APPENDIX [1]

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1 General conclusion.

Definition 1 *Let* $\mathbb{Z}_0 := \mathbb{Z} - \{0\}$. *For any* $0 < s \in \mathbb{N}$ *let* $\mathbf{s} := \{0, ..., s-1\}$. *For any finite sets* A, B *denote by* $A \to B$ *the set of all functions from* A *to* B. *Let* $0 < m, n \in \mathbb{N}$. *For any* $\overrightarrow{z} = \langle z_0, ..., z_{mn-1} \rangle \in (\mathbb{Z}_0)^{mn}$ *define* $\Omega_{m,n}(\overrightarrow{z}) \in \mathbb{N}$ *by*

$$\Omega_{m,n}(\overrightarrow{z}) := \sum_{f \in \mathbf{n} \to \mathbf{m}} \prod_{i<j<n} \left(z_{mi+f(i)} + z_{mj+f(j)} \right)^2 .$$

Definition 2 *Let* $x \ominus y := \max(0, x-y)$ *and* $x \odot y := (1 \ominus (1 \ominus |x|)) \cdot y$. *Let* $0 < s, \ell \in \mathbb{N}$ *and* $\overrightarrow{v} = \langle v_0, ..., v_{s-1} \rangle$ *(variables). Denote by* $\mathbb{QP}_{s,\ell}$ *the* (s,ℓ)*-quasipolynomials, which are as follows. Suppose* $P \in \mathbb{QP}_s$ *arises by applying the following clauses 1-6 at most* ℓ *times, in arbitrary order. Then* $P \in \mathbb{QP}_{s,\ell}$.

1. *Let* $1 \in \mathbb{QP}_s$
2. *For any* $k < s$, *let* $v_k \in \mathbb{QP}_s$
3. *If* $P, Q \in \mathbb{QP}_s$ *then let* $P + Q \in \mathbb{QP}_s$
4. *If* $P, Q \in \mathbb{QP}_s$ *then let* $P - Q \in \mathbb{QP}_s$
5. *If* $P, Q \in \mathbb{QP}_s$ *then let* $P \ominus Q \in \mathbb{QP}_s$
6. *If* $P, Q \in \mathbb{QP}_s$ *then let* $P \odot Q \in \mathbb{QP}_s$

Remark 3 *Obviously, for every* $\Omega_{m,n}$ *there exists a* $P \in \mathbb{QP}_{mn,(n^2-n+4)m^n+mn}$ *such that* $\Omega_{m,n}(\overrightarrow{z}) = 0 \Leftrightarrow P[\overrightarrow{v} := \overrightarrow{z}] = 0$ *holds for all* $\overrightarrow{z} \in (\mathbb{Z}_0)^{mn}$.

Conjecture 4 *For every* $c \in \mathbb{N}$ *there are* $0 < m, n \in \mathbb{N}$ *such that for every* $P \in \mathbb{QP}_{mn,\max(m,n)^c}$ *there is a* $\overrightarrow{z} \in (\mathbb{Z}_0)^{mn}$ *with* $\Omega_{m,n}(\overrightarrow{z}) = 0 \not\Leftrightarrow P[\overrightarrow{v} := \overrightarrow{z}] = 0$.

Theorem 5 *Conjecture 4 implies* $\boldsymbol{P} \neq \boldsymbol{NP}$.

Remark 6 *It would suffice to weaken the conjecture by assuming that* P *in question is in fact determined by* $\langle m, n \rangle$ *while being polynomial in* $\max(m,n)$. *The corresponding weak variant of Conjecture 4 is equivalent to* $\boldsymbol{P} \neq \boldsymbol{NP}$.

Conjecture 7 (Π_2^0 `variant of Conjecture 4`). *For every* $c \in \mathbb{N}$ *there are* $0 < m, n \in \mathbb{N}$ *such that for every* $P \in \mathbb{QP}_{mn,\max(m,n)^c}$ *there is a* $\overrightarrow{z} \in (\mathbb{Z}_{\leq mn})^{mn}$ *with* $\Omega_{m,n}(\overrightarrow{z}) = 0 \not\Leftrightarrow P[\overrightarrow{v} := \overrightarrow{z}] = 0$, *where* $\mathbb{Z}_{\leq s} := \{x \in \mathbb{Z}_0 : |x| \leq s\}$.

Conjecture 8 *(*`Strong` Π_3^0 `variant of Conjecture 4`*). For every* $c \in \mathbb{N}$ *there is a* $N \in \mathbb{N}$ *so large that for every* $N < n \in \mathbb{N}$ *and every* $P \in \mathbb{QP}_{3n,n^c}$ *there is a* $\overrightarrow{z} \in (\mathbb{Z}_{\leq 3n})^{3n}$ *with* $\Omega_{3,n}(\overrightarrow{z}) = 0 \not\Leftrightarrow P[\overrightarrow{v} := \overrightarrow{z}] = 0$.

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[1]See the author's source "Proof theory and Post-Turing analysis" in: Proc. Proof Theory in Computer Science, Dagstuhl 2001, LN in Comp. Sci. 2183 (2001), 130-152