Time Series Analysis

Introduction to EViews

1. Import to EViews the file "DOWJONES.xls" (download from homepage) which contains daily close values of the Dow Jones index.

2. Compute continuously compounded returns $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ (log returns) for the Dow Jones Index and square the computed return series. Note that $r_t \approx \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)$ and that by computing log returns one assumes continuous compounding of the invested capital between $t-1$ and t, i.e. $P_t = \exp(r_t)P_{t-1}$. Log returns are more suitable from a statistical perspective than simple returns $R_t = \frac{P_t}{P_{t-1}}$ as one can compute multi-period returns $r_t(q) = \ln\left(\frac{P_t}{P_{t-q}}\right)$ by adding the single period log returns, whilst for computing multiperiod simple returns $R_t^q = \frac{P_t}{P_{t-q}}$ from single-period returns one has to **multiply** the single period returns. From mathematical statistics more results are available regarding the sum of a sequence of random variables (e.g. laws of large numbers and central limit theorems) than for multiplicative sequences of random variables.

3. Plot the three time series (Dows Jones, log returns, and squared log returns) and provide meaningful axis labels and titles. Export the graphics in your word processor (Word or LaTeX).

4. Compute a histogram of the empirical return distribution and export the resulting figure into your word processor (Word or LaTeX). Use meaningful axis labels and titles for the figure.

5. Compute sample mean, sample variance and sample kurtosis for the sample of log returns. Compute the 0.01 and the 0.05 empirical quantile of the sample distribution of the log returns. Interpret these results.

6. Compute sample autocorrelations for the log return and squared log return series. Use a graphical representation of the sample ACF (Abscissae: Lag (j); Ordinate: $\hat{\rho}_j$). Export the figure into your word processor using meaningful axis labels and titles for the figure. Interpret your results with respect to the predictability of asset returns and squared asset returns.

7. Conduct a unit root test to check if the P_t series is stationary (choose an appropriate Dickey/Fuller specification). Interpret the test result.

8. Compute the first differences of the $\ln(P_t)$ series, i.e. r_t. Conduct a unit root test to check if the r_t series is stationary (choose an appropriate Dickey/Fuller specification).

First set of assignments

1. The stochastic process $\{\varepsilon_t\}(t = 1, 2, \cdots)$ consists of independent random variables $\varepsilon_t \sim N(0, 1)$. Compute the probability $P(\varepsilon_t \leq 0 \cap \varepsilon_{t+1} > 1.96 \cap \varepsilon_{t+2} \leq -1.96)$.

2. Write the joint density $f_{\varepsilon_t \varepsilon_{t+1}}(\varepsilon_t, \varepsilon_{t+1})$. Interpret your result.

3. Write the conditional density $f_{\varepsilon_{t+1}|\varepsilon_t}(\varepsilon_{t+1}|\varepsilon_t)$.

4. Denote a realisation of the stochastic process $\{\varepsilon_t\}$ as $\{x_1, x_2, \cdot\cdot, x_T\}$.
   Write down the joint density function of the random vector $\underline{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \cdot\cdot, \varepsilon_T\}$ evaluated at $\{x_1, x_2, \cdot\cdot, x_T\}$.

   Since the random vector $\underline{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \cdot\cdot, \varepsilon_T\}$ is jointly normally distributed you can use the multivariate normal density which is generally written as

   $$f_{\underline{X}} = (2\pi)^{-n/2} |\Omega|^{-0.5} \exp\left[\frac{(\underline{x} - \underline{\mu})'\Omega^{-1}(\underline{x} - \underline{\mu})}{-2}\right]$$

   What is in our example $n, \underline{x}, \underline{\mu}$ and Ω ?

5. Is the process $\{\varepsilon_t\}$ weakly stationary?

6. Is the process $\{\varepsilon_t\}$ strictly stationary?

7. A new stochastic process $\{Y_t\}$ is generated as $Y_t = a + b \cdot \varepsilon_t$
   The joint distribution of $\underline{Y} = (Y_1, Y_2, \cdot\cdot, Y_T)$ is still the multivariate normal (see 4.)
   What is $\underline{\mu}$ and Ω now?

