

1st set GAUSS assignments Financial Econometrics

The following assignment is based on Hamilton(1994) pp. 409-411

1. Create a t-distributed random variable

GAUSS procedure:

Create a t_ν -distributed random variable Y (sample size $N=10000$) from a normally distributed r.v. X and a χ_ν^2 -distributed r.v. $U = \sum_{i=1}^\nu Z_i^2$ where Z is normally distributed. X and Z have to be independent. For example, Y could be computed as:

$$Y = \frac{X}{\sqrt{U/\nu}}$$

2. Create an objective function to estimate the parameter ν

The density function of a t_ν - distributed r.v. is:

$$f_Y(y; \nu) = \frac{\Gamma[(\nu + 1)/2]}{(\pi\nu)^{1/2}\Gamma(\nu/2)} [1 + (y^2/\nu)]^{-(\nu+1)/2}$$

The unconditional second and fourth moments are:

$$\begin{aligned}\mu_2 \equiv E(Y^2) &= \nu/(\nu - 2) \\ \mu_4 \equiv E(Y^4) &= \frac{3\nu^2}{(\nu - 2)(\nu - 4)}\end{aligned}$$

The sample second and fourth moments are:

$$\begin{aligned}\hat{\mu}_2 &= (1/N) \sum_{i=1}^N y_i^2 \\ \hat{\mu}_4 &= (1/N) \sum_{i=1}^N y_i^4\end{aligned}$$

GAUSS procedure:

Write a procedure which returns the objective function

$$Q(\nu) \equiv g'Wg$$

where

$$g = \begin{bmatrix} \left\{ \hat{\mu}_2 - \frac{\nu}{\nu-2} \right\} \\ \left\{ \hat{\mu}_4 - \frac{3\nu^2}{(\nu-2)(\nu-4)} \right\} \end{bmatrix}$$

3. Evaluating the objective function

GAUSS procedure:

Evaluate the objective function for different values of ν in a grid search. Save the values of ν and the corresponding value of the objective function and plot them. (Hint: Use this procedure as the main procedure which will be called later. Therefore, nest the previous procedure inside.)

Now, call your data generating procedure and use the output as your data set. Use the generated data set for the grid search procedure.