A brief introduction to practical SAT solving

What can I expect from SAT today?

Daniel Le Berre

Joint work with Anne Parrain, Pascal Rapicault, Olivier Roussel, Laurent Simon, and others

CRIL-CNRS UMR 8188 - Université d'Artois

SAT Workshop, 24–25 February 2011,Tübingen

Context : SAT receives much attention since a decade

Why are we all here today ?

- Most companies doing software or hardware verification are now using SAT solvers.
- SAT technology indirectly reaches our everyday life :
  - Intel core I7 processor designed with the help of SAT solvers [Kaivola et al, CAV 2009]
  - Windows 7 device drivers verified using SAT related technology (Z3, SMT solver) [De Moura and Bjorner, IJCAR 2010]
  - The Eclipse open platform uses SAT technology for solving dependencies between components [Le Berre and Rapicault, IWOCE 2009]
- Many SAT solvers are available from academia or the industry.
- SAT solvers can be used as a black box with a simple input/ouput language (DIMACS).
- The consequence of a new kind of SAT solver designed in 2001 (Chaff)

Outline

SAT in theory

SAT in practice : the international SAT competition

SAT solvers for solving “real” problems : the CDCL architecture

Nearby SAT : PBO, MaxSAT, MUS

Using boolean constraints to solve Software Dependency problems

Conclusion

Depending on time : a quick overview of Sat4j

The SATisfiability problem

Definition

Input : A set of clauses built from a propositional language with n variables.

Output : Is there an assignment of the n variables that satisfies all those clauses ?

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Example

$$C_1 = \{\neg a \vee b, \neg b \vee c\} = (\neg a \vee b) \wedge (\neg b \vee c) = (a' + b).(b' + c)$$

$$C_2 = C_1 \cup \{a, \neg c\} = C_1 \wedge a \wedge \neg c$$

For C_1, the answer is yes, for C_2 the answer is no

$$C_1 \models \neg(a \wedge \neg c) = \neg a \vee c$$

Where are clauses coming from ?

My own background : Automated Reasoning/Artificial Intelligence

Suppose :

a *I like beer*

b *I should visit Germany*

c *I should drink German beer*

Then C_1 could represent the beliefs :

- $a \implies b$: *If I like beer, then I should visit Germany.*
- $b \implies c$: *If I visit Germany, then I should drink german beer.*

What happens if I like beer and I do not drink german beer $(a \wedge \neg c)$? This is inconsistent with my beliefs.
From C_1 I can deduce $a \implies c$: *If I like beer, then I should drink german beer.*

Typical application for SAT : reachability analysis

Definition

Given an initial state s_0, a state transition relation ST, a goal state g and a bound k.

Is there a way to reach g from s_0 using ST within k steps ?

Is there a succession of states $s_0, s_1, s_2, ..., s_k = g$ such that $\forall\ 0 \leq i < k\ (s_{i-1}, s_i) \in ST$?

- The problems are generated for increasing k.
- For small k, the problems are usually UNSATISFIABLE
- For larger k, the problems can be either SAT or UNSAT.
- Complete SAT solvers are needed !

1992 - Planning As Satisfiability

H.A. Kautz and B. Selman, Planning as satisfiability, in proceedings of the 10th European Conference on Artificial Intelligence (ECAI'92), pp. 359–363, 1992

$$PAS(S, I, T, G, k) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1}) \wedge \bigvee_{i=0}^{k} G(s_i)$$

where :

S the set of possible states s_i

I the initial state

T transitions between states

G goal state

k bound

If the formula is satisfiable, then there is a plan of length k.

1997 - Software Model Analysis

Daniel Jackson. An Intermediate Design Language and its Analysis Proc. ACM SIGOFT Conf. Foundations of Software Engineering, Orlando, FL, November 1998

$$SMA(S, op, p) = \exists s, s' \in S \; op(s, s') \wedge p(s) \wedge \neg p(s')$$

where :

S	the set of possible states
op	an operation
p	an invariant

If the formula is satisfiable, then there is an execution of the operation that breaks the invariant.

1999 - Bounded Model Checking

Armin Biere, Alessandro Cimatti, Edmund M. Clarke, and Yunshan Zhu. Symbolic model checking without BDDs. In proc. of TACAS, LNCS volume 1579, pages 193–207, 1999.

$$BMC(S, I, T, p, k) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1}) \wedge \bigvee_{i=0}^{k} \neg p(s_i)$$

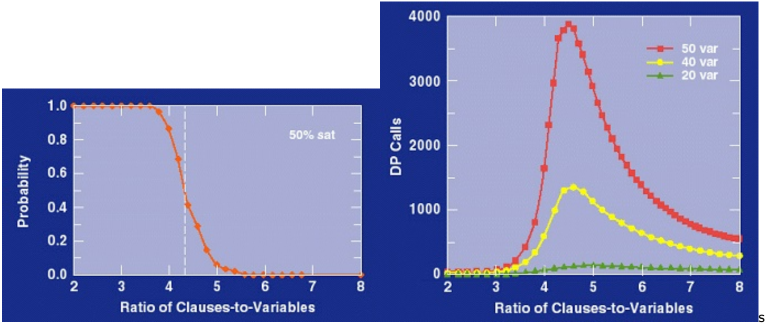
where :

S	the set of possible states s_i
I	the initial state
T	transitions between states
p	is an invariant property
k	a bound

If the formula is satisfiable, then there is a counter-example reachable in k steps.

SAT is important in theory ...

- Canonical NP-Complete problem (Cook, 1971)
- Threshold phenomenon on randomly generated k-SAT instances (Mitchell,Selman,Levesque, 1992)



source : http ://www.isi.edu/ szekely/antsebook/ebook/modeling-tools-and-techniques.htm

... but theory does not always meet practice

- Cannot translate a CNF into 3-CNF and use ratio $\frac{\#clauses}{\#variables}$ to know in advance if a SAT instance is difficult to solve or not.
- Adding more variables can speed up solving time (despite theoretical lower and upper bounds complexity depending on the number of variables n, e.g. 1.473^n).
- Adding more clauses (redondent ones) may also speed up the solving time.
- Solving real problems with SAT solvers requires specific expertise.
- How to find out which approaches work well in practice ?

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- ... give it a try ! :)

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- ... give it a try ! :)
- ... the international SAT competition or SAT Race

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DP 60 and DLL62 : the very first SAT competition !