8. $\{X_t\}$ denotes a stochastic process. We have $E(X_t) = E(X_{t+1}) = 2$
   $cov(X_t, X_{t+1}) = 2$ and $var(X_t) = var(X_{t+1}) = 1$
   using $A = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$ we generate two new random variables Z_1, Z_2 by
   $\underline{Z} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = A \cdot \begin{bmatrix} X_t \\ X_{t+1} \end{bmatrix}$
   compute $E(\underline{Z})$ and $cov(\underline{Z}) = \begin{bmatrix} var\,(Z_1)\, cov\,(Z_1, Z_2) \\ cov\,(Z_1, Z_2)\, var\,(Z_2) \end{bmatrix}$

Solutions to the *first set* of assignments:

1. $P(\varepsilon_t \leq 0) \cdot P(\varepsilon_{t+1} > 1.96) \cdot P(\varepsilon_t \leq -1.96) = 0.5 \cdot 0.025 \cdot 0.025 = 0.0003125$

8. $E(\underline{Z}) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

   $cov(\underline{Z}) = \begin{bmatrix} 1.42 & 1.5 \\ 1.5 & 1.5 \end{bmatrix}$

Second set of assignments

1. Are the following stochastic processes $\{y_t\}$ stationary and ergodic?

   $$\left[\begin{array}{ll} \{\varepsilon_t\} & \text{denotes a Gaussian white noise process} \\ \text{i.e.} & \mathbb{E}(\varepsilon_t)=0, \quad \mathbb{E}(\varepsilon_t^2)=Var(\varepsilon_t)=\sigma^2, \quad \mathbb{E}(\varepsilon_t\cdot\varepsilon_\tau)=0 \quad t\neq\tau \end{array}\right]$$

   $$\begin{array}{llllll} \text{a)} & y_t & = & \varepsilon_t & & \\ \text{b)} & y_t & = & y_{t-1}+\varepsilon_t & \text{with} & y_1=\varepsilon_1 \\ \text{c)} & y_t & = & y_{t-1}-y_{t-2}+\varepsilon_t & \text{with} & y_1=\varepsilon_1 \\ \text{d)} & y_t & = & a\cdot t+\varepsilon_t & \text{with} & a \quad \text{a real number} \end{array}$$

2. Compute $\mathbb{E}(y_t-\mu)(y_{t-j}-\mu)$ [i.e. $cov(y_t, y_{t-j})$] for the stochastic processes b) and d).

3. Check, by writing $\mathbb{E}(y_t)$, $Var(y_t)$ and $cov(y_t, y_{t-j})$ $j \geq 1$, whether a MA(2) process

   $y_t = \mu + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \varepsilon_t$

   is stationary and ergodic.
   Plot the autocorrelation function for a MA(2) where $\theta_1 = 0.5$ and $\theta_2 = -0.3$.

4. Write $\mathbb{E}(y_t)$ and $Var(y_t)$ for a MA(q) process.
   $y_t = \mu + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + ... + \theta_q\varepsilon_{t-q} + \varepsilon_t$

5. The sequence of autocovariances $\{\gamma_j\}_{j=0}^{\infty}$ of a Gaussian process $\{y_t\}$ evolves as $\gamma_j = \theta^j$ where $|\theta| < 1$.
   Is the process ergodic?

6. What do we mean by a Gaussian process?

7. Why is ergodic stationarity such an important property for the purpose of estimating the moments $\mathbb{E}(y_t)$, $Var(y_t)$, $cov(y_t, y_{t-j})$,... of a stochastic process $\{y_t\}$?

   Hint: refer to the ergodic theorem (Hayashi, *Econometrics*, p. 101) and note that if $\{y_t\}$ is stationary and ergodic, so is $\{f(y_t)\}$ where $f(\cdot)$ is a measurable function like $\ln(y_t)$, y_t^2 i.e. a function that produces a new random variable.

8. A MA(∞) is given by
   $y_t = \mu + \theta^2\varepsilon_{t-1} + \theta^4\varepsilon_{t-2} + \theta^6\varepsilon_{t-3} + ...$
   where $|\theta| < 1$.
   Compute $\mathbb{E}(y_t)$ and $Var(y_t)$.

