Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 6 — due 1.12. — T. Piecha

Exercise 1 (6 points)
Find a λ-term that represents the Ackermann–Péter function a, which is defined recursively as follows:

$$a(0,n) = n+1$$
$$a(m+1,0) = a(m,1)$$
$$a(m+1,n+1) = a(m,a(m+1,n))$$

Exercise 2 (8 points)
Show by finding derivations in $\lambda\beta$ that for all λ-terms P,Q,R we have:

(a) $\lambda\beta \vdash \mathbf{K}PQ = P$ (2 points)

(b) $\lambda\beta \vdash \mathbf{S}PQR = PR(QR)$ (3 points)

(c) $\lambda\beta \vdash \mathbf{\Upsilon}x = x(\mathbf{\Upsilon}x)$ (3 points)

Which of these equalities are also provable in $\lambda\beta_{\triangleright}$?

Exercise 3 (6 points)
Prove for all λ-terms M,N:

(a) If $M \triangleright_{1\beta} N$, then $\lambda\beta_{\triangleright} \vdash M = N$. (3 points)

(b) If $\lambda\beta_{\triangleright} \vdash M = N$, then $M \triangleright_{\beta} N$. (3 points)