4th set of assignments Time Series Analysis

1. Identify the following ARMA processes (e.g. ARMA(0,1),..)?

$$
\begin{array}{ll}
(a) & (1-\phi L)(1-L)Y_t = (1+\theta L)\varepsilon_t \\
(b) & Y_t = (1+0.4L+0.3L^2)\varepsilon_t \\
(c) & (1-0.9L)(1-L)Y_t = (1+0.3L)\varepsilon_t \\
(d) & (1-0.3L)(1-0.2L^{12})Y_t = (1+0.2L)(1+0.3L^{12})\varepsilon_t \\
(e) & (1-\phi L)(1-L)Y_t = (1+\theta L)\varepsilon_t \\
(f) & (1-\phi_1 L)(1-\phi_{12}L^{12})Y_t = (1+\theta_1 L)(1+\theta_{12}L^{12})\varepsilon_t
\end{array}
$$

2. Use the eigenvalues of $\mathbf{F}$, to check whether the following AR processes are stationary

$$
(1)\ \mathbf{F} = \begin{pmatrix} 0.6 & -0.4 \\ 1 & 0 \end{pmatrix}, \qquad (2)\ \mathbf{F} = \begin{pmatrix} 0.4 & 0.8 & -0.3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3)\ \mathbf{F} = \begin{pmatrix} 1.2 & -0.1 \\ 1 & 0 \end{pmatrix}
$$

where

$$
\lambda_1 = 0.30 + 0.55677644i \qquad \lambda_2 = 0.30 - 0.55677644i
$$

where

$$
\lambda_1 = 0.91584462 \qquad \lambda_2 = -0.88568851 \qquad \lambda_3 = 0.36984389
$$

where

$$
\lambda_1 = 1.1099020 \qquad \lambda_2 = 0.090098049
$$

3. In the following, $\{\varepsilon_t\}$ denotes a Gaussian White Noise process. Which of the following processes $\{Y_t\}$ is a stationary and ergodic process? Give a brief explanatory statement and describe each process as a special case of an ARMA(p,q) process. For example 'This is a stationary AR(2) process...' et cetera.

$$
\begin{array}{ll}
(a) & (1-0.5L-0.7L^2)Y_t = \varepsilon_t \\
(b) & (1-0.9L-0.1L^2)Y_t = (1+0.3L)\varepsilon_t \\
(c) & Y_t = (1-L)\varepsilon_t \\
(d) & Y_t = (1+0.9L^2)\varepsilon_t \\
(e) & Y_t = c + 0.5Y_{t-1} + 0.3Y_{t-2} + 1.2\varepsilon_{t-1} + \varepsilon_t \\
(f) & Y_t = \dfrac{(1-1.3L^2)}{1-0.8L-0.1L^2}\varepsilon_t \\
(g) & (1-0.9L)Y_t = \varepsilon_t \\
(h) & (1-0.8L-0.1L^2)Y_t = \varepsilon_t \\
(i) & Y_t = (1+0.4L+0.3L^2)\varepsilon_t
\end{array}
$$

4. Give your opinion to the following statements. Answer "Correct, since..." or "Incorrect, rather..."

   (a) Any MA process is a stationary process .

   (b) Any finite Gaussian AR(p) process is stationary .

   (c) Whether an ARMA(p,q) is stationary is solely determined by its MA part.

   (f) A White Noise process is an ergodic process

   (g) Any finite MA(q) is ergodic.