Sektion 1: Logik

On the notion of *assumption* in logical systems

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Logical calculi, in particular natural deduction systems, exhibit a certain asymmetry between assumptions and assertions. There is a variety of rules for asserting a formula depending on which form this formula has (introduction rules) or from which this formula is inferred (elimination rules), but there is just a single trivial rule for making assumptions, namely by asserting a formula A as depending on itself.

This asymmetry can be removed by carrying ideas from the sequent calculus over to natural deduction. The left introduction rules of the sequent calculus might then be read as rules which introduce assumptions in a specific way depending on their form. For example, the rule of $\wedge$-introduction on the left side of the sequent sign can be interpreted in natural deduction as a rule for introducing $A \wedge B$ as an assumption (assuming that derivations with either A or B as assumptions are available). The result is a natural-deduction-style sequent calculus, in which the role of assumptions is symmetric to that of assertions. In this calculus, major premisses of elimination rules only occur in top position (i.e., as assumptions).

Our next step is to investigate what happens when different sorts of assumptions, those introduced in an *unspecific* way (by just stating A as an assumption) and those introduced by a *specific* assumption introduction rule (which depends on the form of A) are kept apart, as they rely on a different sense of "assumption". There are various strategies at hand to achieve this goal. One is to prohibit the contraction of several occurrences of the same formula into a single one, if these occurrences result from different (specific vs. unspecific) ways of making an assumption.

Finally, these strategies are applied to circular reasoning as it takes place in connection with antinomies. It turns out that the different treatment of specific and unspecific assumptions blocks the derivation of contradictions from circular constructions (within minimal logic). This sheds new light on logical aspects of handling contradictions which add to the proof-theoretic peculiarities which arise in the derivation of an outright contradiction $A \wedge \neg A$ (or $\bot$) from the proposition $A \leftrightarrow \neg A$ which just expresses circularity.