In the present paper, a uniform proof procedure for quantification theory is given which is feasible for use with some rather complicated formulas and which does not ordinarily lead to exponentiation. The superiority of the present procedure over those previously available is indicated in part by the fact that a formula on which Gilmore's routine for the IBM 704 causes the machine to compute for 21 minutes without obtaining a result was worked successfully by hand computation using the present method in 30 minutes [Davis and Putnam, 1960].

The well-formed formula (...) which was beyond the scope of Gilmore's program was proved in under two minutes with the present program [Davis et al., 1962]

A brief history of SAT competitive events

Since the beginning the SAT community has been keen to run competitive events

- 1992 : Paderborn
- 1993 : The second DIMACS challenge [standard input format]
  Johnson, D. S., & Trick, M. A. (Eds.). (1996). Cliques, Coloring and Satisfiability : Second DIMACS Implementation Challenge, Vol. 26 of DIMACS Series in Discrete Mathematics and Theoretical Computer Science. AMS.
- 1996 : Beijing
- 1999 : SATLIB
- 2000 : SAT-Ex
- Since 2002 : yearly competition (or Race)

DIMACS common input format

```
p cnf 3 4
-1 2 0
-2 3 0
1 0
-3 0
```

- Not really fun !
- Designed for SAT solver designers, not end users !
  - Need to know in advance the number of variables and clauses (p cnf line)
  - Use only integer to denote variables

SAT technology cannot be used directly by end users (they need a more user friendly layer/API)

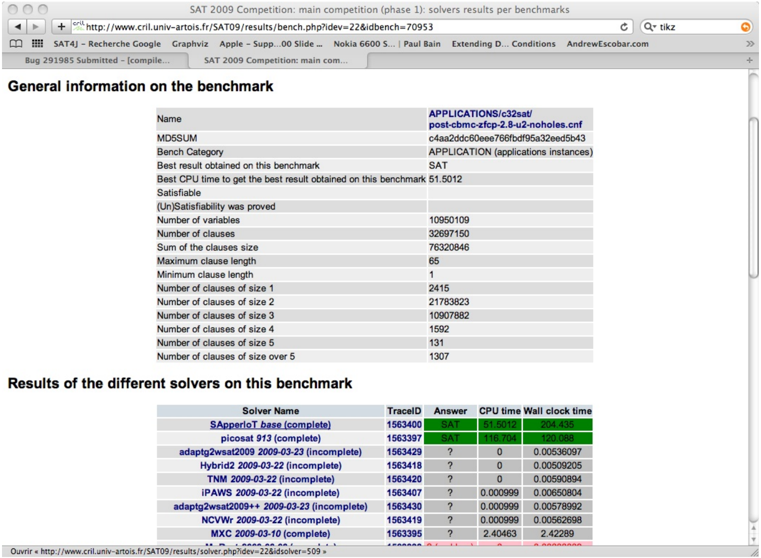
To keep in mind before looking at the results of a competition

Results depends on :

- available solvers and benchmarks.
- hardware (amount of RAM, size of L2 cache, etc).
- operating system (linux)
- competition rules (timeout, source code)
- the amount of computing resources available
- ...

We do not claim to have statistically meaningful results !

Impressive results of the SAT 2009 competition


SAT 2009 Competition: main competition (phase 1): solvers results per benchmarks
http://www.cril.univ-artois.fr/SAT09/results/bench.php?idev=22&idbench=70953
tikz
SAT4J – Recherche Google
Graphviz
Apple – Supp...00 Slide ...
Nokia 6600 S... | Paul Bain
Extending D... Conditions
AndrewEscobar.com
Bug 291985 Submitted – [compile...
SAT 2009 Competition: main com...

General information on the benchmark

Name	APPLICATIONS/c32sat/ post-cbmc-zfcp-2.8-u2-noholes.cnf
MD5SUM	c4aa2ddc60eee766fbdf95a32eed5b43
Bench Category	APPLICATION (applications instances)
Best result obtained on this benchmark	SAT
Best CPU time to get the best result obtained on this benchmark	51.5012
Satisfiable	
(Un)Satisfiability was proved	
Number of variables	10950109
Number of clauses	32697150
Sum of the clauses size	76320846
Maximum clause length	65
Minimum clause length	1
Number of clauses of size 1	2415
Number of clauses of size 2	21783823
Number of clauses of size 3	10907882
Number of clauses of size 4	1592
Number of clauses of size 5	131
Number of clauses of size over 5	1307

Results of the different solvers on this benchmark

Solver Name	TraceID	Answer	CPU time	Wall clock time
SApperloT *base* (complete)	1563400	SAT	51.5012	204.435
picosat *913* (complete)	1563397	SAT	116.704	120.088
adaptg2wsat2009 *2009-03-23* (incomplete)	1563429	?	0	0.00536097
Hybrid2 *2009-03-22* (incomplete)	1563418	?	0	0.00509205
TNM *2009-03-22* (incomplete)	1563420	?	0	0.00590894
iPAWS *2009-03-22* (incomplete)	1563407	?	0.000999	0.00650804
adaptg2wsat2009++ *2009-03-23* (incomplete)	1563430	?	0.000999	0.00578992
NCVWr *2009-03-22* (incomplete)	1563419	?	0.000999	0.00562698
MXC *2009-03-10* (complete)	1563395	?	2.40463	2.42289

Ouvrir « http://www.cril.univ-artois.fr/SAT09/results/solver.php?idev=22&idsolver=509 »

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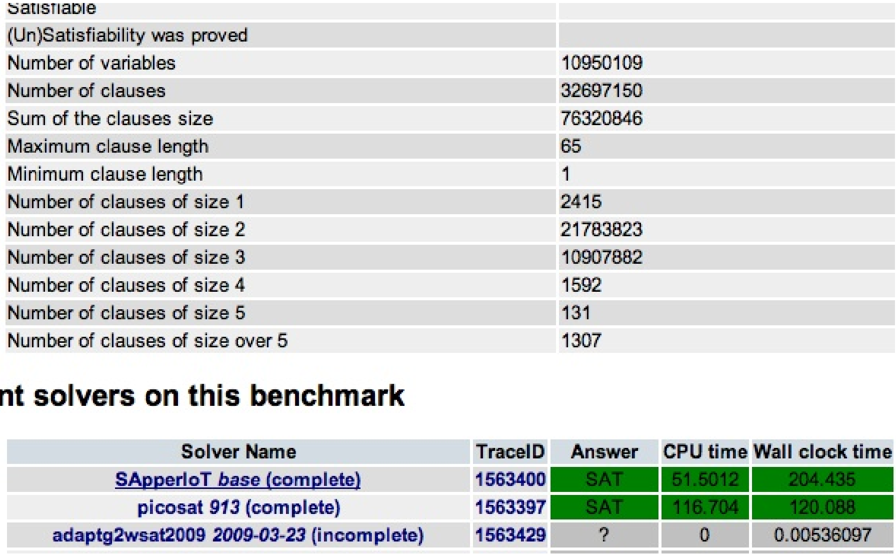

