UNIVERSITÄT TÜBINGEN
Wirtschaftswissenschaftliche Fakultt
Abteilung Statistik und Ökonometrie
Professor Dr. Joachim Grammig

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Final Exam: TIME SERIES ANALYSIS (SS 2005)

Task 1

Calculate $\mathrm{E}(Y_t)$ for the following stationary stochastic processes with $\varepsilon_t \sim N(0,1)$:

1-1) $(1-0.5L)(1+0.6L^4)Y_t = c + \varepsilon_t$ **(3)**

2-2) $Y_t = c + (1-0.8L)(1+0.2L^4)\varepsilon_t$ **(3)**

3-3) $(1-0.9L-0.3L^2+0.8L^3)Y_t = c + (1+0.2L+0.4L^2)\varepsilon_t$ **(3)**

Task 2

Suppose, you have the following bivariate system with $\varepsilon_{i,t} \sim N(0,\sigma_i)$, $Cov(\varepsilon_{i,t}, \varepsilon_{i,t-j}) = 0 \; \forall \; j$ and $Cov(\varepsilon_{i,t}, \varepsilon_{j,t}) = 0$ for $i \neq j$:

$$\begin{aligned} z_t &= \omega_1 + \beta_{12} v_t + \gamma_{11} z_{t-1} + \gamma_{12} v_{t-1} + \tilde{\gamma}_{11} z_{t-2} + \tilde{\gamma}_{12} v_{t-2} + \varepsilon_{1,t} \\ v_t &= \omega_2 + \beta_{21} z_t + \gamma_{21} z_{t-1} + \gamma_{22} v_{t-1} + \tilde{\gamma}_{21} z_{t-2} + \tilde{\gamma}_{22} v_{t-2} + \varepsilon_{2,t} \end{aligned}$$

2-1) How do we call such a representation? Can you consistently estimate the structural parameters β_i and γ_{ij} when using the above specification? If not, explain why not. **(5)**

2-2) Collect z_t and v_t in a vector $X_t = (z_t, v_t)'$ and transfer the above system in the standard VAR form. **(5)**

2-3) What is the crucial difference between the composite shocks in the VAR in standard form and the original shocks in the above VAR in primitive form? **(3)**

2-4) Why do we need the Cholesky decomposition and how does that work? Why is the ordering of variables in the Cholesky decomposition important? **(5)**

2-5) What would the null be hypothesis if you want to test whether v_t does not Granger cause z_t? **(3)**

2-6) What restriction on β_{12} and β_{21} would be imposed if z_t does not Granger cause v_t? **(3)**

Task 3

A linear projection of Y_{t+1} on X_t is computed as $\hat{E}(Y_{t+1}) = \alpha' X_t$ where $\alpha = E(X_t X_t')^{-1} E(X_t Y_{t+1})$.

3-1 Derive the expression for α. **(5)**

3-2 What conditions have to hold for a linear projection to deliver an optimal forecast? **(2)**

3-3 Explain how ordinary least squares estimation is related to linear projection. **(3)**

3-4 Under which circumstances do OLS estimators provide consistent estimates of linear projection coefficients? **(2)**

3-5 What does the term loss function mean for forecast evaluation and for which specific loss function do the results in 3-1 apply? **(3)**

Task 4

	ARMA(1,0)	ARMA(0,1)	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)
C	—	—	—	—	—	—
S.E.	—	—	—	—	—	—
AR(1)	0.689	—	0.496	0.586	0.217	-0.193
S.E.	0.032	—	0.052	0.136	0.111	0.120
AR(2)	—	—	—	-0.079	—	0.262
S.E.	—	—	—	0.105	—	0.089
MA(1)	—	0.668	0.412	0.332	0.722	1.125
S.E.	—	0.033	0.054	0.132	0.109	0.111
MA(2)	—	—	—	—	0.249	0.386
S.E.	—	—	—	—	0.082	0.058
SBC	2.979	3.036	2.995	2.896	2.900	2.906
LL	-740.153	-750.981	-716.213	-714.298	-714.158	-711.227
p(Q)	0.000	0.000	0.043	0.049	0.514	0.781

4-1 In the above table you find estimation results for different specifications of a univariate time series regression. SBC denotes the Schwartz/Bayes criterion, LL the value of the Log-Likelihood function and $p(Q)$ is the p-value of the Ljung-Box statistic computed for the estimated residuals for 20 lags. Provide a comprehensive discussion to select an appropriate specification. **(10)**

4-2 Sketch graphically, not analytically the theoretical autocorrelation function (ACF) for

a) an ARMA(1,0) process with $\phi = 0.5$ **(2)**

b) an ARMA(0,3) process with $\theta_1 = 0.8$, $\theta_2 = 0.4$ and $\theta_3 = -0.2$ **(2)**

c) an ARMA(0,0) process **(2)**

Task 5

Discuss in short the following issues:

5-1 It is always feasible to estimate an MA(q) process with conditional maximum likelihood techniques. **(3)**

5-2 X_t and Y_t are two I(1) variables. If the R^2 is high in a regression of Y_t on X_t we can rely on the estimation results. **(3)**

5-3 For a stationary Gaussian process $\{Y_t\}$, only the first sample moment converges in probability to $E(Y_t)$. **(3)**

5-4 An AR(p) process is said to be stationary, if all eigenvalues of the F-matrix lie outside or on the unit circle. **(3)**

5-5 Solely the MA part of an ARMA(p,q) process determines, whether the process is stationary. **(3)**

5-6 If you want to explore the dynamics of three I(1) variables it is always the best strategy to estimate a structural VAR in first differences. **(5)**

Task 6

The Daimler-Chrysler (DCX) stock is traded in Frankfurt and in New York. Denote with DCX_{NY} the stock price in New York and with DCX_X the stock price traded on the XETRA trading platform in Frankfurt. Further, denote with FX the exchange rate of US-Dollar and Euro (in EUR/USD).

6-1 Formulate a suitable economic model to test the hypothesis that the law of one price holds. **(5)**

6-2 What do you have to do before choosing the estimation strategy? **(3)**

6-3 The Dickey/Fuller test for the level of your three series delivers τ-values $\left(\frac{\hat{\rho}-1}{\hat{\sigma_\rho}}\right)$ of -0.5, -0.65 and -0.8. How do you conclude? **(3)**

6-4 Write down all the necessary steps in order to estimate your model. Outline the implicit assumptions. **(8)**

6-5 The Dickey/Fuller test for the residual of a regression of DCX_{NY} on FX and DCX_x delivers a p-value of 0.015. Your conclusion? **(3)**

6-6 Would you expect a specific cointegrating vector a priori? If yes, which one? **(3)**

6-7 How could you test which market reacts faster to deviations from the equilibrium price? **(3)**