Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 9 — due 22.12. — T. Piecha

Exercise 1 (6 points)
Construct combinators **O** and **C** with the following properties:

(a) $\mathbf{O}XY =_w Y(XY)$ (3 points)

(b) $\mathbf{C}XYZ =_w XZY$ (3 points)

Do this by using Corollary 2.11 (combinatorial completeness) and Definition 2.9 (to resolve $[x]$).

Exercise 2 (2 points)
Consider again the weak equality $\mathbf{O}XY =_w Y(XY)$.

Prove: U is a fixed-point combinator $\iff$ U is a fixed point of **O**.

Remarks: A combinator U is a *fixed-point combinator* if $UX =_w X(UX)$.
To prove the direction "$\Longrightarrow$" you may have to assume extensionality.

Exercise 3 (8 points)
Prove the following:

(a) If X is a fixed point of **K**, then X is a fixed point of itself, i.e. $XX =_w X$. (2 points)

(b) If $\mathbf{K}X$ is a fixed point of itself, then X is a fixed point of **K**. (2 points)

(c) If $\mathbf{K}X$ is a fixed point of **K**, then X is a fixed point of **K**. (2 points)

(d) If $\mathbf{K}X =_w \mathbf{K}Y$, then $X =_w Y$. (2 points)

Exercise 4 (4 points)
Let M and N be λ-terms.

(a) Show: $M \triangleright_{\beta\eta} N \implies M_{\mathrm{CL}} \succ\!\!- N_{\mathrm{CL}}$. (2 points)

(b) Show that the converse of (a), i.e. $M_{\mathrm{CL}} \succ\!\!- N_{\mathrm{CL}} \implies M \triangleright_{\beta\eta} N$, does *not* hold. (2 points)