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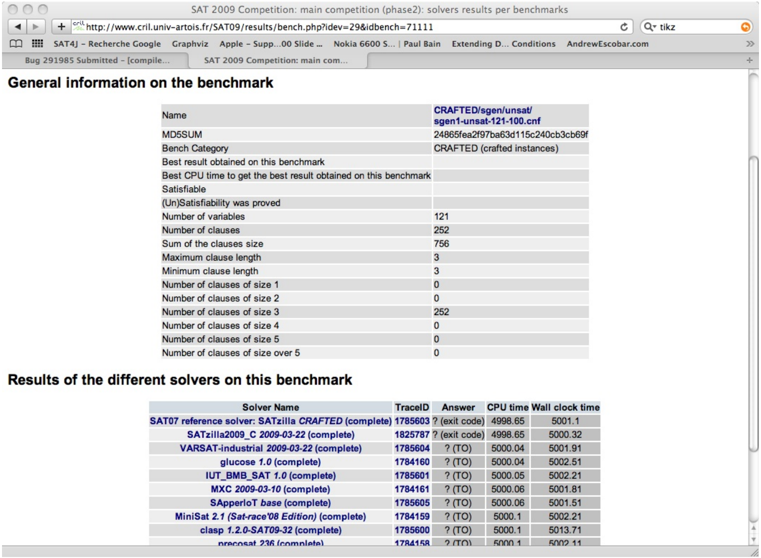
SAT 2009 Competition: main competition (phase2): solvers results per benchmarks
http://www.cril.univ-artois.fr/SAT09/results/bench.php?idev=29&idbench=71111
tikz
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Nokia 6600 S... | Paul Bain
Extending D... Conditions
AndrewEscobar.com
Bug 291985 Submitted – [compile...
SAT 2009 Competition: main com...

General information on the benchmark

Name	CRAFTED/sgen/unsat/ sgen1-unsat-121-100.cnf
MD5SUM	24865fea2f97ba63d115c240cb3cb69f
Bench Category	CRAFTED (crafted instances)
Best result obtained on this benchmark	
Best CPU time to get the best result obtained on this benchmark	
Satisfiable	
(Un)Satisfiability was proved	
Number of variables	121
Number of clauses	252
Sum of the clauses size	756
Maximum clause length	3
Minimum clause length	3
Number of clauses of size 1	0
Number of clauses of size 2	0
Number of clauses of size 3	252
Number of clauses of size 4	0
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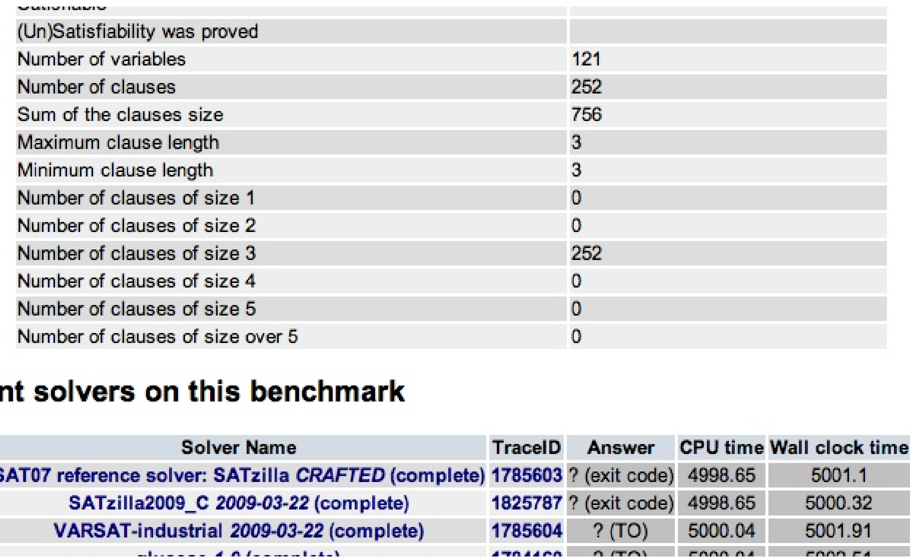
Solver Name	TraceID	Answer	CPU time	Wall clock time
SAT07 reference solver: SATzilla *CRAFTED* (complete)	1785603	? (exit code)	4998.65	5001.1
SATzilla2009_C *2009-03-22* (complete)	1825787	? (exit code)	4998.65	5000.32
VARSAT-industrial *2009-03-22* (complete)	1785604	? (TO)	5000.04	5001.91
glucose *1.0* (complete)	1784160	? (TO)	5000.04	5002.51
IUT_BMB_SAT *1.0* (complete)	1785601	? (TO)	5000.05	5002.21
MXC *2009-03-10* (complete)	1784161	? (TO)	5000.06	5001.81
SApperloT *base* (complete)	1785605	? (TO)	5000.06	5001.51
MiniSat *2.1 (Sat-race'08 Edition)* (complete)	1784159	? (TO)	5000.1	5002.21
clasp *1.2.0-SAT09-32* (complete)	1785600	? (TO)	5000.1	5013.71
precosat *236* (complete)	1784158	? (TO)	5000.1	5002.11

