EBERHARD KARLS UNIVERSITÄT TÜBINGEN

WIRTSCHAFTS- UND SOZIALWISSENSCHAFTLICHE FAKULTÄT

Chair of Econometrics, Statistics and Empirical Economics

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Preparatory Course for Mathematical Methods in Economics and Business

4. Exercise Sheet

Exercise 1 (Quadratic Functions)

Determine the equation of the parabola $y = ax^2 + bx + c$ which runs through the three points $(1, -3)$, $(0, -6)$, and $(3, 15)$.

Exercise 2 (Composite Functions)

Given are the two functions $f(x) = 2x + 4$ and $g(x) = \ln(x)$. For the following compositions, provide the functional equation. Specify the natural domain of the composite function (give a short explanation).

(a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ f)(x)$

(d) $(g \circ g)(x)$ (e) $f(x) \cdot g(x)$

Exercise 3 (Polynomial Division)

By means of polynomial division, determine the terms q(x) and r(x) of the following equations: $P(x) = q(x)Q(x) + r(x)$.

(a) $P(x) = x^4 + 1$ $Q(x) = x^2 + 1$

(b) $P(x) = x^5 + 3x^3 + 7x^2 - 3$ $Q(x) = x^2 + 2x + 1$

For $P(x)$, $Q(x)$, $q(x)$ and $r(x)$ provide the degree of the polynomial.

Exercise 4 **(Logarithmic Laws)**

You don't have a pocket calculator at your disposal but you know that $\log_{10} 5.2 = 0.716$ applies with sufficient accuracy. Provide the following expressions:

(a) $\log_{10} 52$

(b) $\log_{10} 520$

(c) $\log_{10} 5.2^2$

(d) $\log_{10} 5200^7$

Exercise 5 **(Logarithmic Laws)**

Determine the following logarithms:

(a) $\log_{0.5\pi} 1$

(b) $\log_{100} 5.2$

(c) $\log_2(1/8)$

(d) $\log_{1/2} 4$

Generalize the results from d), by showing that it generally applies: $\log_{1/a} x = -(\log_a x)$.

Exercise 6 **(Exponential and Logarithmic Functions)**

Exponential functions can be easily transformed to another base:
Convert a^x into e^{cx}. How does c have to be defined such that it holds $a^x = e^{cx}$? Use this result to transform 10^z and $2^{(0.5y)}$ to the base e.

Exercise 7 **(Inverse Functions)**

Check whether for $y = f(x)$ an inverse function $x = f^{-1}(y)$ exists and provide it if possible. ($D_f = \mathbb{R}$, in case not stated explicitly).

(a) $y = a + b \cdot x$ (b) $y = x^2$

(c) $y = (1 - x)^2 \quad D_f =]-1, 1]$ (d) $y = \dfrac{1}{1 + e^{-x}}$