Abstracts of the 9th International Congress of Logic, Methodology and Philosophy of Science, Uppsala, Sweden, 7-14 August 1991

An asymmetry between introduction and elimination inferences

Peter Schroeder-Heister, Universität Tübingen/SNS, Germany
e-mail: schroeder-heister@mailserv.zdv.uni-tuebingen.de

The symmetry between introduction (I) and elimination (E) inferences in natural deduction or between right-introduction ($\vdash *$) and left-introduction ($* \vdash$) inferences in sequent calculi is normally considered a central feature of Gentzen systems. Both philosophical and mathematical investigations have tried to point out a uniform relationship or duality between I or $\vdash *$ and E or $* \vdash$ inferences, always in connection with normalization and cut elimination. This is not being questioned here. However, a certain characteristic asymmetry will be pointed out that has to do with the notion of discharging assumptions. Let $X[A]$ express that the formula A occurs at a certain place in a list X of formulae, and let $X[Y]$ denote the result of replacing this occurrence of A in X by the list Y. Then, for example, the schema of implication introduction in sequent-style natural deduction should be formulated as

$$\frac{X, A \vdash B}{X \vdash A \to B}$$

and *not* as

$$\frac{X[A] \vdash B}{X \vdash A \to B},$$

whereas the schema of disjunction elimination should be formulated as

$$\frac{X \vdash A \vee B \quad Y[A] \vdash C \quad Y[B] \vdash C}{Y[X] \vdash C}$$

and *not* as

$$\frac{X \vdash A \vee B \quad Y, A \vdash C \quad Y, B \vdash C}{Y, X \vdash C}.$$

Similarly, in the multiple-conclusion sequent calculus the schema of implication introduction on the right should be formulated as

$$\frac{X, A \vdash B, Y}{X \vdash A \to B, Y}$$

and *not* with A or B bracketed, whereas the schema of disjunction introduction on the left should be formulated as

$$\frac{Y[A] \vdash C \quad Y[B] \vdash C}{Y[A \vee B] \vdash C}$$

rather than

$$\frac{Y, A \vdash C \quad Y, B \vdash C}{Y, A \vee B \vdash C},$$

and analogously for other connectives.
These claims are based on the following principles:

1. Rules for logical constants should be uniform and independent of the structural principles assumed.

2. Normalization (for natural deduction systems) and cut elimination (for sequent calculi) should hold.

3. In the multiple-conclusion case symmetry should not be forced by providing a mechanism that permits to move formulae between the two sides of a sequent.