Time Series Analysis

Second set of assignments:

1. Are the following stochastic processes $\{y_t\}$ stationary and ergodic?

   $$\left[\begin{array}{ll} \{\varepsilon_t\} & \text{denotes a Gaussian white noise process} \\ \text{i.e.} & \mathbb{E}(\varepsilon_t)=0, \quad \mathbb{E}(\varepsilon_t^2)=Var(\varepsilon_t)=\sigma^2, \quad \mathbb{E}(\varepsilon_t\cdot\varepsilon_\tau)=0 \quad t\neq\tau \end{array}\right]$$

   a) $y_t = \varepsilon_t$

   b) $y_t = y_{t-1}+\varepsilon_t$ with $y_1=\varepsilon_1$

   c) $y_t = y_{t-1}-y_{t-2}+\varepsilon_t$ with $y_1=\varepsilon_1$

   d) $y_t = a\cdot t+\varepsilon_t$ with a a real number

2. Compute $\mathbb{E}(y_t-\mu)(y_{t-j}-\mu)$ [i.e. $cov(y_t,y_{t-j})$] for the stochastic processes b) and d).

3. - Check, by writing $\mathbb{E}(y_t)$, $Var(y_t)$ and $cov(y_t,y_{t-j})$ $j\geq 1$, whether a MA(2) process

   $$y_t=\mu+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\varepsilon_t$$

   is stationary and ergodic.

   - Plot the autocorrelation function for a MA(2) where $\theta_1=0.5$ and $\theta_2=-0.3$.

4. Write $\mathbb{E}(y_t)$ and $Var(y_t)$ for a MA(q) process.

   $y_t=\mu+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+...+\theta_q\varepsilon_{t-q}+\varepsilon_t$

5. The sequence of autocovariances $\{\gamma_j\}_{j=0}^{\infty}$ of a Gaussian process $\{y_t\}$ evolves as $\gamma_j=\theta^j$ where $|\theta|<1$.
   Is the process ergodic?

6. What do we mean by a Gaussian process?

7. Why is ergodic stationarity such an important property for the purpose of estimating the moments $\mathbb{E}(y_t)$, $Var(y_t)$, $cov(y_t,y_{t-j})$,... of a stochastic process $\{y_t\}$?

   Hint: refer to the ergodic theorem (Hayashi, *Econometrics*, p. 101) and note that if $\{y_t\}$ is stationary and ergodic, so is $\{f(y_t)\}$ where $f(\cdot)$ is a measurable function like $\ln(y_t)$, y_t^2 i.e. a function that produces a new random variable.

8. A MA(∞) is given by

$y_t = \mu + \theta^2 \varepsilon_{t-1} + \theta^4 \varepsilon_{t-2} + \theta^6 \varepsilon_{t-3} + ...$

where $|\theta| < 1$.

Compute $\mathbb{E}(y_t)$ and $Var(y_t)$.