Abstract for the 1st World Congress on the Square of Opposition,
Montreux, Switzerland, 1-3 June 2007.
Full paper under the title "Definitional reasoning in proof-theoretic semantics and the square of opposition" in: The Square of Opposition: A General Framework for Cognition. Ed. by Jean-Yves Béziau and Gillman Payette. Bern: Peter Lang 2012, pp. 323-349. https://doi.org/10.15496/publikation-72333

Direct negation in proof-theoretic semantics and the square of opposition

Peter Schroeder-Heister
Wilhelm-Schickard-Institut, Universität Tübingen
Sand 13, 72076 Tübingen, Germany
www-ls.informatik.uni-tuebingen.de/psh

The standard approach to negation in proof-theoretic semantics is via its intuitionistic interpretation using *falsum* as a logical constant. The inference rule *ex falso quodlibet* is then obtained from the fact that no canonical way of proving *falsum* is available, so that it is vacuously true that every canonical proof of *falsum* can be transformed into a proof of any proposition whatsoever. While this point is itself related to the interpretation of the *square of opposition* (see Wagner de Campos Sanz' contribution to this conference), I would like to relate the *square* to the treatment of direct or explicit negation in proof-theoretic semantics. By *direct negation* I mean negation given through explicit denial rules governing the refutation of propositions, in contradistinction to the indirect treatment via a *falsum* constant.

Suppose a rule-based definition is given, consisting of clauses with positive heads ('assertion clauses') and clauses with negative heads ('denial clauses'). They are called clauses for *primary assertion and denial*. Then by a procedure very close to *inversion* or *definitional reflection*, corresponding inferences for *secondary assertion and denial* can be generated, the secondary denial of A saying that all canonical conditions for the primary assertion of A can be refuted, whereas the secondary assertion of A says that all of the canonical conditions for the primary denial of A are refutable. The system as a whole is called *balanced*, when secondary assertion and denial can be inferred from primary assertion and denial, respectively.

In my very tentative talk, I would like reach a result of the following kind: Primary assertion and denial are contraries, secondary assertion and denial are subcontraries, secondary assertion and denial are subalterns to the corresponding primary judgements, and (primary assertion)/(secondary denial) and (primary denial)/(secondary assertion) are contradictories.