Impressive results of the SAT 2009 competition



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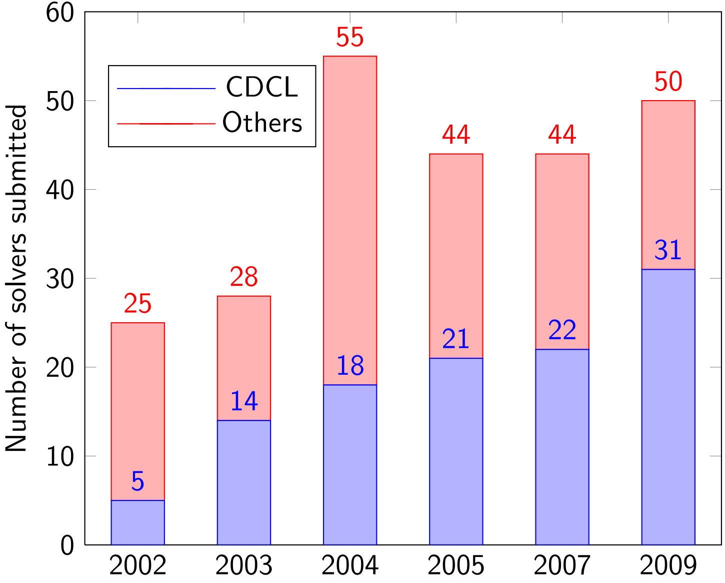
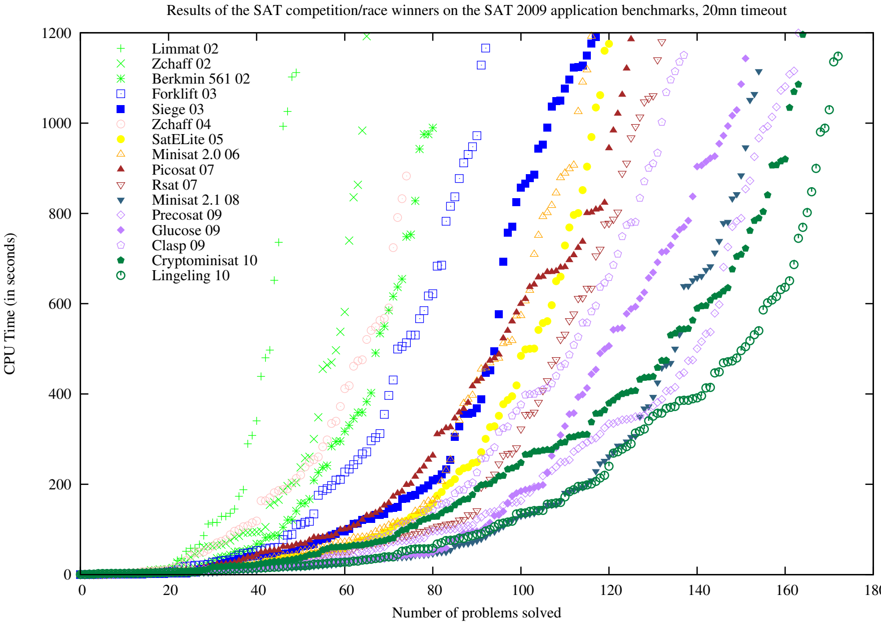
Huge success in the community !

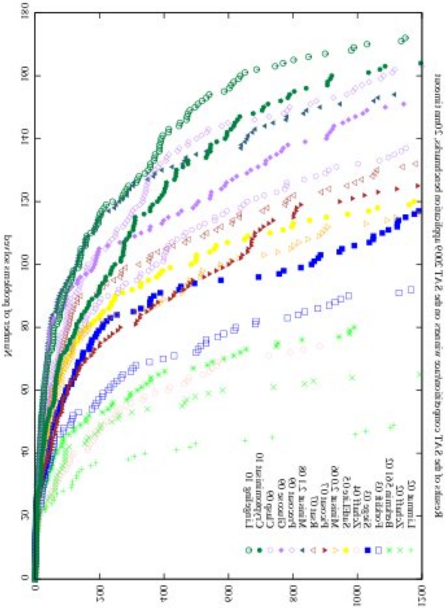


Evolution of SAT competition winners 2002-2010



Understanding “Cactus Plots”

Same data, “probabilistic” view



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Which solver to choose for solving “real” SAT problems

1990s Local Search
Fast boolean enumeration, built-in randomization
Main application : Planning as Satisfiability

2000s Conflict Driven Clause Learning
Fast Conflict Analysis, Cheap clause learning penalty, adaptive heuristics
Main application : Bounded Model Checking

2010s ? ? ? ? ? ? ?
My guess : moving to optimization problems, not simply decision problems

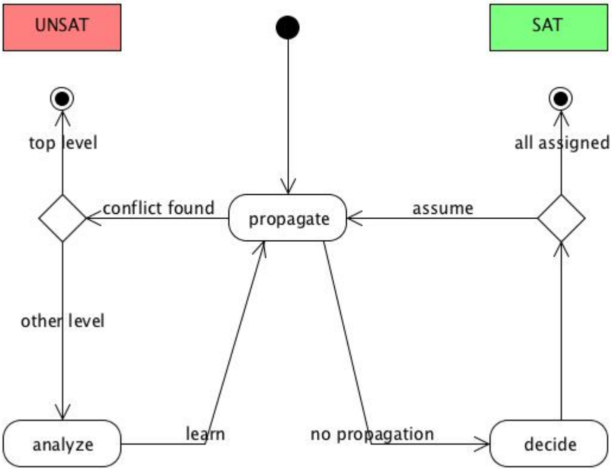
The breakthrough : Chaff

Chaff : Engineering an Efficient SAT Solver by M. Moskewicz, C. Madigan, Y. Zhao, L. Zhang, S. Malik, 39th Design Automation Conference (DAC 2001), Las Vegas, June 2001.

- 2 order of magnitude speedup on unsat instances compared to existing approaches on BMC (Velev) benchmarks.
- Immediate speedup for SAT based tools : BlackBox "Supercharged with Chaff"
- Based on careful analysis of GRASP internals [Marques-Silva and Sakallah, 1996]
- 3 key features :
  - New lazy data structure : Watched literals
  - New adaptative heuristic : Variable State Independent Decaying Sum
  - New conflict analysis approach : First UIP
- Taking into account randomization

The Conflict Driven Clause Learning approach

CDCL is not heuristic binary search !



Another way to represent how it works

Go from 0000000.... to 11111111..... such that

- Proceed from left to right
- Decisions are represented by 0
- Propagations are represented by 1
- When a conflict occurs, backtrack, i.e. go back to a 0 and change it by a 1

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01

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011

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0111

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01110110111101 conflict

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... and so on until either...

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11110111101111101111011 sat

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11111111111111111111111 unsat

Chaff : a highly coupled set of features

- Learning does not degrade solver performance thanks to the watched literals
- The VSIDS heuristics does not need a complete picture of the reduced formula, i.e. is compatible with the lazy data structure.
- VSIDS take advantage of the conflict analysis to spot important literals.
- VSIDS provides different orders of literals at each restart
- VSIDS adapt itself to the instance !

Since then ...

- Minisat simplified architecture, heuristic scheme, preprocessor and conflict minimization strategy.
- Specific data structures for binary and ternary clauses (Siege ?)
- Phase caching (RSAT) and Rapid restarts (many authors and solvers)
- Aggressive learned clauses deletion strategy ? (Glucose)
- Dynamic restarts (many authors)
- ...

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- ...

All those reasons are correct.

