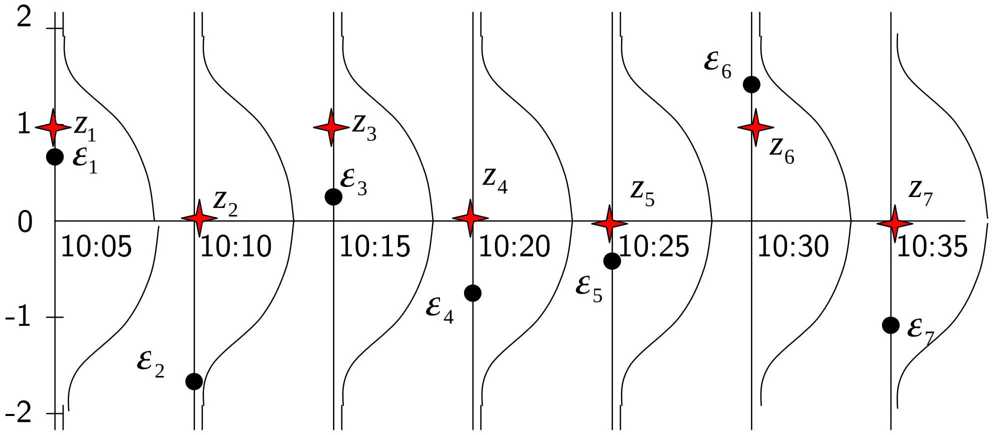
1. Statistical Basics and their use in time series analysis

Economic and financial time series are conceived as realisations of stochastic processes



Statistics in a nutshell: Two roads diverted in the wood

Probability

Frequentist (classic)
repeat experiment many times:
frequency of occurrence
of event → probability

Bayesian view: subjective probability

Kolmogorov's axioms:

1) $0 \le P(\text{Event}) \le 1$

2) $P(\text{Certain Event}) = 1$

3) If events A and B exclusive, then:

$$P(\text{A occurs or B occurs}) =$$

Statistics in a nutshell: Independence and conditional probability

Independent events:

$$P(\text{A occurs and B occurs}) =$$

Conditional probability:

$$P(\text{event A occurs} \mid \text{event B occurs}) = \frac{P(\text{A occurs and B occurs})}{P(\text{B occurs})}$$

Statistics in a nutshell: Random Variables

A random variable is neither random nor a variable

$$\{\text{Dow Jones} \uparrow \text{in } t+1\} \rightarrow 1$$
$$\{\text{Dow Jones} \sim\downarrow \text{in } t+1\} \rightarrow 0$$

$$Z(\text{Dow Jones} \uparrow \text{in } t+1) = 1$$

$$Z(\text{Dow Jones} \sim\downarrow \text{in } t+1) = 0$$

$$P(Z=1) = f_Z(1) = p$$
$$P(Z=0) = f_Z(0) = 1-p$$

Recall: Distributions are described by the expectation operator

Discrete case

$$E(X) = \sum_i x_i \cdot f_X(x_i) = \mu_x \qquad \hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} x_t$$