Third set of assignments

1. An AR(1) process is given by

   $Y_t = 0.5 + 0.9\,Y_{t-1} + \varepsilon_t$ where $\{\varepsilon_t\}$ is Gaussian White Noise $\varepsilon_t \sim N(0,9)$

   Compute $E(Y_t)$ and $Var(Y_t)$. Compute the first 5 auto covariances $\gamma_1, \gamma_2, \cdot\cdot, \gamma_5$ and plot the corresponding autocorrelations $\rho_1, \rho_2, \cdot\cdot, \rho_5$.

   Hint $\rho_j = \frac{Cov(Y_t, Yt-j)}{\sqrt{Var(Y_t)}\sqrt{Var(Y_{t-j})}} = \frac{\gamma_j}{\gamma_0}$

2. Show by applying the "brute force" method that the sequence of autocovariances for an AR(1) process

   $Y_t = c + \phi Y_{t-1} + \varepsilon_t$

   can be written as

   $\gamma_j = \frac{\phi^j}{1-\phi^2}\,\sigma^2$

3. Express the stochastic process in 1) in an alternative representation that has the change of Y_t(i.e. $Y_t - Y_{t-1}$) on the left hand side and the difference of the lagged value of Y_t(i.e. Y_{t-1}) and $E(Y_t)$ on the right hand side ("Ohrnstein-Uhlenbeck-representation" $\Rightarrow$ lecture notes)
   Using this representation: What is the expected change $E(Y_t - Y_{t-1})$ given a deviation of $Y_{t-1} - E(Y_t) = 10$ in the previous period?
   What is the variance of $Y_t - Y_{t-1}$ given $Y_{t-1} - E(Y_t) = 10$?

4. From the course page you can download the EVIEWS workfile svar.wf1. The file contains macroeconomic variables at a quarterly frequency. The series ZS3MLIBQ contains an interest rate series, the 3-month Swiss France LIBOR (1974-2002). The series BIPNSA contains the nominal gross domestic product (seasonally adjusted)of Switzerland (1974-2002). The series WKUSDQ contains the Swiss France/US dollar exchange rate (1974-2002).
   Select and estimate an ARMA(p,q) model for
   a) the series ZS3MLIBQ
   b) the log-difference(natural logs) of the BIPNSA series
   c) the log-difference of the WKUSDQ series.

Let the significance of the parameter estimates, the Akaike and Schwartz Information criteria and the sample autocorrelations guide your specification search.

Solutions to the *third set* of assignments:

1. $E(y_t) = 5$; $var(y_t) = 47.368$; $\gamma_1 = 42.632$; $\gamma_2 = 38.368$; $\gamma_3 = 34.532$; $\gamma_4 = 31.078$; $\gamma_5 = 27.971$

3. $E(y_t - y_{t-1}) = -1$; $var(y_t - y_{t-1}) = 81$

Fourth set of assignments

1. Identify the following ARMA processes (e.g. ARMA(0,1),..)?

$$\begin{array}{ll} (a) & (1-\phi L)(1-L)Y_t = (1+\theta L)\varepsilon_t \\ (b) & Y_t = (1+0.4L+0.3L^2)\varepsilon_t \\ (c) & (1-0.9L)(1-L)Y_t = (1+0.3L)\varepsilon_t \\ (d) & (1-0.3L)(1-0.2L^{12})Y_t = (1+0.2L)(1+0.3L^{12})\varepsilon_t \\ (e) & (1-\phi L)(1-L)Y_t = (1+\theta L)\varepsilon_t \\ (f) & (1-\phi_1 L)(1-\phi_{12}L^{12})Y_t = (1+\theta_1 L)(1+\theta_{12}L^{12})\varepsilon_t \end{array}$$