But there is a more fundamental reason too ...

CDCL has a better proof system than DPLL !

Proof theory strikes back !

- ... thanks to many others before ...
- Bonet, M. L., & Galesi, N. (2001). Optimality of size-width tradeoffs for resolution. Computational Complexity, 10(4), 261-276.
- Beame, P., Kautz, H., and Sabharwal, A. Towards understanding and harnessing the potential of clause learning. JAIR 22 (2004), 319-351.
- Van Gelder, A. Pool resolution and its relation to regular resolution and dpll with clause learning. In LPAR'05 (2005), pp. 580-594.
- Hertel, P., Bacchus, F., Pitassi, T., and Van Gelder, A. Clause learning can effectively p-simulate general propositional resolution. In Proc. of AAAI-08 (2008), pp. 283-290.
- Knot Pipatsrisawat, Adnan Darwiche : On the Power of Clause-Learning SAT Solvers with Restarts. CP 2009 : 654-668

CDCL has a better proof system than DPLL !

Proof theory strikes back !

Definition

p-simulation Proof system S p-simulates proof system T , if, for every unsatisfiable formula φ, the shortest refutation proof of φ in S is at most polynomially longer than the shortest refutation proof of φ in T.

Theorem 1 [Pipatsrisawat, Darwiche 09]. CLR with any asserting learning scheme p-simulates general resolution.

Outline

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Nearby SAT : PBO, MaxSAT, MUS

- MaxSat
- Pseudo-Boolean Optimization
- MUS

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Conclusion

Depending on time : a quick overview of Sat4j

Extending SAT 1 : ~~MaxSat~~ MinUnsat

- Associate to each constraint (clause) a weight (penalty) w_i taken into account if the constraint is violated : Soft constraints ϕ.
- Special weight (∞) for constraints that cannot be violated : hard constraints α
- Find a model I of α that minimizes $weight(I, \phi)$ such that :
  - $weight(I, (c_i, w_i)) = 0$ if I satisfies c_i, else w_i.
  - $weight(I, \phi) = \sum_{wc \in \phi} weight(I, wc)$

Weight	∞	denomination
∞	yes	Sat
k	no	MaxSat
k	yes	Partial MaxSat
$\mathbb{N}$	no	Weighted MaxSat
$\mathbb{N}$	yes	Weighted Partial MaxSat

Extending SAT 2 : Pseudo-Boolean problems

Linear Pseudo-Boolean constraint

$$-3x_1 + 4x_2 - 7x_3 + x_4 \leq -5$$

- variables x_i take their value in $\{0,1\}$
- $\overline{x_1} = 1 - x_1$
- coefficients and degree are integral constants

Pseudo-Boolean decision problem : NP-complete

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \\ (c) & x_1 + \overline{x_2} + x_5 \geq 1 \end{cases}$$

Plus an objective function : Optimization problem, NP-hard

$$min : 4x_2 + 2x_3 + x_5$$

Solving Pseudo Boolean Optimization problems with a SAT solver

- Pseudo-Boolean constraints express a boolean formula → that formula can be expressed by a CNF
- One of the best Pseudo-Boolean solver in 2005 was Minisat+, based on that idea : Niklas Eén, Niklas Sörensson : Translating Pseudo-Boolean Constraints into SAT. JSAT 2(1-4) : 1-26 (2006)
- Handling those constraints natively in a CDCL solver isn't hard either (Satire, Satzoo, Minisat, ...) : simplifies the mapping from domain constraints and model constraints, explanations.
- One can easily use a SAT solver to solve an optimization problem using either linear or binary search on the objective function.

Optimization using strengthening (linear search)

```
input  : A set of clauses, cardinalities and pseudo-boolean
         constraints setOfConstraints and an objective function
         objFct to minimize
output: a model of setOfConstraints, or UNSAT if the problem is
         unsatisfiable.

answer $\leftarrow$ isSatisfiable (setOfConstraints);
if answer is UNSAT then
    return UNSAT
end
repeat
    model $\leftarrow$ answer ;
    answer $\leftarrow$ isSatisfiable (setOfConstraints $\cup$
                                   $\{$objFct $<$ objFct (model)$\}$);
until (answer is UNSAT);
return model ;
```

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Objective function

$$\min : \quad 4x_2 + 2x_3 + x_5$$

Optimization algorithm

Formula :

$$\left\{\begin{array}{ll} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{array}\right.$$

Model

$\overline{x_1}, x_2, \overline{x_3}, x_4, x_5$

Objective function

$$\min : \quad 4x_2 + 2x_3 + x_5$$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Model

$\overline{x_1}, x_2, \overline{x_3}, x_4, x_5$

Objective function

min : $4x_2 + 2x_3 + x_5$

$<$

Objective function value

5

Optimization algorithm

Formula :

$$\left\{\begin{array}{lr} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{array}\right.$$

Objective function

$$\min : \quad 4x_2 + 2x_3 + x_5 \qquad < \qquad 5$$

Optimization algorithm

Formula :

$$\left\{\begin{array}{lr} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{array}\right.$$

Model

$$x_1, \overline{x_2}, x_3, \overline{x_4}, x_5$$

Objective function

$$\text{min} : \quad 4x_2 + 2x_3 + x_5 \quad < \quad 5$$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Model

$x_1, \overline{x_2}, x_3, \overline{x_4}, x_5$

Objective function

min : $4x_2 + 2x_3 + x_5$

$<$

Objective function value

$3 < 5$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Objective function

$$\min : \quad 4x_2 + 2x_3 + x_5 \qquad < \qquad 3$$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Model

$x_1, \overline{x_2}, \overline{x_3}, x_4, x_5$

Objective function

min : $4x_2 + 2x_3 + x_5 \quad < \quad 3$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Model

$x_1, \overline{x_2}, \overline{x_3}, x_4, x_5$

Objective function

min : $4x_2 + 2x_3 + x_5$

$<$

Objective function value

$1 < 3$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Objective function

$$\text{min :} \quad 4x_2 + 2x_3 + x_5 \qquad\qquad < \qquad\qquad 1$$

Optimization algorithm

Formula :

$$\begin{cases} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{cases}$$

Objective function

min : $\boxed{4x_2 + 2x_3 + x_5 \qquad < \qquad 1}$

Optimization algorithm

Formula :

$$\left\{\begin{array}{lr} (a_1) & 5x_1 + 3x_2 + 2x_3 + 2x_4 + x_5 \geq 8 \\ (a_2) & 5\overline{x_1} + 3\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} + \overline{x_5} \geq 5 \\ (b) & x_1 + x_3 + x_4 \geq 2 \end{array}\right.$$

Objective function

$$\text{min} : \quad 4x_2 + 2x_3 + x_5$$

The objective function value 1 is optimal for the formula.

