(1) *The symbol* "|" *is interpreted as the honest divisibility relation*:

$$T \models \forall xy(x \mid y \leftrightarrow \exists z(y = x \cdot y)).$$

(2) *The following* ***divisibility property*** *holds in* T:

(DP) $$\forall xy(|x| \leq |y| \rightarrow y \mid x).$$

If T *admits quantifier-elimination in* $\mathscr{L}$, *then* $T = RCVR$.

The approximation technique of [2] and the omitting types theorem are used in the proof. We also have the following characterization of models of (DP):

PROPOSITION. *Let* A *be a linearly ordered commutative domain. The following are equivalent*:

(1) $A \models DP$.

(2) *Every subset of* A *defined by a quantifier-free* $\mathscr{L}$-*formula is a finite union of convex subsets of* A.

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KOSTA DOŠEN, *Weak propositional logics (sequent-systems and groupoid models for a family of propositional logics weaker than Heyting's)*.

The purpose of this paper is to connect the proof theory and the model theory of a family of propositional logics weaker than Heyting's. This family includes systems analogous to the Lambek calculus of syntactic categories, systems of relevant logic, systems related to BCK algebras, and finally, Johansson's and Heyting's logic. First, sequent-systems are given for these logics, and cut-elimination results are proved. In these sequent-systems the rules for the logical constants are never changed: all changes are made in the structural rules. Next, Hilbert-style formulations of these logics are given, and algebraic completeness results are demonstrated with respect to residuated lattice-ordered groupoids. Finally, model structures related to relevant model structures (of Urquhart, Fine, Routley-Meyer, and Maksimova) are given for our logics. These model structures are based on groupoids, analogous both to the algebraic models and to the pattern exhibited by the sequent-systems. This paper lays the ground for a kind of correspondence theory for implicational axioms, analogous to the correspondence theory of normal modal logics.

KOSTA DOŠEN and PETER SCHROEDER-HEISTER, *A general interpolation and definability theorem.*

Let L be a set of formulae of an arbitrary language. A *consequence relation* $\vdash$ over L is a subset of $2^L \times L$ which satisfies $\{A\} \vdash A$, $X \vdash A \Rightarrow X \cup Y \vdash A$, and $((\forall A \in Y)X \vdash A \,\&\, Y \cup V \vdash B) \Rightarrow X \cup V \vdash B$. It is called *compact* if $X \vdash A \Rightarrow (\exists Y \text{ finite} \subseteq X)Y \vdash A$. Let $\vdash_1$ and $\vdash_2$ be consequence relations over $L_1 \subseteq L$ and $L_2 \subseteq L$, respectively, which agree on $L_0 = L_1 \cap L_2$ in the sense that $(\forall X \subseteq L_0)(\forall A \in L_0)$ $(X \vdash_1 A \Leftrightarrow X \vdash_2 A)$. Let $\vdash_c$ be the minimal consequence relation over L which is an extension of $\vdash_1$ and $\vdash_2$. If $\vdash_1$ and $\vdash_2$ are compact, let $\vdash_{cc}$ be the minimal compact consequence relation over L which is an extension of $\vdash_1$ and $\vdash_2$.

INTERPOLATION THEOREM.

$$(\forall X \subseteq L_1)(\forall A \in L_2)(X \vdash_c A \Rightarrow (\exists Y \subseteq L_0)(\forall B \in Y)(X \vdash_1 B \,\&\, Y \vdash_2 A)),$$

$$(\forall X \subseteq L_2)(\forall A \in L_1)(X \vdash_c A \Rightarrow (\exists Y \subseteq L_0)(\forall B \in Y)(X \vdash_2 B \,\&\, Y \vdash_1 A)).$$

In the compact case, this holds for $\vdash_{cc}$, *and* Y *can be chosen finite.*

Let now L_1 differ from L_2 only in having a constant α, of an arbitrary syntactical category, where L_2 has a constant α' of the same syntactical category as α, and vice versa. Let $\vdash_1$ and $\vdash_2$ be consequence relations over L_1 and L_2, respectively, which upon uniform substitution of α for α', and vice versa, become identical.

DEFINABILITY THEOREM. *If* $A(\alpha) \in L_1$, *then*

$$\{A(\alpha)\} \vdash_c A(\alpha') \quad \textit{iff} \quad (\exists Y \subseteq L_0)(\forall B \in Y)(\{A(\alpha)\} \vdash_1 B \,\&\, Y \vdash_1 A(\alpha)).$$

In the compact case, this holds for $\vdash_{cc}$, *and* Y *can be chosen finite.*

Unlike Craig's and Beth's theorems, these results only involve the structural aspects of logic as captured by the notion of a consequence relation. In particular, the definability theorem is not restricted to the category of nonlogical constants. Beth's theorem can be shown to be a consequence of our general definability theorem.

ANTONINO DRAGO, *The relevance of constructive mathematics to physical theories.*

By both philosophical and mathematical analysis one can show that: 1) dimensional theory in physics has no constructive version unless the dimensions are restricted to a suitable subset of real constructive numbers, for example to rational numbers, just as the physicists do in their practice of calculations; 2) only few thermodynamics formulations have constructive versions: the Carnot-Kelvin-Clausius and the Brønsted ones; and 3) in classical mechanics the Newtonian formulation has no constructive version; on the contrary, the L. Carnot (1803), Eisenbud (1958), and Hood (1969) ones have. Furthermore, constructive versions of physical theories lead us to see them no more as deductive systems of our hypostatized ideas but as operational (or heuristic) instruments for solving problems, just as L. Carnot's formulations of mathematical and physical theories are, as well as the formulation of thermodynamics by S. Carnot.

P. ECSEDI-TÓTH, *First order properties preserved under lattice-like operators.*

1. Finite meet of structures has been introduced in [1], where the following preservation theorem was established: An equality-free first order theory T is preserved under finite meets iff T has a set of equality-free universal Horn axioms. The meet of structures gives rise to a lattice-ordering and so to such concepts as "filter of structures". In [2] we have characterized by syntactical means those equality-free first-order theories the models of which form a filter of structures. It was also demonstrated that neither these results nor the method of proofs can be generalized directly to theories with equality.

2. Our aims in the present contribution are twofold:

(a) To give the analogue of the preservation theorem mentioned above for arbitrary first order theories (maybe with equality) and to investigate the similar problem for infinite meets.

(b) To give a (partial) answer to the natural question of how far the analogy between lattice theory and the theory of structures can be extended. In particular, we define join of structures and derive preservation theorems under joins for theories with and without equality.

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M. EYTAN, *Universal grammar, intensional logic and* **Set**[I]

In his ***Universal grammar*** Montague proposes a "universal syntax and semantics" for both natural and artificial languages that has been the object of a great number of papers by formal linguists. However much one may contend some of Montague's assertions, it may be useful to present a synthetic view of his construction, with the hindsight given by what we know now.

First Montague defines the language L_0 of intensional logic, for which he then proceeds to give a semantics. We show that this is essentially just the logic of the topos **Set**[I] (together with a modal operator, i.e. together with a topology) that has been formalized since, in a more general setting, by Scott and Fourman.

Then Montague defines the language L_1 in what is essentially a categorial grammar, the syntactical equivalent, as has been shown by Lambek, of a cartesian closed category, **C**.

Finally Montague gives a semantics for the language L_1 (assumed to give a nice description of a fragment of English) indirectly by defining a "translation" that induces a semantics for L_1 as soon as we have one for L_0. Thus we get a "realization" of the cartesian closed category **C**: this is a functor, some of whose properties we sketch.

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