Assignments first week time series

1. The stochastic process $\{\varepsilon_t\}(t = 1, 2, \cdots)$ consists of independent random variables $\varepsilon_t \sim N(0, 1)$. Compute the probability $P(\varepsilon_t \leq 0 \cap \varepsilon_{t+1} > 1.96 \cap \varepsilon_{t+2} \leq -1.96)$.

2. Write the joint density $f_{\varepsilon_t \varepsilon_{t+1}}(\varepsilon_t, \varepsilon_{t+1})$. Interpret your result.

3. Write the conditional density $f_{\varepsilon_{t+1}|\varepsilon_t}(\varepsilon_{t+1}|\varepsilon_t)$.

4. Denote a realisation of the stochastic process $\{\varepsilon_t\}$ as $\{x_1, x_2, \cdot\cdot, x_T\}$.
   Write down the joint density function of the random vector $\underline{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \cdot\cdot, \varepsilon_T\}$ evaluated at $\{x_1, x_2, \cdot\cdot, x_T\}$.

   Since the random vector $\underline{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \cdot\cdot, \varepsilon_T\}$ is jointly normally distributed you can use the multivariate normal density which is generally written as

   $$f_{\underline{X}} = 2\pi^{-n/2} |\Omega|^{-0.5}\, exp \left[\frac{(\underline{x} - \underline{\mu})' \Omega^{-1} (\underline{x} - \underline{\mu})}{-2} \right]$$

   What is in our example $n, \underline{x}, \underline{\mu}$ and Ω ?

5. Is the process $\{\varepsilon_t\}$ weakly stationary?

6. Is the process $\{\varepsilon_t\}$ strictly stationary?

7. A new stochastic process $\{Y_t\}$ is generated as $Y_t = a + b \cdot \varepsilon_t$
   The joint distribution of $\underline{Y} = (Y_1, Y_2, \cdot\cdot, Y_T)$ is still the multivariate normal (see 4.)
   What is $\underline{\mu}$ and Ω now?

8. $\{X_t\}$ denotes a stochastic process. We have $E(X_t) = E(X_{t+1}) = 2$
   $cov(X_t, X_{t+1}) = 2$ and $var(X_t) = var(X_t) = 1$
   using $A = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$ we generate two new random variables Z_1, Z_2 by
   $\underline{Z} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = A \cdot \begin{bmatrix} X_t \\ X_{t+1} \end{bmatrix}$
   compute $E(\underline{Z})$ and $cov(\underline{Z}) = \begin{bmatrix} var\,(Z_1)\, cov\,(Z_1, Z_2) \\ cov\,(Z_1, Z_2)\, var\,(Z_2) \end{bmatrix}$