$x_1, \overline{x_2}, \overline{x_3}, x_4, x_5$ is an optimal solution.

Extending SAT 3 : Minimally Unsatisfiable Subformula

- Let C be an inconsistent set of clauses.
- $C' \subseteq C$ is an unsat core of C iff C' is inconsistent.
- $C' \subseteq C$ is a MUS of C iff C' is an unsat core of C and no subset of C' is an unsat core of C.

Extending SAT 3 : Minimally Unsatisfiable Subformula

- Let C be an inconsistent set of clauses.
- $C' \subseteq C$ is an unsat core of C iff C' is inconsistent.
- $C' \subseteq C$ is a MUS of C iff C' is an unsat core of C and no subset of C' is an unsat core of C.
- Computing a MUS (set of clauses) is equivalent to computing the set of literals L such that :
  1. L satisfies $\{k_i \vee C_i | C_i \in C\}$
  2. $L \cap K$ is subset minimal

Solvers are available for those problems

Some competitive events are organized for those problems :

- Pseudo Boolean since 2005
- MAX-SAT since 2006
- MUS this year

As such, a common input format exists, together with a bunch of solvers.

Generalized use of selector variables

The minisat+ syndrom : is a SAT solver sufficient for all our needs ?

Selector variable principle : satisfying the selector variable should satisfy the selected constraint.

clause simply add a new variable

$\bigvee l_i \quad \Rightarrow \quad s \vee \bigvee l_i$

cardinality add a new weighted variable

$\sum l_i \geq d \quad \Rightarrow \quad d \times s + \sum l_i \geq d$

The new constraints is PB, no longer a cardinality !

pseudo add a new weighted variable

$\sum w_i \times l_i \geq d \quad \Rightarrow \quad d \times s + \sum w_i \times l_i \geq d$

if the weights are positive, else use

$(d + \sum_{w_i<0} |w_i|) \times s + \sum w_i \times l_i \geq d$

From Weighted Partial Max SAT to PBO

Once cardinality constraints, pseudo boolean constraints and objective functions are managed in a solver, one can easily build a weighted partial Max SAT solver

- Add a selector variable s_i per soft clause C_i : $s_i \vee C_i$
- Objective function : minimize $\sum s_i$
- Partial MAX SAT : no selector variables for hard clauses
- Weighted MAXSAT : use a weighted sum instead of a sum.
  Special case : do not add new variables for unit weighted clauses $w_k l_k$
  Ignore the constraint and add simply $w_k \times \overline{l_k}$ to the objective function.

From Unsat Core computation to MaxSat

Recent advances in practical Max Sat solving rely on unsat core computation [Fu and Malik 2006] :

- Compute one unsat core C' of the formula C
- Relax it by replacing C' by $\{ r_i \vee C_i | C_i \in C'\}$
- Add the constraint $\sum r_i \leq 1$ to C
- Repeat until the formula is satisfiable
- If $MinUnsat(C) = k$, requires k loops.

Many improvement since then (PM1, PM2, MsUncore, etc) : works for Weighted Max Sat, reduction of the number of relaxation variables, etc.

Selector variables + assumptions = explanation (MUS)

- Assumptions available from the beginning in Minisat 1.12 (incremental SAT)
- Add a new selector variable per constraint
- Check for satisfiability assuming that the selector variables are falsified
- if UNSAT, analyze the final root conflict to keep only selector variables involved in the inconsistency
- Apply a minimization algorithm afterward to compute a minimal explanation
- Advantages :
  - no changes needed in the SAT solver internals
  - works for any kind of constraints !
- See in action during the MUS track of the SAT 2011 competition !

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Current softwares are composite !

- Linux distributions : made of packages
- Eclipse application : made of plugins
- Any complex software : made of libraries
- There are requirements between the diverse components

Dependency Management Problem

P a set of packages

depends requirement constraints

$$depends : P \rightarrow 2^{2^P}$$

conflicts impossible configurations

$$conflicts : P \rightarrow 2^P$$

Definition (consistency of a set of packages)

$Q \subseteq P$ is consistent with $(P, depends, conflicts)$ iff
$\forall q \in Q, (\forall dep \in depends(q), dep \cap Q \neq \emptyset) \wedge (conflicts(q) \cap Q = \emptyset)$.

Dependency Management Problem

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depends requirement constraints

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Definition (consistency of a set of packages)

$Q \subseteq P$ is consistent with $(P, depends, conflicts)$ iff $\forall q \in Q, (\forall dep \in depends(q), dep \cap Q \neq \emptyset) \wedge (conflicts(q) \cap Q = \emptyset)$.

What is the complexity of finding if a Q containing a specific package exists ?

Just as hard as SAT : NP-complete !

See how to decide satisfiability of $(\neg a \lor b \lor c) \land (\neg a \lor \neg b \lor c) \land a \land \neg c$

```
package: a
version: 1
conflicts: a = 2

package: a
version: 2
conflicts: a = 1

package: b
version: 1
conflicts: b = 2

package: b
version: 2
conflicts: b = 1

package: c
version: 1
conflicts: c = 2

package: c
version: 2
conflicts: c = 1
```

```
package: clause
version: 1
depends: a = 2 | b = 1 | c = 1

package: clause
version: 2
depends: a = 2 | b = 2 | c = 1

package: clause
version: 3
depends: a = 1

package: clause
version: 4
depends: c = 2

package: formula
version: 1
depends: clause = 1, clause = 2,
         clause = 3, clause = 4

request: satisfiability
install: formula
```

Dependencies expressed by clauses

- Dependencies can easily be translated into clauses :

```
package: a
version: 1
depends: b = 2 | b = 1, c = 1
```

$$a_1 \rightarrow (b_2 \vee b_1) \wedge c_1$$

$$\neg a_1 \vee b_2 \vee b_1, \neg a_1 \vee c_1$$

- Conflict can easily be translated into binary clauses :

```
package: a
version: 1
conflicts: b = 2, d = 1
```

$$\neg a_1 \vee \neg b_2, \neg a_1 \vee \neg d_1$$

From decision to optimization

- NP-complete, so we can use a SAT solver to solve it
- Finding a solution is usually not sufficient !
  - Minimizing the number of installed packages
  - Minimizing the size of installed packages
  - Keeping up to date versions of packages
  - Preferring most recent packages to older ones
  - ...
- In practice an aggregation of various criteria
- Need a more expressive representation language than plain CNF !