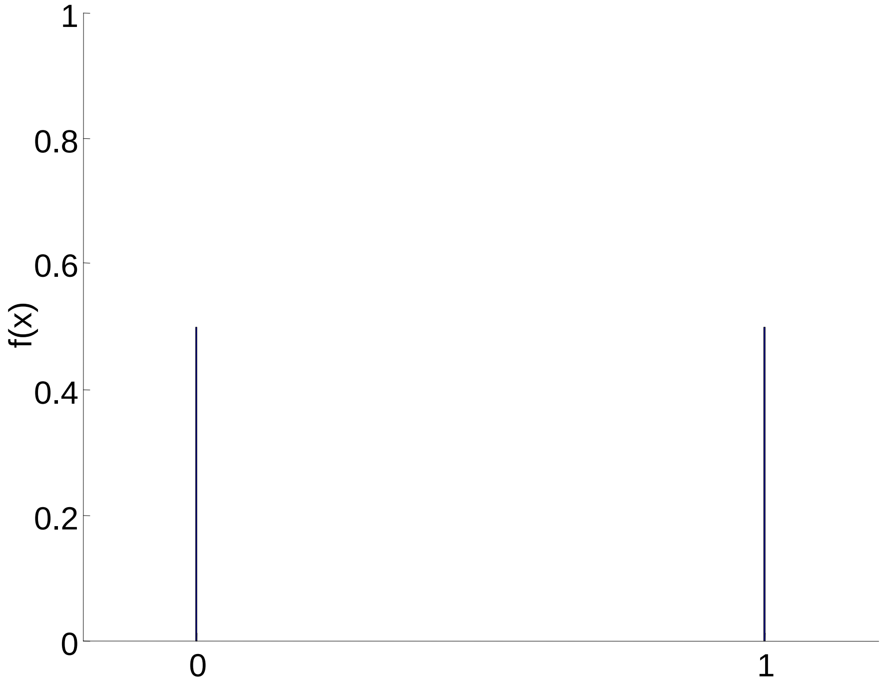
$f_X(x_i) \downarrow$ $P(X = x_i)$

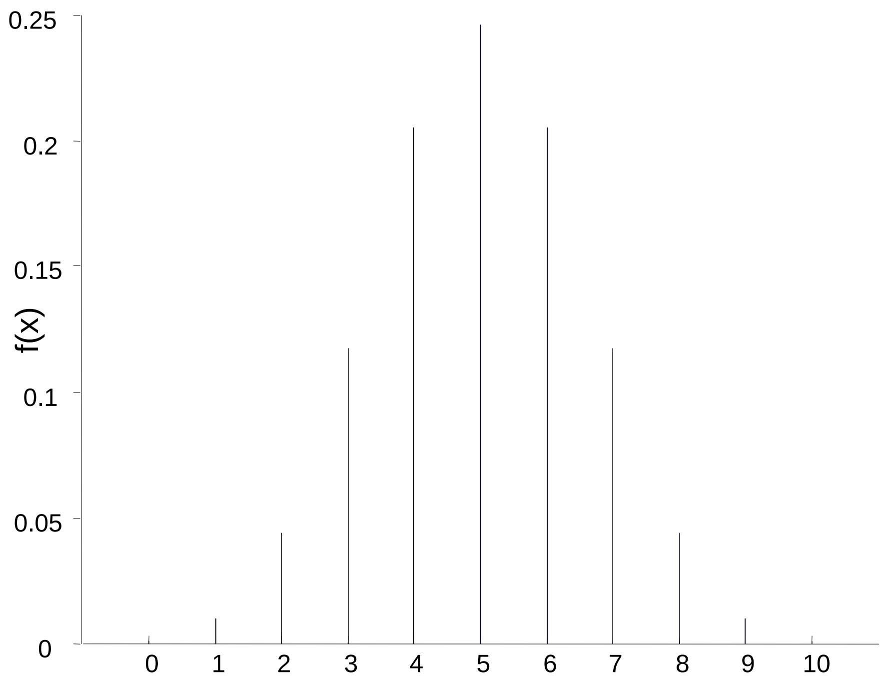
$$E(g(X)) = \sum_i g(x_i) \cdot f_X(x_i)$$

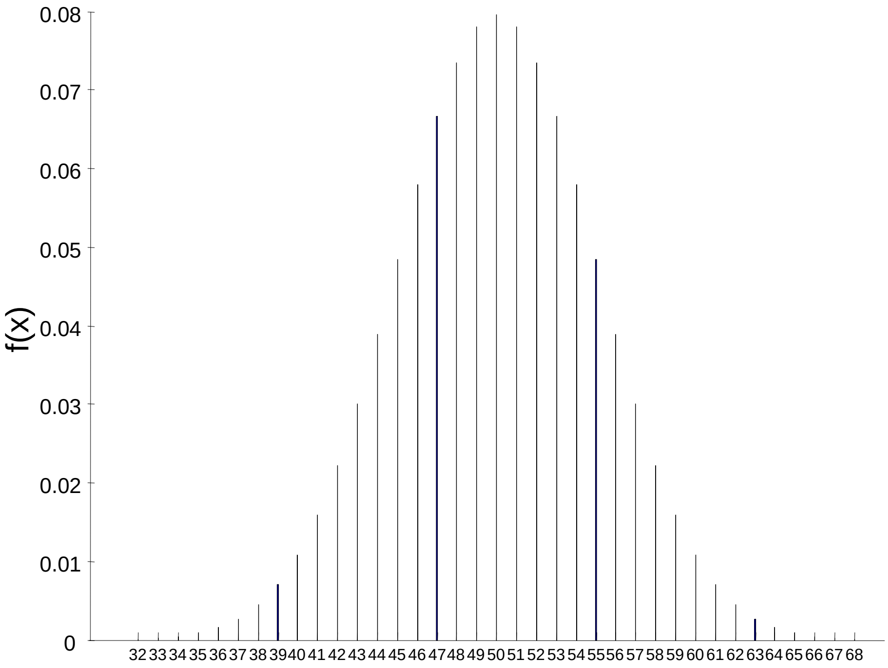
e.g. $E(X^2) = \sum_i x_i^2 \cdot f_X(x_i)$

or $E((X-\mu_x)^2) = \sum_i (x_i - \mu_x)^2 \cdot f_X(x_i) = Var(X)$ $\quad STD(X) = \sqrt{Var(x)}$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N} (x_n - \mu_x)^2$$


