Are those processes stationary and ergodic ?

Process 1 :

Independent draws at each point of time t from a standard normal distribution

$\{\varepsilon_t\}$ $\qquad \varepsilon_t \sim N(0,1)$ for all $t = 1, 2, \ldots$

used as a notation for stochastic process

Check $E(\varepsilon_t),\ Var(\varepsilon_t),\ Cov(\varepsilon_t, \varepsilon_{t-j})$ for all j

Process 2 :

$\{Y_t\}$ $\qquad Y_t = Y_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0,1)$

How can we test for serial independence ?

serial independence $\Rightarrow$ $Cov(Z_t, Z_{t-j}) = E((Z_t - E(Z_t))(Z_{t-j} - E(Z_{t-j}))) = 0$ for all $j \neq 0$

For a stationary process : $Cov(Z_t, Z_{t-j}) = E((Z_t - \mu)(Z_{t-j} - \mu)) = \gamma_j = 0$ for all $j \neq 0$

$$Var(Z_t) = E(Z_t - \mu)(Z_t - \mu) = \gamma_0$$

Equivalent:

Correlation : $\rho_{XY} = \dfrac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$ $\quad -1 \leq \rho_{XY} \leq 1$

Autocorrelation : $\rho_{Z_t Z_{t-j}} = \dfrac{Cov(Z_t, Z_{t-j})}{\sqrt{Var(Z_t)}\sqrt{Var(Z_{t-j})}}$

for stationary process $\rho_{Z_t Z_{t-j}} = \dfrac{\gamma_j}{\sqrt{\gamma_0}\sqrt{\gamma_0}} = \dfrac{\gamma_j}{\gamma_0} = \rho_j$

serial independence

No linear predictability

How can we test for serial independence ?

If stochastic process is weakly stationary and ergodic

$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} z_t$ consistent estimator for

$\hat{\gamma}_0 = \frac{1}{T}\sum_{t=1}^{T} (z_t - \hat{\mu})^2$ consistent estimator for

$\hat{\gamma}_j = \frac{1}{T-j}\sum_{t=j+1}^{T} (z_t - \hat{\mu})(z_{t-j} - \hat{\mu})$ consistent estimator for

$\hat{\rho}_j = \frac{\hat{\gamma}_j}{\hat{\gamma}_0}$ consistent estimator for

Compute $\hat{\rho}_j$ from data

if serial independence (no linear predictability)

$\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \ldots$ should be close to zero

How can we test for serial independence?

Assume: computation of $\hat{\rho}_1$ yields 0.8 for T=1000

$\Rightarrow$ would you accept hypothesis that $\rho_1 = 0$?

Computation of $\hat{\rho}_1$ yields -0.003 for T=1,000

would you accept hypothesis that $\rho_1 = 0$?

How likely or unlikely is the outcome assuming the (Null) Hypothesis is true?

You can commit 2 type of errors

type 1 : reject null hypothesis despite that it is true

type 2 : do not reject null hypothesis despite that it is false

Significance level (α) Maximal probability of committing type 1 error you are willing to accept in your decision to reject or maintain your null hypothesis

Remember the paradigm of statistical testing (Neyman-Person)

α (significane level) = 0.001 (0.1%)

Assume hypothesis is true (null hypothesis). If the probability that outcome occurs ('under the null hypothesis') is less than 0.1% you reject the null hypothesis.

α (significance level) = 0.2 (20 %)

Assume hypothesis is true (null hypothesis). If the probability that outcome occurs ('under the null hypothesis') is less than 20 % you reject the null hypothesis

Remember the paradigm of statistical testing (Neyman-Person)

a) Fix significance level α

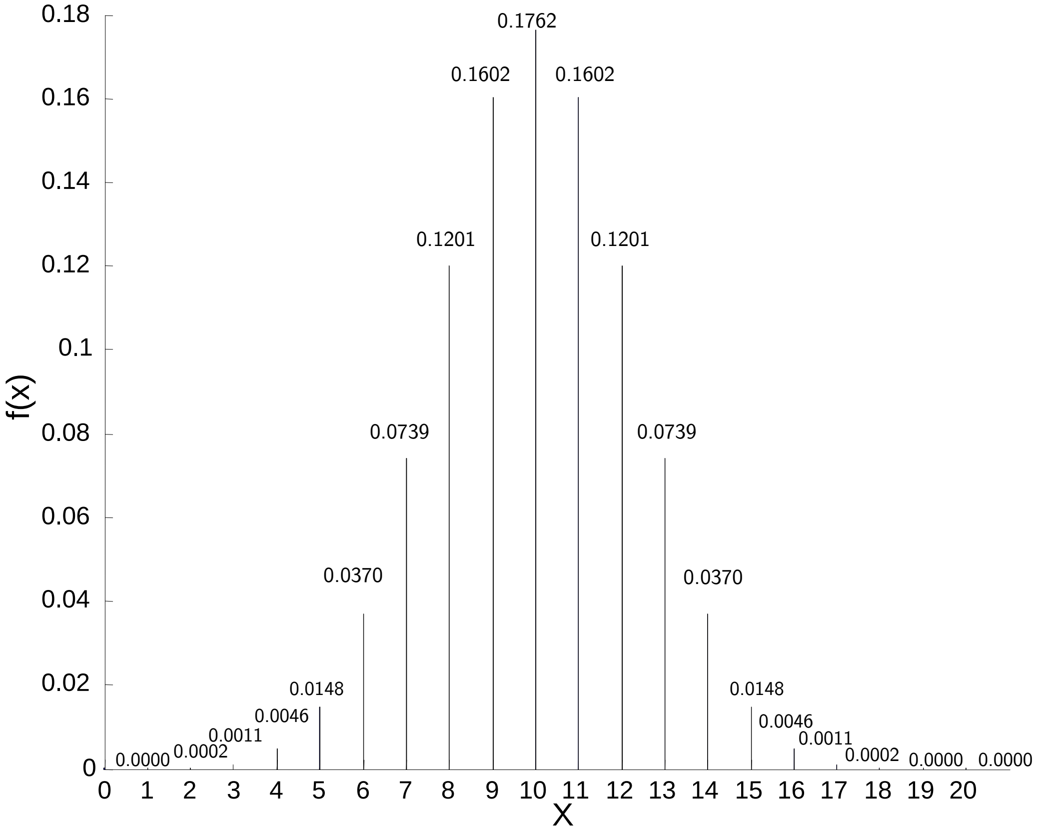
b) Find a test statistic (computable from data) with known distribution assuming

$\underbrace{\text{Null-Hypothesis is true}}$

Under the Null-Hypothesis the distribution of the test statistic is....(Normal or Chi-square or Binomial or ....)

c) From a) and b) determine acceptance and rejection regions

Remember the paradigm of statistical testing (Neyman-Person)



Remember the paradigm of statistical testing (Neyman-Person)