25 Easy Pieces in MATHSTAT

1. Write the expectation of a random variable (r.v.) Z, $E(Z)$, extensively
a) for a discrete random variable
b) for a continuous random variable

2. $Var(Z)$ can be written as $E(Y)$. What is Y?

3. Write $Var(Z)$ extensively
a) for a discrete random variable
b) for a continuous random variable

4. What does the cumulative density function or cumulative distribution function (c.d.f.) tell you?
$F_X(x) =$

5. X is a continuous r.v.. How are the c.d.f. $F_X(x)$ and the density function (d.f.) $f_X(x)$ related?

6. $Cov(X, Y)$ can be written as $E(Z)$. What is Z?

7. Write $Cov(X, Y)$ extensively for X and Y
a) as discrete r.v.s.
b) as continuous r.v.s.

8. Express $E_{XY}(XY)$ as a function of $Cov(X, Y)$

9. Write $E_{XY}(XY)$ extensively for X and Y
a) as discrete r.v.s.
b) as continuous r.v.s.

10. $g(X)$ denotes a measurable function of the r.v. X (like e.g. X^2, $ln(X)$). Write extensively $E(g(X))$ for the continuous r.v. X

11. X and Y are cont. r.v.s.. $Z = g(X, Y)$ is a measurable function. Write extensively $E(g(X, Y))$

12. X and Y are cont. r.v.s.. What does the joint c.d.f. $F_{XY}(x, y)$ tell you? Write the c.d.f. extensively. What does the joint p.d.f. $f_{XY}(x, y)$ tell you? (discrete case)

13. How are $F_{XY}(x, y)$ and $f_{XY}(x, y)$ (joint density) related? (X and Y are cont. r.v.s.)

14. If X and Y are independent:
$F_{XY}(x, y) =$
$f_{XY}(x, y) =$

15. If X and Y are independent:
$E_{XY}(X \cdot Y) =$
$Cov(X, Y) =$

16. If X and Y are independent:
$E_{XY}(h(X) * g(Y)) =$

17. $E_{XY}(X + Y) =$
$E_{XYZ}(X + Y + Z) =$
$Var(X + Y) =$

18. Write extensively for X and Y as discrete r.v.s. and X and Y as continuous r.v.s.
$f_{X|Y}(X|Y = y)$
$E_{X|Y}(X|Y = y)$
$E_{X|Y}(X^2|Y = y)$

19. $E(aX) =$
$Var(aX) =$
(a is a nonrandom scalar)

20. For $\underline{X} = (X_1, X_2, \ldots, X_n)'$
$E(\underline{X}) = \mu, \mu =?$
$Var(\underline{X}) = \Sigma, \Sigma =?$
$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}$$
(A is a nonrandom matrix)

$\underline{Z} = A * \underline{X}$
$E(\underline{Z}) =$
$Var(\underline{Z}) =$

21. $Y = a + b * X$
$E(Y) =$
$E(Y|X = x) =$

22. Given joint density $f_{XY}(x, y)$. How do you get $f_X(x)$ and $f_Y(y)$?
a) as discrete r.v.s.
b) as continuous r.v.s.

23. Under which circumstances can you get $f_{XY}(x, y)$ from $f_X(x)$ and $f_Y(y)$?

24. X and Y are jointly normally distributed
$\begin{pmatrix} X \\ Y \end{pmatrix} \sim BVN(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho_{XY})$.
What is the relation of parameters and moments?
$X \sim$
$Y \sim$
$X|(Y = y)$
$Y|(X = x)$
$E(X|Y = y) =$
$Var(X|Y = y) =$

25. X, Y and Z are normally distributed.
$W = a * X + b * Y + c * Z \sim$
How is W distributed?