Representing optimization criteria with MaxSat ?

$\alpha \equiv \bigwedge_{p_v \in P}(p_v \rightarrow (\bigwedge_{dep \in depends(p_v)} dep), \infty) \wedge$
$\bigwedge_{conf \in conflicts(p_v)}(p_v \rightarrow \neg conf, \infty,) \wedge (q, \infty)$
denote the formula to satisfy for installing q.

Minimizing the number of installed packages (Partial MaxSat) :

$$\phi \equiv (\bigwedge_{p_v \in P, p_v \neq q} (\neg p_v, k)) \tag{1}$$

Minimizing the size of installed packages (Weighted Partial MaxSat) :

$$\phi \equiv (\bigwedge_{p_v \in P, p_v \neq q} (\neg p_v, size(p_v))) \tag{2}$$

Those problems are really Binate Covering Problems (CNF + objective function).

Representing optimization criteria using pseudo-boolean optimization

- We can now rewrite the previous optimization criteria in a simpler manner :
  - Minimizing the number of installed packages :

$$min : \sum_{p_v \in P, p_v \neq q} p_v$$

  - Minimizing the size of installed packages :

$$min : \sum_{p_v \in P, p_v \neq q} size(p_v) \times p_v$$

- We can express easily that only one version of package *libnss* can be installed :
  $libnss_1 + libnss_2 + libnss_3 + libnss_4 + libnss_5 \leq 1$

The full encoding

- Contains Eclipse specific features (patches, root packages, etc).
- Has a specific optimization function tailored for two years thanks to user feedback.
- Up and running since Eclipse 3.4 (June 2008).
- For both :
  - Building Eclipse-based products
  - Updating and Eclipse installation

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Conclusion

- Many efficient SAT solvers available
- Can be used to solve either decision or optimization problems
- Success stories for decision problems (Intel Core i7, Windows 7)
- Optimization problems receiving more and more attention (MaxSat, PBO, WBO, SMT OPT, etc)
- Efficient optimization on "real" test cases often rely on raw SAT solvers (encoding Minisat+, unsat core for MaxSat)
- Reusing SAT engines is key for solving efficiently those problems
- because SAT engines are still more and more efficient
- Not many SAT library fully featured to offer to the end user state-of-the-art boolean constraints solver and optimizer.

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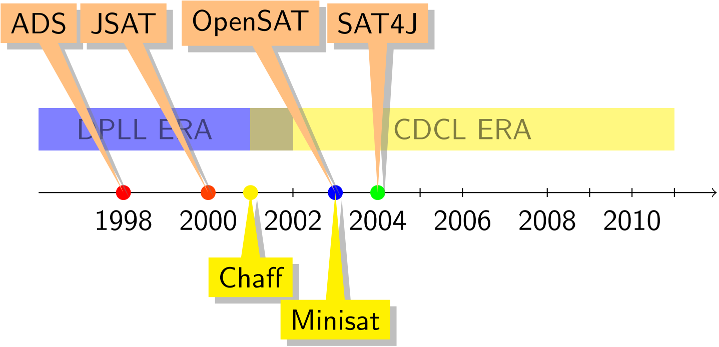
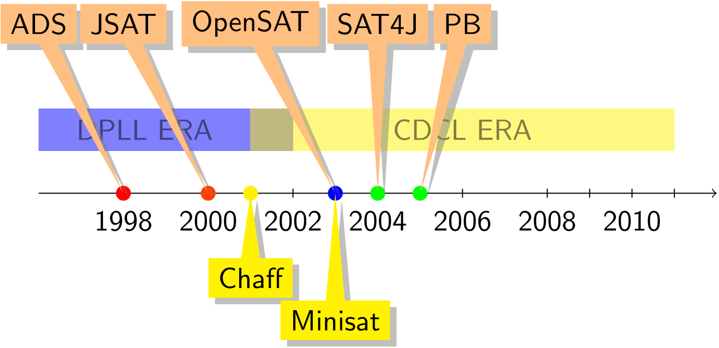
Sat4j, from ADS to p2cudf



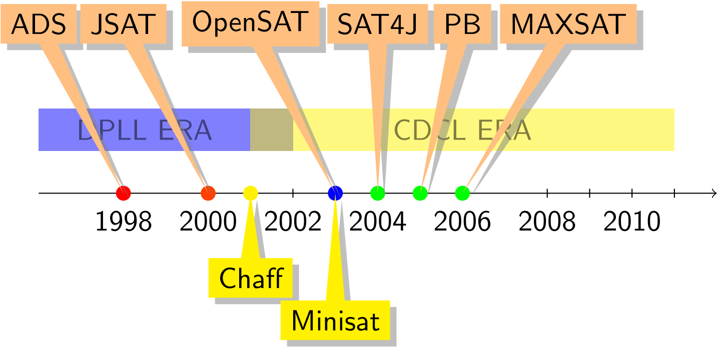
- Birth of SAT4J, Java implementation of Minisat.
- Open Source (licensed under GNU LGPL).

Sat4j, from ADS to p2cudf



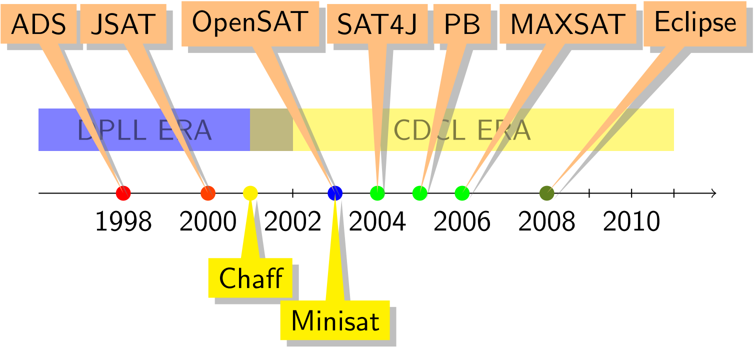
- Joint project with INESC → PB evaluation + Sat4j PB
- First CSP competition → release of Sat4j CSP

