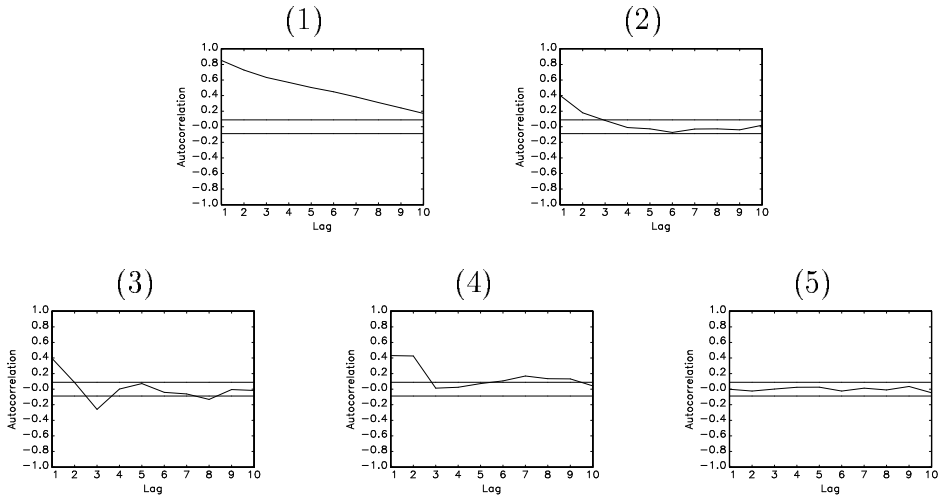
A 1: Assign a stochastic process to each of the five ACF graphs (5).



a) $Y_t = 0.5\varepsilon_{t-4}$ _____ b) $Y_t = 0.5\varepsilon_{t-1} + 0.7\varepsilon_{t-2} + \varepsilon_t$ _____

c) $Y_t = 0.4Y_{t-1} + \varepsilon_t$ _____ d) $Y_t = 0.9Y_{t-1} + \varepsilon_t$ _____

e) $Y_t = \varepsilon_t$ _____ f) $Y_t = 0.7\varepsilon_{t-1} + 0.6\varepsilon_{t-2} - 0.3\varepsilon_{t-3} + \varepsilon_t$ _____

A 2: Are the following stochastic processes stationary? Argue why (or not). (8)

(1) $(1 - 0.9L - 0.1L^2)Y_t = \varepsilon_t$

(2) $(1 - 0.3L)Y_t = (1 + 0.3L)\varepsilon_t$

(3) $(1 - 0.4L - 0.2L^2)Y_t = (1 + 0.1L + 0.05L^2)\varepsilon_t$

(4) $(1 - L)Y_t = \varepsilon_t$

(5) $Y_t = (1 + 0.3L + 0.2L^2 + 0.1L^3)\varepsilon_t$

Use the eigenvalues of $\mathbf{F}$, to check whether the following AR processes are

stationary (8)

$$(1)\ \mathbf{F} = \begin{pmatrix} 0.6 & -0.4 \\ 1 & 0 \end{pmatrix}, \qquad (2)\ \mathbf{F} = \begin{pmatrix} 0.4 & 0.8 & -0.3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3)\ \mathbf{F} = \begin{pmatrix} 1.2 & -0.1 \\ 1 & 0 \end{pmatrix}$$

where

$$\lambda_1 = 0.30 + 0.55677644i$$
$$\lambda_2 = 0.30 - 0.55677644i$$

where

$$\lambda_1 = 0.91584462$$
$$\lambda_2 = -0.88568851$$
$$\lambda_3 = 0.36984389$$

where

$$\lambda_1 = 1.1099020$$
$$\lambda_2 = 0.090098049$$

A 3: Select the suitable ARMA(p,q) process based on the following estimation results. Defend your choice. (8)

	ARMA(0,0)	ARMA(1,0)	ARMA(0,1)	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)
C	0.129	—	—	—	—	—	—
S.E.[a]	0.066	—	—	—	—	—	—
AR(1)	—	0.689	—	0.496	0.586	0.217	-0.193
S.E.[a]	—	0.032	—	0.052	0.136	0.111	0.120
AR(2)	—	—	—	—	-0.079	—	0.262
S.E.[a]	—	—	—	—	0.105	—	0.089
MA(1)	—	—	0.668	0.412	0.332	0.722	1.125
S.E.[a]	—	—	0.033	0.054	0.132	0.109	0.111
MA(2)	—	—	—	—	—	0.249	0.386
S.E.[a]	—	—	—	—	—	0.082	0.058
SBC[b]	3.614	2.979	3.036	2.895	2.906	2.900	2.906
logL	-900.291	-740.153	-755.981	-716.213	-714.298	-714.158	-711.227
p(Q)[c]	0.000	0.000	0.000	0.153	0.196	0.514	0.781

[a] S.E. $\triangleq$ standard error of parameter

[b] SBC $\triangleq$ Schwartz Bayes Criterion

[c] p-value of the Q-statistic was calculated with 10 degrees of freedom

A 4: Compute $E(Y_t)$, $Var(Y_t)$ and $Cov(Y_t, Y_{t-1})$ for the following stochastic processes, where $\{\varepsilon_t\}$ i.i.d N(0,1). (8)

$$(1) \quad (1 - 0.9L)Y_t = \varepsilon_t$$
$$(2) \quad (1 - 0.8L - 0.1L^2)Y_t = \varepsilon_t$$
$$(3) \quad Y_t = (1 + 0.4L + 0.3L^2)\varepsilon_t$$

Compute $E(Y_t)$ and $Var(Y_t)$ for the following stochastic processes (5)

$$(4) \quad (1 - 0.9L)Y_t = (1 - 0.3L)\varepsilon_t$$
$$(5) \quad (1 - L)Y_t = \varepsilon_t$$

A 5: Identify the following ARMA processes (e.g. ARMA(0,1),..)? (5)

$$(1) \quad (1 - \phi_1 L)(1 - \phi_{12} L)Y_t = (1 + \theta_1 L)(1 + \theta_4 L)\varepsilon_t$$
$$(2) \quad (1 - \phi L)(1 - L)Y_t = (1 + \theta L)\varepsilon_t$$
$$(3) \quad Y_t = (1 + 0.4L + 0.3L^2)\varepsilon_t$$

A 6: Give your opinion to the following statements. Answer "Correct, since..." or "Incorrect, rather .."

a) Any MA process is a stationary process (3).

b) Any finite Gaussian AR(p) process is stationary (3).

c) Whether an ARMA(p,q) is stationary is solely determined by its MA part(3).

d) Assuming that the data is generated by a non-stationary process, one can use a weak law of large numbers and estimate consistently expectations by sample means.(3)

f) A White Noise process is an ergodic process (3)

g) Any finite MA(q) is ergodic. (3)

A 7: Multiply the lag polynomials and verbally describe the respective stochastic process. (8)

$$(1) \quad (1 - 0.9L)(1 - L)Y_t = (1 + 0.3L)\varepsilon_t$$

$$(2) \quad (1 - 0.3L)(1 - 0.2L^{12})Y_t = (1 + 0.2L)(1 + 0.3L^{12})\varepsilon_t$$

A 8: Describe the basic approach towards Maximum Likelihood Estimation of stationary ARMA processes. What are main the problems that we encounter?(8)

A 9: Describe the difference between exact ML estimation and conditional ML estimation of an AR(p) process. Explain why the conditional ML approach is equivalent to an OLS approach.(8)

A 10: Have a look at figures (1)-(3) and propose a suitable stochastic process to model these data. Defend your choice.(5)