EXAMPLE. The extension of a Peano-system with axioms

$$\mathrm{Apply}_k((x_1,\ldots,x_k)\colon e, a_1,\ldots,a_k) = e^{x_1\cdot\cdot x_k}_{a_1\cdot\cdot a_k},$$

$$\mathrm{PR}(a,(y,z)\colon b,n) = \mathrm{ifelse}(n = 0, a, \mathrm{Apply}_2((y,z)\colon b, \mathrm{pred}(n), \mathrm{PR}(a,(y,z)\colon b, \mathrm{pred}(n))))$$

allows representation of each primitive recursive function by a single term.

As to the logical axioms, the usual schemes of predicate logic may be adapted, but "bound variables" are also generated by other symbols than quantifiers. The equality axioms become:

$$\cdots \wedge \left(\forall_{q_{i1}} \cdot\cdot \forall_{q_{ir(i)}}\; a_{i\cdot\cdot q_{ij}\cdot\cdot}^{\cdot\cdot u_{ij}\cdot\cdot} = b_{i\cdot\cdot q_{ij}\cdot\cdot}^{\cdot\cdot v_{ij}\cdot\cdot}\right) \wedge \cdots \to \mathrm{op}\Big(\ldots, \underset{\text{if } r(i)>0}{[(\tilde{u}_i)\colon]}\, a_i, \ldots\Big) = \mathrm{op}\Big(\ldots, \underset{\text{if } r(i)>0}{[(\tilde{v}_i)\colon]}\, b_i, \ldots\Big)$$

[$\tilde{u}_i$ is $u_{i1},\ldots,u_{ir(i)}$ and $\tilde{v}_i$ is $v_{i1},\ldots,v_{ir(i)}$].

The main problem is the suitable definition of the semantics. An interpretation of "op" if $k > 0$ and if there is one i with $r(i) > 0$ by a functional:

$$M(\mathrm{op})\colon \prod_{i:1\cdot\cdot k} V_i \to M(\gamma), \qquad V_i = \begin{cases} M(\alpha_i) & \text{if } r(i) = 0, \\ \mathrm{Map}(\prod_{j:1\cdot\cdot r(i)} M(\beta_{ij}), M(\alpha_i)) & \text{if } r(i) > 0 \end{cases}$$

($M(\delta)$ is the range of sort δ and $\mathrm{Map}(X, Y) = \{f \mid f\colon X \to Y\}$) turns out to determine values to more functions of $V_i = \mathrm{Map}(\prod_{j:1\cdot\cdot r(i)} M(\beta_{ij}), M(\alpha_i))$ than essential to a model. A restriction of the argument ranges V_i is needed—a system of subsets to each $\mathrm{Map}(\prod_{j:1\cdot\cdot r(i)} M(\beta_{ij}), M(\alpha_i))$ is to be added as a new constituent of a model. The sets must contain each constant function and projection and must be closed by some operations similar to classes of (sub-)recursive functions. For constructing a model to a consistent formal theory, extension to a complete Henkin theory is still applicable.

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PETER SCHROEDER-HEISTER, *Judgements of higher levels in Martin-Löf's logical theory.*

Ordinary natural deduction can be extended by admitting rules as assumptions which may themselves be discharged by the application of rules of higher levels. This idea can be naturally carried over to Martin-Löf's logical and type-theoretical systems, where rules of higher levels are philosophically interpreted as hypothetical judgements of higher levels. Apart from the questions (1) of a standardized schema for introduction and elimination inferences and (2) of functional completeness of the intuitionistic operators, iterated hypothetical judgements play an independent role. This is so because they cannot in all cases be reduced to categorical judgements.

A sequent-style formalism for the logical part of Martin-Löf's system involving judgements of higher levels is presented. General rules of inference, which are only based on the explanations of the different forms of judgements, are distinguished from special rules of inference governing the logical constants. The question of whether certain inverses of the formation rules should be admitted as primitive rules of inference is discussed. Finally, it is argued that the elimination rules proposed for disjunction and existential quantification, which permit conclusions of the form "A prop", are both philosophically and technically more appropriate than the weaker versions given by Martin-Löf.

DASHARATH SINGH, *A remark on tautology from the computational point of view.*

In this paper we point out that the occurrence of "auxiliary" parts in a sentence *does not contribute anything to the formula* "2^n" for the number of rows in its truth table. Moreover, the idea of substituting an atomic constituent (in all its occurrences) in a tautological sentence by any other atomic sentence *only* without imparting any effect can be carried on to nontautological sentences also if the substitutes do not already occur in the sentence. Nonauxiliary independent constituents behave like atomic sentences in a tautology. Further, we conjecture that no tautology is possible with "nonauxiliary independent constituents" only.