Sat4j, from ADS to p2cudf



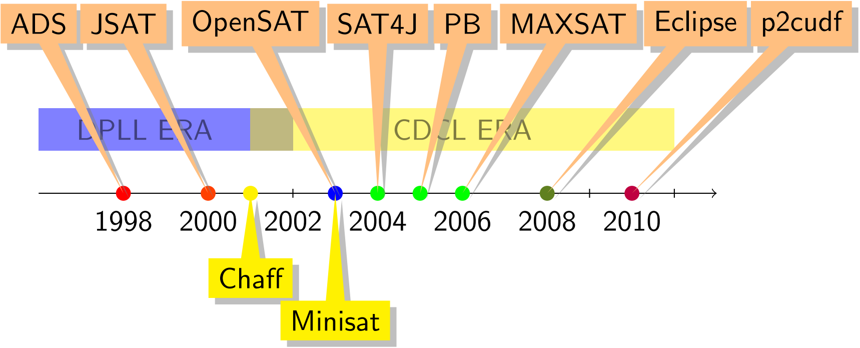
- First MaxSAT evaluation
- Birth of Sat4j MaxSAT

Sat4j, from ADS to p2cudf



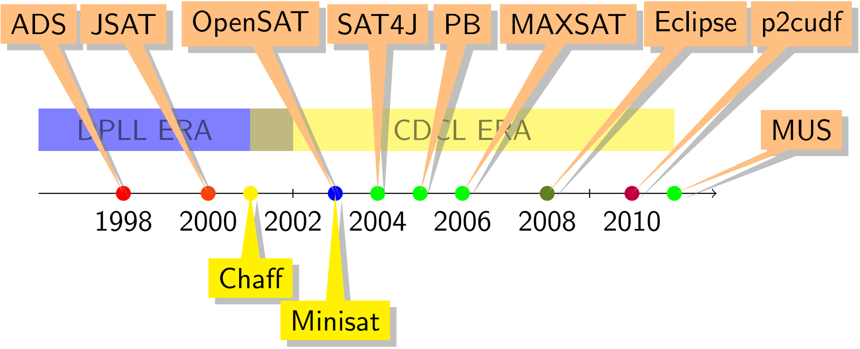
- Integration within Eclipse [IWOCE09]
- Sat4j relicensed under both EPL and GNU LGPL

Sat4j, from ADS to p2cudf



- Application to Linux dependencies : p2cudf [LoCoCo 2010]
- Licensed under EPL

Sat4j, from ADS to p2cudf

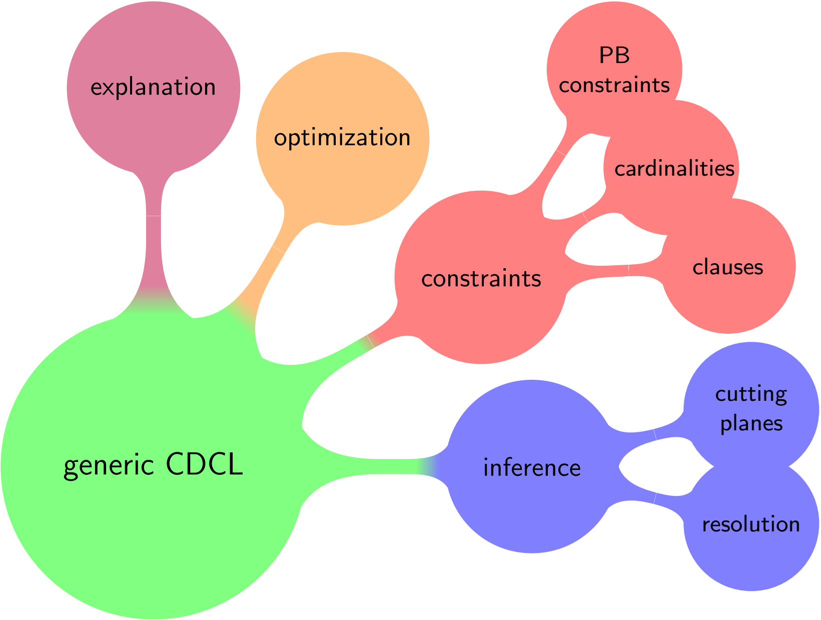


UNIVERSITÉ D'ARTOIS

Initial aim of the project

- Providing 100% Java SAT technology
- With an Open Source software
- Flexible enough to experiment our ideas
- Efficient enough to solve real problems
- Designed to be used in academia or production software

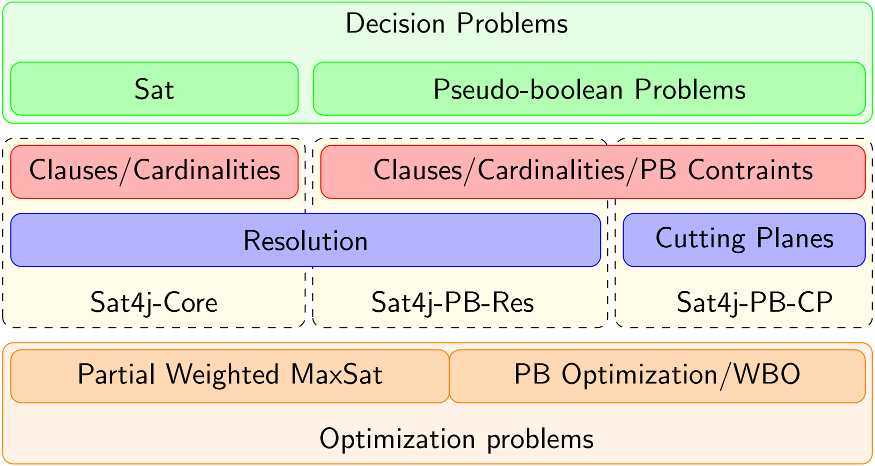
A flexible framework for solving propositional problems



SAT4J today

- SAT4J MAXSAT considered state-of-the-art on Partial [Weighted] MaxSAT application benchmarks (2009).
- SAT4J PB (Res, CP) are not very efficient, but correct (arbitrary precision arithmetic).
- SAT4J SAT solvers can be found in various software from academia (Alloy 4, Forge, ...) to commercial applications (GNA.sim).
- SAT4J PB Res solves Eclipse plugin dependencies since June 2008 (Eclipse 3.4, Ganymede)
  - SAT4J ships with every product based on the Eclipse platform (more than 13 millions downloads per year from Eclipse.org since June 2008)
  - SAT4J helps to build Eclipse products daily (e.g. nightly builds on Eclipse.org, IBM, SAP, etc)
  - SAT4J helps to update Eclipse products worldwide daily

Summary of SAT4J functionalities



Decision Problems

Sat

Pseudo-boolean Problems

Clauses/Cardinalities

Clauses/Cardinalities/PB Contraints

Resolution

Cutting Planes

Sat4j-Core

Sat4j-PB-Res

Sat4j-PB-CP

Partial Weighted MaxSat

PB Optimization/WBO

Optimization problems