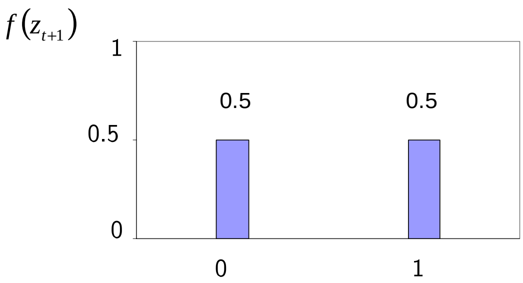
Do not confuse conditional and marginal distribution (1)

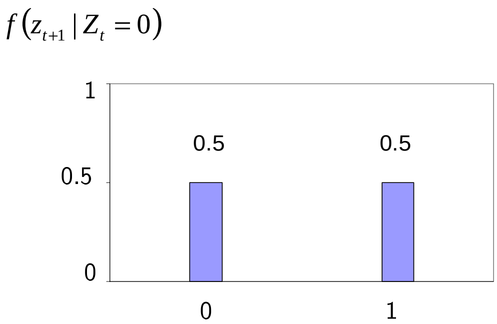
- marginal distribution

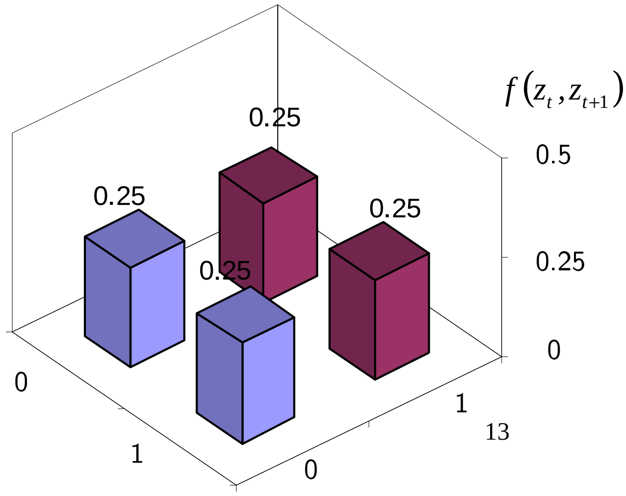


- joint distribution

$f(z_t, z_{t+1})$	$Z_{t+1} = 1$	$Z_{t+1} = 0$
$Z_t = 1$	0.25	0.25
$Z_t = 0$	0.25	0.25

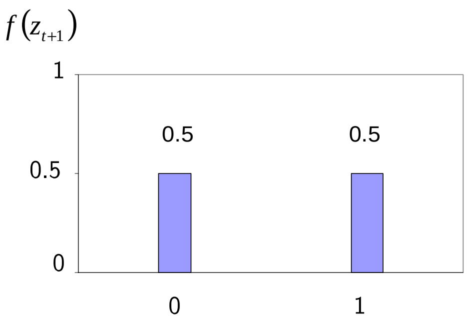
- conditional distribution





Do not confuse conditional and marginal distribution (2)

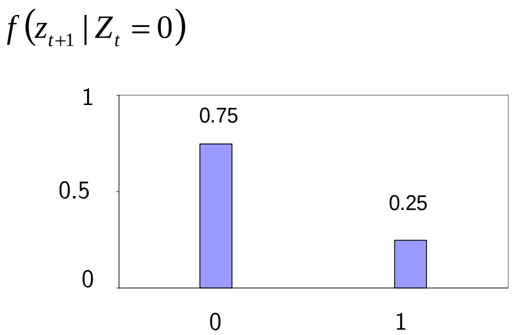
- marginal distribution

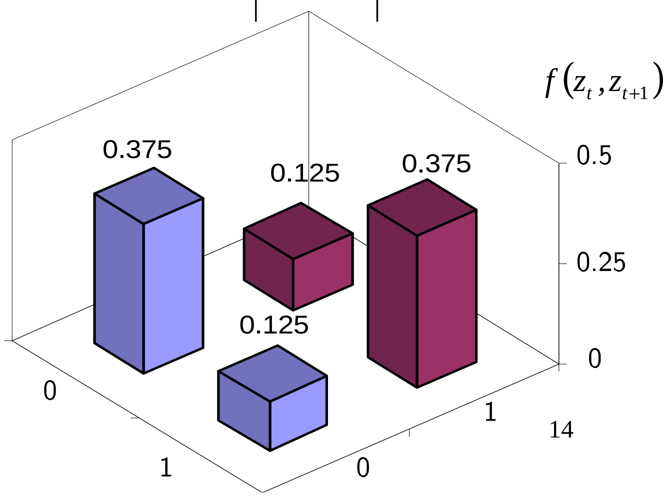


- joint distribution

$f(z_t, z_{t+1})$	$Z_{t+1} = 1$	$Z_{t+1} = 0$
$Z_t = 1$	0.375	0.125
$Z_t = 0$	0.125	0.375

