w^4 = 0$. Consider $\mathbb{R}^3$ as the set of all points (x, y, z), then

$$X = \{(x, y, z) \in \mathbb{R}^3 \text{ such that } x^2 + y^3 + z^4 = 0\}$$

is a singular algebraic surface, which has a singular point at the origin, see Fig. 4. In the last century, the connections of Platonic solids to singular algebraic surfaces were studied further, for example via resolution of singularities and in the theory of Lie groups.

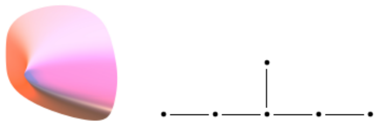

Figure 4: The algebraic surface $x^2 + y^3 + z^4 = 0$ (left), which is called an E_6-singularity. This name comes from its so-called resolution graph (right), which is the E_6 Dynkin diagram.

3 Snap: Reflections!

The Canadian mathematician H.S.M. Coxeter (1907–2003) looked at groups consisting of reflections in hyperplanes, called *reflection groups*, see [1]. One

[7] A transformation φ in the group sends a point $P = (p_1, p_2, p_3) \in \mathbb{R}^3$ to a point $\varphi(p_1, p_2, p_3) \in \mathbb{R}^3$. In this context one says that $SO_3(\mathbb{R})$ (or any of its subgroups) *acts* on a polynomial $f(x_1, x_2, x_3)$ via $f(\varphi^{-1}(x_1, x_2, x_3))$. Similarly for complex rotations in the plane on polynomials in two variables.

Was sind Schnappschüsse?

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- Und die mathematikinteressierte Öffentlichkeit

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Sophia Jahns
Daniel Kronberg
Johannes Niediek
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- Direktor: Prof. Dr. Gerhard Huisken, Universität Tübingen
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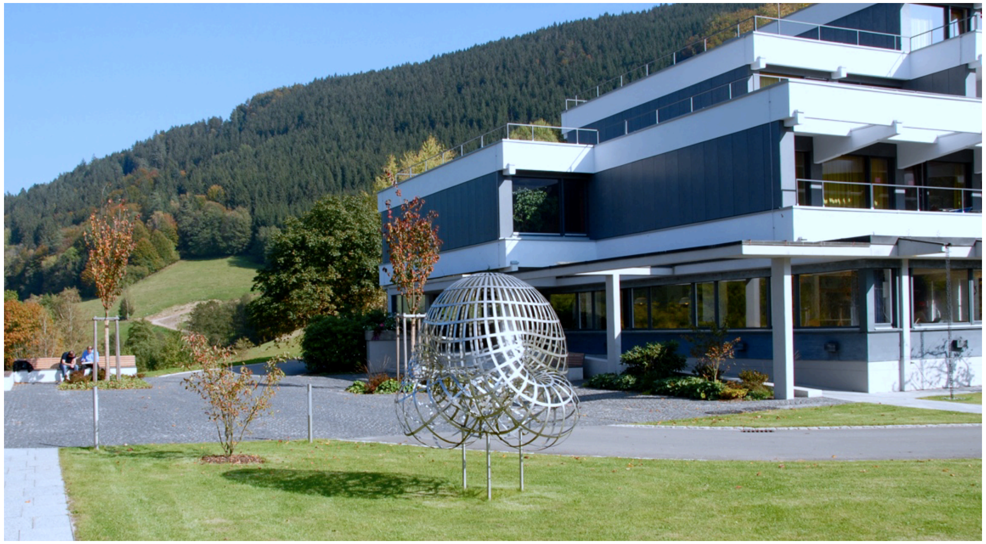
- Herausgeber der *Schnappschüsse moderner Mathematik*



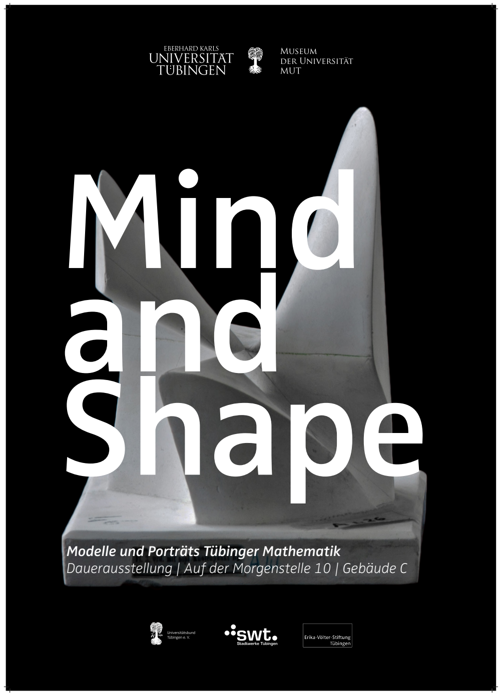
Ausstellungsguides

➢ Leitung: Sophia Jahns

Studentische Guides:

- Armin Fabian
- Anja Fetzer
- Penelope Gehring
- Kevin Tuan-Fi Nguyen
- Oliver Schön
- Yannic Senkel
- Markus Wolff

Vielen Dank!



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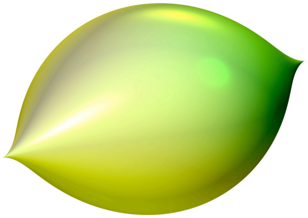

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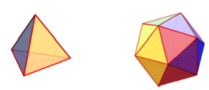

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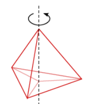

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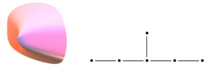

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