

**Exercise 1** (3 points)

Prove that there is no CL-term  $X$  with  $XY =_w [x].Y$ . (In other words,  $[x].Y$  must be a metalinguistic operation.)

*Hint:* One can prove this by showing that  $[x].Y$  conflicts with substitution. That is, one has to find terms  $U, V$  such that for  $x \neq y$  we have  $([x].U)[V/y] \neq [x].(U[V/y])$ .

**Exercise 2** (11 points)

Consider the following  $\lambda$ -terms:

(i)  $\Omega := (\lambda x.xx)(\lambda x.xx)$

(ii)  $M := (\lambda xy.x)(\lambda xyz.xz(yz))$

(iii)  $N := (\lambda xy.yx)((\lambda x.x)x)$

- (a) Provide corresponding CL-terms  $\Omega_{CL}$ ,  $M_{CL}$  and  $N_{CL}$ . (6 points)
- (b) To which terms can  $\Omega_{CL}$ ,  $M_{CL}$  and  $N_{CL}$  be weakly reduced? (3 points)
- (c) To which terms can  $M$  and  $N$  be  $\beta$ -reduced? (2 points)

**Exercise 3** (6 points)

Find counterexamples to disprove the following statements:

- (a) If  $M \triangleright_{\beta} N$ , then  $M_{CL} \triangleright_w N_{CL}$ . (3 points)
- (b) If  $M_{CL} \triangleright_w N_{CL}$ , then  $M \triangleright_{\beta} N$ . (3 points)