tems studied earlier by the present author [1]. The semiformal system S_2^* of rules of deduction consists of thirteen rules of deduction RD2.1–RD2.13.

In this paper, we claim to establish the dependence of each rule RD2.*m*, where $m = 6, 7, 13$, and the independence of each of the rules RD2.*n*, where $n = 3-5$ or $8-12$. Each rule RD2.*m*, where $m = 6, 7, 13$, is dependent in S_2^* in the following sense: For every two closed formulas A and B in the language $Я_2$, if B is deducible from A in S_2^*, then B is deducible from A in $S_2^*\backslash$RD2.*m*. The independence of the rule RD2.*n* (where $n = 3, 5, 8-12$) in S_2^* may be proved by finding two closed formulas A and B of the language $Я_2$ such that B is deducible from A in S_2^* but not in $S_2^*\backslash$RD2.*n*. The independence of the rule RD2.4 in S_2^* may be proved by finding some property P such that each rule from the system $S_2^*\backslash$RD2.4 preserves P but RD2.4 does not always preserve it.

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THEODORE HAILPERIN, *The development of probability logic. Part I.*
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The subject was first envisioned by Leibniz: "... I maintain that the study of the degrees of probability would be very valuable and is still lacking, and that this is a serious shortcoming in our treatises of logic. For when we cannot absolutely settle a question one could still establish the degree of likelihood on the evidence, and so one could judge rationally which side is the most plausible."

In making a rational judgement from an argument two aspects are involved: (i) the probability of the chosen premises, and (ii) the manner in which the probability of the premises devolves through the argument to provide a probability for the conclusion. Our study is concerned only with aspect (ii), for we consider determination of the probability of premises, like that of the truth of premises in ordinary logic, to be a nonlogical matter.

We appeal to only the simplest formal properties of probability, those which are common to essentially all theories of probability whatever their nature. Conditional probability is also included. The conditional probability of C given A can be interpreted as the probability of (A and A-implies-C), using a renormalized probability measure whose unit is the probability of A.

In this Part I we analyse the contributions to probability logic which occur in the works of Jacob Bernoulli, Lambert, De Morgan, Boole, Peirce, MacColl, and Keynes.

Added in Proof. The substance of the talk is included in the author's paper, *The development of probability logic from Leibniz to MacColl*, ***History and Philosophy of Logic***, vol. 9 (1988), pp. 131–191.

LARS HALLNÄS AND PETER SCHROEDER-HEISTER, *Logic programming with higher-level rules.*
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We propose a proof-theoretic view of logic programs, according to which a clause of a definite Horn clause program is a rule $A_1, \ldots, A_n \Rightarrow B$ rather than a logically complex formula $(A_1 \mathbin{\&} \cdots \mathbin{\&} A_n) \supset B$. A program can then be viewed as defining a calculus for the derivation of atomic formulae. A query asks for which joint substitutions certain atomic formulae are derivable in that calculus.

As a generalization of Horn clauses we consider rules of higher levels in the sense of [1], which are obtained by permitting the iteration of the rule arrow "$\Rightarrow$" to the left. A generalized logic program is considered a finite set of higher-level rules. It can again be viewed as defining a calculus, which is now a natural deduction or sequent type with rules allowed as assumptions. Queries concern the derivability in that calculus. An extended version of SLD-resolution can be defined which is shown to be sound and complete.

In an additional extension, program rules are considered introduction rules for the predicates with which their heads start. Correspondingly, a general elimination schema is added to the formal system defined by the program. In the resulting formalism an "intrinsic" notion of negation is available, the rule of "negation as finite failure" being derivable within the system.

Further generalizations concern the explicit use of quantifiers in higher-level rules as in [2], yielding an evaluation procedure in which substitution of variables is blocked in certain cases.

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JAIME IHODA AND SAHARON SHELAH, *Martin's axioms, measurability and equiconsistency results.*
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We deal with the consistency strength of ZFC + variants of MA + suitable sets of reals are measurable (and/or Baire, and/or Ramsey). We improve the theorem of Harrington and Shelah repairing the asymmetry between measure and category, obtaining also the same result for Ramsey. We then prove parallel theorems with weaker versions of Martin's axiom (MA(σ-centered), (MA(σ-linked)), MA($\aleph_0$), MA(K)), getting Mahlo, inaccessible and weakly compact cardinals respectively. We prove that if there exists $r \in \mathbf{R}$ such that $\omega_1^{L[r]} = \omega_1$ and MA holds then there exists a Δ_3^1-selective filter on ω, and from the consistency of ZFC we build a model for ZFC + MA(I) + every Δ_3^1-set of reals is Lebesgue measurable, has the property of Baire and is Ramsey.

C. IMPENS, *Microcontinuity for nonstandard polynomials.*
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For nonstandard polynomials, the monadic concept of microcontinuity is supplemented with the stronger notion of absolute microcontinuity. Relations with the coefficients are established, and it is shown that, whereas microcontinuity may be confined to isolated monads, absolute microcontinuity invariably propagates itself over noninfinitesimal distances. In particular, infinite partial sums of standard power series are examined, revealing a strong intrinsic difference between the notions of convergence (absolute or not) and of microcontinuity (absolute or not).

IDA JOKISZ, *On interpretations of many-sorted structures.*
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The main notion of this work is interpretations for many-sorted structures. The notion of interpretation in our sense, but for structures with one universe, was introduced in [1]. For one-sorted structures it was proved in [2] that any finite composition of elementary interpretations may be represented by a composition of the form $QCTE$, where Q, C, T, E are interpretations defined as defining some new relation or forgetting a relation, treating n-tuples of elements of the old universe as elements of the new universe, restricting the universe to a definable subset, or introducing a new definable equality. For many-sorted structures the problem of choosing elementary interpretations to be such generators is more complicated.

The aim of this paper is to show how the choice of generators determines the normal form of the composition of elementary interpretations for many-sorted structures, and some other consequences of the above choice.

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