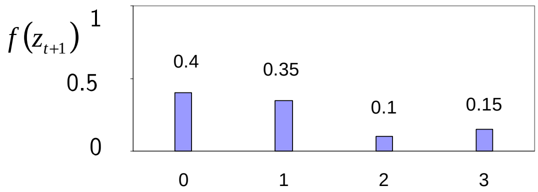
- conditional distribution



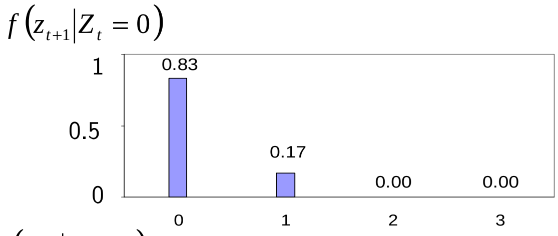


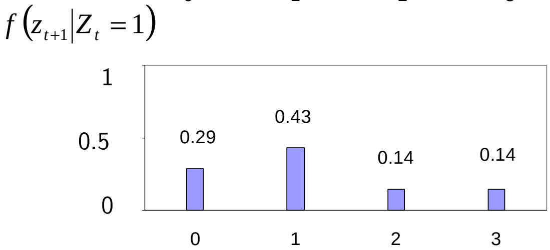
Do not confuse conditional and marginal distribution (3)

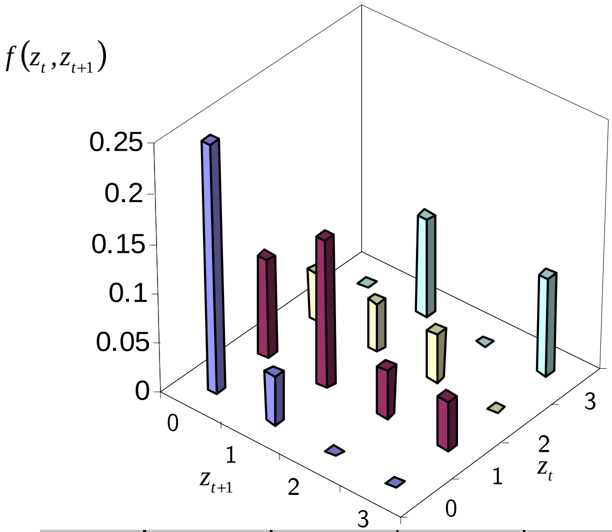
marginal distribution



conditional distributions



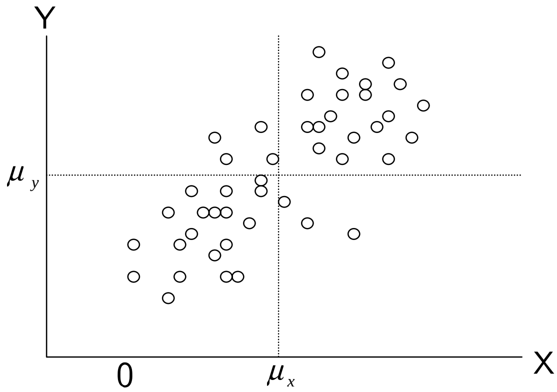




$f(z_t, z_{t+1})$	$Z_{t+1} = 0$	$Z_{t+1} = 1$	$Z_{t+1} = 2$	$Z_{t+1} = 3$
$Z_t = 0$	0.25	0.05	0	0
$Z_t = 1$	0.1	0.15	0.05	0.05
$Z_t = 2$	0.05	0.05	0.05	0
$Z_t = 3$	0	0.1	0	0.1

A measure of linear dependence in time series: Autocovariance

Covariance $E\left((X-\mu_x)\cdot(Y-\mu_y)\right) = Cov(X,Y) = \sum_i \sum_j (x_i-\mu_x)\cdot(y_i-\mu_y)\cdot f(x_i,y_i)$



Autocovariance

$$E\left[(Z_t - E(Z_t))(Z_{t+1} - E(Z_{t+1}))\right] \qquad \text{(order 1)}$$

$$E\left[(Z_t - E(Z_t))(Z_{t+j} - E(Z_{t+j}))\right] \qquad \text{(order j)} \qquad j \neq 0$$

Positive autocovariance :

if Z_t deviates positively from its mean then Z_{t+1} is also expected to deviate positively from its mean.

Linear dependence, time series predictability, and autocovariances

Zero autocovariances : $Cov(Z_t, Z_{t+j}) = 0$ for all j $\Rightarrow$
Future realisations of Z_{t+j} cannot be predicted with linear functions of previous realisations $z_t, z_{t-1}, z_{t-2}, \ldots$

Serial independence : Arbitrary functions of future Z_{t+j} (like Z_{t+j}^2) cannot be predicted by arbitrary functions of past realisations $h(z_t, z_{t-1}, z_{t-2}, \ldots)$

Serial independence $\Rightarrow$ zero autocovariance.

Can we estimate expectations (mean, variances, covariances) by sample means using only a single realisation (time series)?

Economics & finance : We observe only one realisation of a stochastic process (exception: experiments)

Can we estimate - means and variances

- covariances

- distribution parameters

from a single realisation ? e.g. can we estimate $E(Z_t)$ by $\frac{1}{T}\sum_{t=1}^{T} Z_t = \hat{\mu}$

and

$Var(Z_t)$ by $\frac{1}{T}\sum_{t=1}^{T}\left(Z_t - \mu\right)^2$?

The concept of stationarity restricts the heterogeneity of a stochastic process

Understanding the concept of stationarity is crucial. Weak stationarity requires:

A) that a the mean of the process does not change over time

$$E(Z_t) = \mu \qquad \text{for all } t$$

B) that variances do not change over time

$$Var(Z_t) = \sigma^2 \qquad \text{for all } t$$

C) that covariances only depend on the time interval between the two random variables

$$Cov(Z_t Z_{t+j}) = E((Z_t - \mu) \cdot (Z_{t+j} - \mu)) = \gamma_j \qquad \text{for all } t \quad \text{and any } j$$