2. Use the eigenvalues of $\mathbf{F}$, to check whether the following AR processes are stationary

$$(1)\ \mathbf{F} = \begin{pmatrix} 0.6 & -0.4 \\ 1 & 0 \end{pmatrix}, \qquad (2)\ \mathbf{F} = \begin{pmatrix} 0.4 & 0.8 & -0.3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (3)\ \mathbf{F} = \begin{pmatrix} 1.2 & -0.1 \\ 1 & 0 \end{pmatrix}$$

where

$$\lambda_1 = 0.30 + 0.55677644i$$
$$\lambda_2 = 0.30 - 0.55677644i$$

where

$$\lambda_1 = 0.91584462$$
$$\lambda_2 = -0.88568851$$
$$\lambda_3 = 0.36984389$$

where

$$\lambda_1 = 1.1099020$$
$$\lambda_2 = 0.090098049$$

3. In the following, $\{\varepsilon_t\}$ denotes a Gaussian White Noise process. Which of the following processes $\{Y_t\}$ is a stationary and ergodic process? Give a brief explanatory statement and describe each process as a special case of an ARMA(p,q) process. For example 'This is a stationary AR(2) process...' et cetera.

$$\begin{array}{ll} (a) & (1-0.5L-0.7L^2)Y_t = \varepsilon_t \\ (b) & (1-0.9L-0.1L^2)Y_t = (1+0.3L)\varepsilon_t \\ (c) & Y_t = (1-L)\varepsilon_t \\ (d) & Y_t = (1+0.9L^2)\varepsilon_t \\ (e) & Y_t = c + 0.5Y_{t-1} + 0.3Y_{t-2} + 1.2\varepsilon_{t-1} + \varepsilon_t \\ (f) & Y_t = \dfrac{(1-1.3L^2)}{1-0.8L-0.1L^2}\varepsilon_t \\ (g) & (1-0.9L)Y_t = \varepsilon_t \\ (h) & (1-0.8L-0.1L^2)Y_t = \varepsilon_t \\ (i) & Y_t = (1+0.4L+0.3L^2)\varepsilon_t \end{array}$$

4. Give your opinion to the following statements. Answer "Correct, since..." or "Incorrect, rather..."

   (a) Any MA process is a stationary process .

   (b) Any finite Gaussian AR(p) process is stationary .

   (c) Whether an ARMA(p,q) is stationary is solely determined by its MA part.

   (f) A White Noise process is an ergodic process

   (g) Any finite MA(q) is ergodic.

Solutions to the *fourth set* of assignments:

1. (a),(e) ARIMA(1,1,1) (b) MA(2) (c) ARMA(2,1) (d),(f) ARIMA(0,1,1)(0,1,1)$_{12}$
2. (1) stationary (2) stationary (3) not stationary
3. (a) $\lambda_1 = 1.123$ $\lambda_2 = -0.623 \rightarrow$ not stationary;
   (b) finite MA(q) stationary, Check AR part: $\lambda_1 = 1$ $\lambda_2 = -0.1 \rightarrow$ not stationary;
   (c),(d),(i) finite MA(q) stationary
   (e) $\lambda_1 = -0.352$ $\lambda_2 = 0.852 \rightarrow$ stationary;
   (f),(h) $\lambda_1 = 0.910$ $\lambda_2 = -0.110 \rightarrow$ stationary
   (g) stationary

Fifth set of assignments

1. You want to construct the exact likelihood function of an AR(2) process

   $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2)$ and i.i.d.

   a) Write down the joint density of the first two observations $f_{Y_1,Y_2}(y_1, y_2)$

   b) Using the conditional density of the third observation $f_{Y_3|Y_2,Y_1}(y_3|y_2, y_1)$ write down the joint density of the first three observations $f_{Y_1,Y_2,Y_3}(y_1, y_2, y_3)$

2. Are the following $MA(q)$ processes invertible?

   $Y_t = c - 0.9\varepsilon_{t-1} + \varepsilon_t$
   $Y_t = c + \varepsilon_{t-1} + \varepsilon_t$
   $Y_t = c + 1.2\varepsilon_{t-1} + \varepsilon_t$
   $Y_t = c + (1 + 0.7L + 0.4L^2)\varepsilon_t$
   $Y_t = c + (1 + 0.2L + 0.4L^2)\varepsilon_t$

3. a) Write down the joint density of the first three observations of the $MA(3)$ process

   $Y_t = c + 0.3\varepsilon_{t-1} + 0.2\varepsilon_{t-2} - 0.1\varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2)$ and ε_t i.i.d. $N(0, \sigma^2)$

   b) Suppose you want to set up the conditional likelihood function of this process. You condition on pre-sample values $\varepsilon_0, \varepsilon_{-1}, \varepsilon_{-2}$. Write down the first three elements of the conditional likelihood function.