1
0.8
0.6
0.4
0.2
0
f(x)
0
1


0.25
0.2
0.15
0.1
0.05
0
f(x)
0 1 2 3 4 5 6 7 8 9 10



Statistics in a nutshell: Random Vectors and Time Series

$$Z_t = \begin{cases} 1, \text{ if Dow Jones} \uparrow \text{ in } t \\ 0, \text{ else} \end{cases}$$

$$P(Z_t = 1) = 0.5$$
$$P(Z_t = 0) = 0.5$$

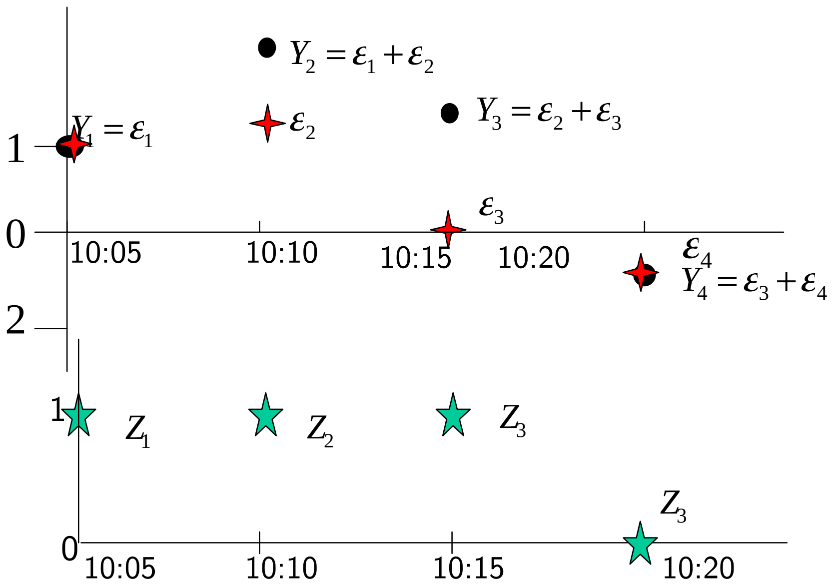
$$Z_{t+1} = \begin{cases} 1, \text{if Dow Jones} \uparrow \text{ in } t+1 \\ 0, \text{else} \end{cases}$$

$$P(Z_{t+1} = 1) = 0.5$$
$$P(Z_{t+1} = 0) = 0.5$$

$$P(Z_t = 1 \text{ and } Z_{t+1} = 1) = P(Z_t = 1) \cdot P(Z_{t+1} = 1) = 0.25$$?????

$$\neq 0.25$$?

Example of a time series with serial dependence



$$Z_t = \begin{cases} 1, \text{if } Y_t > 0 \\ 0, \text{else} \end{cases}$$

$$\underbrace{P(Z_2 = 1 \text{ and } Z_3 = 1)}_{=0.375} \neq \underbrace{P(Z_2 = 1)\cdot P(Z_3 = 1)}_{=0.25}$$

$$\overbrace{P(Z_2 = 0 \text{ and } Z_3 = 0)} \neq \overbrace{P(Z_2 = 0)\cdot P(Z_3 = 0)}$$

Statistics in a nutshell: Continuous Random Variables

$X\{\text{Dow Jones value in } t+1\} \rightarrow X_{t+1}$

$P(X = a) = \ ?$

$P(X \le a) = F_X(a)$

Two random variables X and Y are independent if

$P(X \le x \text{ and } Y \le y) = P(X \le a)\cdot P(Y \le b) = F_X(a) \cdot F_Y(b)$

For time series:

$$\left. \begin{aligned} R_t &= ln\left(\frac{X_t}{X_{t-1}}\right) \\ R_{t+1} &= ln\left(\frac{X_{t+1}}{X_t}\right) \end{aligned} \right\} \Longrightarrow \quad P(Y_t \le a \text{ and } Y_{t+1} \le b) = F_t(a) \cdot F_{t+1}(b) \qquad \textbf{??}$$