

## 4th set assignments Introductory Econometrics

### Task 1

#### Confidence Intervals:

Suppose you have estimated a parameter vector  $\mathbf{b} = (0.55 \ 0.37 \ 1.46 \ 0.01)'$  with an estimated variance-covariance matrix

$$\widehat{Var}(\mathbf{b}|\mathbf{X}) = \begin{bmatrix} 0.01 & 0.023 & 0.0017 & 0.0005 \\ 0.023 & 0.0025 & 0.015 & 0.0097 \\ 0.0017 & 0.015 & 0.64 & 0.0006 \\ 0.0005 & 0.0097 & 0.0006 & 0.001 \end{bmatrix}$$

- Compute the 95% confidence interval each parameter  $b_k$ .
- What does the specific confidence interval computed in a) tell you?
- Why are the bounds of a confidence interval for  $\beta_k$  random variables?
- Another estimation yields an estimated  $b_k$  with the corresponding standard error  $se(b_k)$ . You conclude from computing the t-statistic  $t_k = \frac{b_k - \bar{\beta}_k}{se(b_k)}$  that you can reject the null hypothesis  $H_0 : b_k = \bar{\beta}_k$  on the  $\alpha\%$  significance level. Now, you compute the  $(1 - \alpha)\%$  confidence interval. Will  $\bar{\beta}_k$  lie inside or outside the confidence interval?

### Task 2

#### More about confidence intervals:

Suppose, computing the lower bound of the 95% confidence interval yields  $b_k - t_{\alpha/2}(n - K)se(b_k) = -0.01$ . The upper bound is  $b_k + t_{\alpha/2}(n - K)se(b_k) = 0.01$  Which of the following statements are correct?

- With probability of 5% the true parameter  $\beta_k$  lies in the interval -0.01 and 0.01.
- The null hypothesis  $H_0 : \beta_k = \bar{\beta}_k$  cannot be rejected for values  $(-0.01 \leq \bar{\beta}_k \leq 0.01)$  on the 5% significance level.
- The null hypothesis  $H_0 : \beta_k = 1$  can be rejected on the 5% significance level.
- The true parameter  $\beta_k$  is with probability  $1 - \alpha = 0.95$  greater than -0.01 and smaller than 0.01.
- The stochastic bounds of the  $1 - \alpha$  confidence interval overlap the true parameter with probability  $1 - \alpha$ .
- If the hypothesized parameter value  $\bar{\beta}_k$  falls within the range of the  $1 - \alpha$  confidence interval computed from the estimates  $b_k$  and  $se(b_k)$  then we do not reject  $H_0 : \beta_k = \bar{\beta}_k$  at the significance level of 5%.

### Task 3

#### Goodness of fit:

- a) Show that if the regression includes a constant:

$$y_i = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_K x_{iK} + \varepsilon_i$$

then the variance of the dependent variable can be written as:

$$\frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2 = \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - \bar{\hat{y}})^2 + \frac{1}{N} \sum_{i=1}^N e_i^2$$

Hint:  $\bar{y} = \bar{\hat{y}}$

- b) Take your result from a) and formulate an expression for the coefficient of determination  $R^2$ .
- c) Suppose, you estimated a regression with an  $R^2 = 0.63$ . Interpret this value.
- d) Suppose, you estimate the same model as in c) without a constant. You know that you cannot compute a meaningful centered  $R^2$ . Therefore, you compute the uncentered  $R_{uc}^2$ :

$$R_{uc}^2 = \frac{\hat{\mathbf{y}}' \hat{\mathbf{y}}}{\mathbf{y}' \mathbf{y}} = 0.84$$

Compare the two goodness of fit measures in c) and d). Would you conclude that the constant can be excluded because  $R_{uc}^2 > R^2$ ?

#### Task 4

##### Regression with EViews:

In a hedonic price model the price of an asset is explained with its characteristics. In the following we assume that housing pricing can be explained by its size *sqrft* (measured as *square feet*), the number of bedrooms *bdrms* and the size of the lot *lotsize* (also measured as *square feet*). Therefore, we estimate the following equation with OLS:

$$\log(\text{price}) = \beta_0 + \beta_1 \log(\text{sqrft}) + \beta_2 \text{bdrms} + \beta_3 \log(\text{lotsize})$$

Results of the estimation can be found in the following table:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.29704	0.65128	-1.99152	0.0497
LSQRFT	0.70023	—	7.54031	0.0000
BDRMS	0.03696	0.02753	—	—
LLOTSIZE	0.16797	0.03828	4.38771	0.0000
R-squared	0.64297	Mean dependent var		5.6332
Adjusted R-squared	0.63021	S.D. dependent var		0.3036
S.E. of regression	0.18460	Akaike info criterion		-0.4968
Sum squared resid	2.86256	Schwarz criterion		-0.3842
Log likelihood	25.86066	F-statistic		—
Durbin-Watson stat	2.08900	Prob(F-statistic)		0.0000

- Interpret the estimated coefficients  $\hat{\beta}_1$  und  $\hat{\beta}_2$ .
- Compute the missing values for *Std. Error* and *t-Statistic* in the table and comment on the statistical significance of the estimated coefficients ( $H_0: \beta_j = 0$  vs.  $H_1: \beta_j \neq 0$ ,  $j = 0, 1, 2, 3$ ).
- Test the null hypothesis  $H_0: \beta_1 = 1$  vs.  $H_1: \beta_1 < 0$ .
- Estimate the p-value for  $\hat{\beta}_2$  as close as possible and interpret.
- What is the null hypothesis of this specific *F-Statistic*? Compute the missing value and interpret the result.
- Interpret the value of *R-squared*.
- An alternative specification of the model that excludes the lot size as an explanatory variable provides you with values for the Akaike information criterion of -0.313 and a Schwartz criterion of -0.229. Which specification would you prefer?