   $f_{Y_1|\varepsilon_0=0,\varepsilon_{-1}=0,\varepsilon_{-2}=0} =$
   $f_{Y_2|Y_1,\varepsilon_0=0,\varepsilon_{-1}=0,\varepsilon_{-2}=0} =$
   $f_{Y_3|Y_1,Y_2,\varepsilon_0=0,\varepsilon_{-1}=0,\varepsilon_{-2}=0} =$

   c) Which condition has to hold in order to make the Conditional Maximum-Likelihood work?

4. You have succeeded in providing Maximum-Likelihood estimates of the parameters of an $ARMA(2, 2)$ process.

   $(1 - L\phi_1 - L\phi_2)Y_t = c + (1 + \theta_1 L + \theta_2 L^2)\varepsilon_t \qquad \varepsilon_t \sim \; i.i.d.N(0, \sigma^2)$

   The (conditioned) Maximum-Likelihood estimates are

$$\begin{array}{llll} \hat{c} & = 0.2 & \hat{\theta}_1 = & 0.2 \\ \hat{\phi}_1 & = 0.6 & \hat{\theta}_2 = & -0.1 \\ \hat{\phi}_2 & = 0.1 & \hat{\sigma}^2 = & 0.8 \end{array}$$

The value of the log likelihood function evaluated at these estimates is -1432.6.

Suppose you want to test the null hypothesis

$H_0 : \theta_1 = 0.5$ against $H_A : \theta_1 \neq 0.5$
and $H_0 : \theta_1 = 0$ against $H_A : \theta_1 \neq 0$

Perform and interpret the appropriate tests.
An estimate of the variance-covariance matrix of the estimates $\hat{\theta} = (\hat{c}, \hat{\phi}_1, \hat{\phi}_2, \hat{\theta}_1, \hat{\theta}_2, \hat{\sigma}^2)$ is given by

$$\widehat{Var}(\hat{\theta}) = \left[- \frac{\partial^2 lnL(\theta)}{\partial\theta\partial\theta'} \bigg|_{\hat{\theta}} \right]^{-1} = \begin{bmatrix} 0.007 & \cdots & & & & \vdots \\ 0.001 & 0.005 & & & & \\ 0.002 & 0.001 & 0.003 & & & \\ 0.003 & 0.002 & 0.001 & 0.01 & & \\ 0.001 & 0.003 & 0.004 & 0.001 & 0.002 & \\ 0.001 & 0.0001 & 0.0001 & 0.0001 & 0.00002 & 0.0001 \end{bmatrix}$$

$\theta = (c, \phi_1, \phi_2, \theta_1, \theta_2, \sigma^2)'$

You have also estimated an $ARMA(2,0)$ i.e. an $AR(2)$ model. The estimation of this restricted model yields a log likelihood value equal to -1434.3.
Compute and interpret a likelihood ratio statistic to test the hypothesis that the restrictions implied by the $ARMA(2,0)$ specification are correct. Here the $ARMA(2,2)$ specification is the unrestricted model, the $ARMA(2,0)$ is the restricted model.

As another alternative you have estimated an $MA(2)$ model. The log likelihood evaluated at the maximum likelihood estimates is -1442.2. Perform a test of the $ARMA(2,2)$ specification against the MA(2) model.

Solutions to the *fifth set* of assignments:

2. (1) invertible (2) not invertible (3) not invertible (4) not invertible (5) invertible

4. test statistic: first null hypothesis $t_1 = \frac{0.2-0.5}{\sqrt{0.01}} = -3$
   second null hypothesis $t_2 = \frac{0.2}{\sqrt{0.01}} = 2$

   Likelihood ratio test statistic for ARMA(2,2) vs. AR(2): $LR_1 = 3.4$
   Likelihood ratio test statistic for ARMA(2,2) vs. MA(2): $LR_2 = 19.2$
   critical value: $\chi^2(2) = 5.99$

Sixth set of assignments

1. Estimate a suitable ARIMA(p,d,q) model for the seasonally adjusted consumer price index (variable `pcqsa` in the dataset `svar.wf1`).

    - First, take the logarithm and conduct a unit root test to check if differencing the series is necessary.
    - Choose your specification of the ARMA(p,q) model by looking at the correlogram and checking the information criteria for different orders of p and q (up to ARMA(2,2)).
    - For estimation, use the sample up to the first quarter of the year 2000.
    - Then, forecast the consumer price index (in levels) from 2000:2 to 2002:1.
    - Save the forecast values and their standard errors in order to compute a 95% confidence interval.
    - Finally, plot your result.

2. Use an ARIMA(0,1,1)$(0,1,1)_4$ model to estimate and forecast the nominal GDP (variable `bipn` in the dataset `svar.wf1`). The above notation reads as follows (see Enders (1995) pp. 111-118 for details):

    ARIMA(p,d,q)$(P,D,Q)_s$, where

p and q	=	the nonseasonal ARMA coefficients
d	=	number of nonseasonal differences
P	=	number of multiplicative autoregressive coefficients
D	=	number of seasonal differences
Q	=	number of multiplicative moving average coefficients
s	=	seasonal period

    In algebraic terms, the model looks like:

    $$(1-L)(1-L^4)Y_t = (1-\theta_1 L)(1-\theta_4 L^4)\varepsilon_t$$

    - For estimation, use the sample up to the first quarter of the year 1997
    - In order to forecast the level of the series, use `d(log(bipn)-log(bipn(-4)))` as the dependent variable in the equation (forecast horizon: 1997:2-2002:1).
    - Save the forecast values and their standard errors in order to compute a 95% confidence interval.
    - Finally, plot your result.

Seventh set of assignments: VAR

1. You want to analyze the interdependence between South East Asian stock markets. Therefore, estimate a VAR model with EViews (workfile: `book_var.wf1`) for the returns of five stock markets over the sample period from 1/01/1985 to 4/29/1999: Hong Kong (`hrt1`), Japan (`trt2`), Singapore (`srt3`), Korea (`krt4`) and Thailand (`brt5`).

   - First, conduct a unit root test to check if the series are stationary (choose an appropriate Dickey/Fuller specification).
   - Create a VAR object in Eviews.
   - Include a constant `c` as an exogenous variable and use the return for the five stock markets as endogenous variables.
   - The Akaike information criterion suggests a lag length of four.

2. In order to examine the effect of a shock in one market on the other markets. Compute impulse response functions and conduct a variance decomposition.

   - Impulse response functions: click the `IMPULSE` button bar in the VAR object.
   - Create combined or multiple graphs for the impulse response functions. Choose for period 10. Try several Cholesky orderings of the variables. e.g. `hrt1 trt2 srt3 krt4 brt5` or `brt5 krt4 srt3 trt2 hrt1` etc. Describe and compare patterns of impulse response function by answering following questions: How big/small are the responses to shocks, i.e. the response of a variable to its 'own' shock and to the shock of other variables? How does the response alter if the Cholesky ordering is changed? How long do shocks persist in the series of returns?
   - Variance Decomposition: click `VIEW` button bar above the VAR object and choose `Variance Decomposition`.
   - Create combined graphs that depict the variance decomposition of the five stock market indices investigated for 20 periods. Choose several Cholesky orderings of the variables. e.g. as above in the Impulse Response Analysis. Discuss the results and draw a conclusion from the plots of the variance decomposition. In particular analyze the market in Singapore and Hong Kong e.g. How is the proportion of the movements in a sequence due to its 'own' shocks versus shocks to the other variables? How do the results change with changed Cholesky orderings?

Eighth set of assignments: Cointegration

Purchase Power Parity (PPP): PPP states that the equilibrium or long run exchange rate between two countries is equal to the ratio of their relative price level. This application analyzes if the PPP is empirically supported or not using French and German monthly data for the period January 1974 - July 2000. Test for cointegration and model the cointegrated system using the EViews workfile `ppp.wfl`. `lexc` is the log of the exchange rate (ε), expressed in French Francs (FF) per German Mark (DM), `lfr` and `lge` are logs of the French and German consumer price series, respectively.

- Check whether the log series are nonstationary (choose an appropriate Dickey/ Fuller specification).
- If the unit root null hypothesis can not be rejected for the three series, create a new equation object and run the regression: `lexc c lfr lge`. The residuals of this regression are stored in the object `RESID`. A new series `regresid` has to be generated to keep this residual series. `series regresid = resid`
- First, plot the `regresid` series and guess if the series appears to be stationary. Afterwards, conduct a Dickey/Fuller test to support or disprove your assumption (choose an appropriate Dickey/Fuller specification). Hint: Use the appropriate critical values for unit root tests for residuals in Figure 1. If you can reject the null hypothesis of a unit root work under the assumption that the `lexc`, `lfr` and `lge` series are cointegrated, i.e. there exists a linear combination of the logs of French and German consumer price and the exchange rate that is stationary. Thus, an error correction model can be estimated. Otherwise this provides evidence that the series are not cointegrated and the PPP theory does not hold.

Ninth set of assignments: Cointegration

Additional assignment for the PPP. Check whether the PPP is empirically supported for an asset that is traded on parallel markets, i.e. the same stock is traded at the home market and at the NYSE. Test for cointegration and model the cointegrated system using the EViews workfile `coint_small.wfl`. `homeask/homebid` is the home market ask/bid price in home currency (HC), `nyaskadr/nybidadr` is the ask/bid price at the NYSE in dollar ($=FC) and `exchask/exchbid` is the ask/bid exchange rate (ε) that is expressed in dollar per home currency, i.e. in $\frac{\$}{HC}$.

- PPP states: $\frac{\varepsilon * P_{HC}}{P_{FC}} = 1$. Taking log yields: $\log(P_{FC}) - \log(\varepsilon) - \log(P_{HC}) = 0$
- Check whether the log midquote series are nonstationary (choose an appropriate Dickey/Fuller specification).
- If the unit root null hypothesis can not be rejected for the three series, assume that the three series are cointegrated.
- Alternative 1: Engle/Granger Two Step Method: Create a new equation object and run the regression of the three midquote series. `midnyadr c midex midhome`. The residuals of this regression are stored in the object `RESID`. A new series `regresid` has to be generated to keep this residual series. `series regresid = resid`. Alternatively, click on `PROCS` in the equation object. Choose `Make Residuals` and name the series `regresid`.
- First, plot the `regresid` series and guess if the series appears to be stationary. Afterwards, conduct a Dickey/Fuller test to support or disprove your assumption (choose an appropriate Dickey/Fuller specification). Hint: Use the appropriate critical values for unit root tests for residuals in Figure 1! If you can reject the null hypothesis of a unit root work under the assumption that the log mitquote series are cointegrated, i.e. there exists a linear combination of the logs of home stock price, dollar stock price and the exchange rate that is stationary.
- Estimate the VECM by running three regressions. You have to set up three equations with each variable in first differences on the left hand side. Include on the right hand side two lags of each variable also in first differences and the regression residuals `regresid`. Then you can estimate the parameters of each of the three VECM equations by OLS containing only stationary variables.
- Alternative 2: PPP theory delivers a cointegrating vector [1 -1 -1]. Check whether the resulting `cointreg` series from the linear combination `cointresid` $= \log(P_{FC}) - \log(\varepsilon) - \log(P_{HC})$ is stationary. If you can reject the null hypothesis of a unit root the this provides evidence that the residuals series is stationary and log mitquote series are cointegrated.

- Conduct a Johansen cointegration test in EViews in order to determine the number of cointegrating vectors. In EViews you can conduct the test by creating a group of the three variables and clicking on `VIEW` in the group object. Choose `Cointegration Test`, four lags and *Intercept (no trend) in CE and VAR*. Interpret the results and discuss the number of cointegrating vectors.

- Estimate an error correction model with two lags. Click on `ESTIMATE` in the VAR object and choose in the `Cointegration` card *Intercept (no trend) in CE and VAR*.

- Create combined graphs that depict the impulse response function and the variance decomposition of home and US market. Choose two Cholesky orderings: `midhome midnyadr midex` and `midnyadr midhome midex`. For the impulse response function each market is shocked and the reaction of both markets is depicted in a figure. Discuss the results and draw a conclusion from the plots of the impulse response function and the variance decomposition.

Tenth set of assignments: Cointegration

You are interested to model the effects of Swiss Monetary policy. Use the EViews workfile `svar.wf1`. You consider the following Swiss variables for your analysis: the consumer price index `pcqsa`, the GDP in 1990 Swiss francs `biprsa`, the money stock M1 variable `m1qsa`, and the quarterly average of three month Swiss franc LIBOR rate of interest `zs3mlibq`. Your task is to suggest an appropriate model.

- Start your analysis by taking logs of the series (except the interest rate).
- Check whether the log series are nonstationary (choose an appropriate Dickey/ Fuller specification).
- Check whether the series are cointegrated by a Johansen test. Discuss the number of cointegrating relations in the system and test hypotheses up to $(n-1)$ relations. In EViews you can conduct the test by creating a group of the four variables and clicking on `VIEW` in the group object. Choose `Cointegration Test`, four lags and *Intercept (no trend) in CE and VAR.*
- Alternative 1: If you identify cointegrating relations, estimate a vector error correction model with four lags and a rank equal to the number of cointegrating vectors. Click on `ESTIMATE` in the VAR object and choose in the `Cointegration` card *Intercept (no trend) in CE and VAR.*
- Alternative 2: If you do not identify cointegrating relations model the system by a SVAR as in Assignment 7. Hint: Do not forget to take first differences of all of your series!
- In order to determine the effects of the monetary policy analyze impulse response functions and the variance decompositions. Create combined graphs that depict the impulse response function and the variance. Choose an economic plausible Cholesky ordering, e.g. `m1qsa zs3mlibq pcqsa biprsa`. Analyze by impulse response functions the effects of a money shock on the other investigated variables. Discuss the results and draw a conclusion from the plots of the impulse response function and the variance decomposition.

Figure 1: Asymptotic Critical Values for Cointegration Tests

Test Statistic	1%	2.5%	5%	10%	97.5%
$m = 2$					
τ_c	−3.90	−3.59	−3.34	−3.04	−0.30
τ_{ct}	−4.32	−4.03	−3.78	−3.50	−1.03
τ_{ctt}	−4.69	−4.40	−4.15	−3.87	−1.52
z_c	−28.3	−23.9	−20.6	−17.1	−0.7
z_{ct}	−35.8	−31.1	−27.3	−23.4	−3.2
z_{ctt}	−42.6	−37.5	−33.4	−29.1	−5.8
$m = 3$					
τ_c	−4.29	−4.00	−3.74	−3.45	−0.85
τ_{ct}	−4.66	−4.37	−4.12	−3.84	−1.39
τ_{ctt}	−4.99	−4.70	−4.45	−4.17	−1.81
z_c	−35.2	−30.4	−26.7	−22.7	−2.4
z_{ct}	−42.0	−36.9	−32.8	−28.5	−5.0
z_{ctt}	−48.5	−43.0	−38.7	−34.0	−7.6
$m = 4$					
τ_c	−4.64	−4.35	−4.10	−3.81	−1.30
τ_{ct}	−4.97	−4.68	−4.43	−4.15	−1.73
τ_{ctt}	−5.27	−4.98	−4.73	−4.45	−2.09
z_c	−41.6	−36.5	−32.4	−28.1	−4.5
z_{ct}	−48.1	−42.6	−38.2	−33.5	−7.0
z_{ctt}	−54.3	−48.5	−43.9	−38.9	−9.8
$m = 5$					
τ_c	−4.96	−4.66	−4.42	−4.13	−1.68
τ_{ct}	−5.25	−4.96	−4.72	−4.43	−2.04
τ_{ctt}	−5.53	−5.24	−4.99	−4.72	−2.36
z_c	−47.8	−42.3	−38.0	−33.3	−6.7
z_{ct}	−54.0	−48.2	−43.5	−38.5	−9.3
z_{ctt}	−60.0	−53.9	−49.0	−43.7	−12.1
$m = 6$					
τ_c	−5.25	−4.96	−4.71	−4.42	−2.01
τ_{ct}	−5.52	−5.23	−4.98	−4.70	−2.32
τ_{ctt}	−5.77	−5.49	−5.24	−4.96	−2.61
z_c	−53.8	−48.0	−43.4	−38.4	−9.1
z_{ct}	−59.7	−53.7	−48.8	−43.5	−11.8
z_{ctt}	−65.5	−59.2	−54.1	−48.6	−14.6