

## 5th set assignments Introductory Econometrics

### Task 1

Practical example: Nerlove(1963): Returns to Scale in Electricity Supply

A complete discussion of the example can be found in Hayashi(2000), pp.60. To summarize the problem: We want to estimate a microeconomic cost function for the electricity supply sector. Therefore, consider the output of firm  $i$  to be determined by a Cobb-Douglas production function:

$$Q_i = A_i x_{i1}^{\alpha_1} x_{i2}^{\alpha_2} x_{i3}^{\alpha_3}$$

where  $Q_i$  is firm  $i$ 's output,  $x_{i1}$  is labor input for firm  $i$ ,  $x_{i2}$  is capital input and  $x_{i3}$  is fuel.  $A_i$  captures unobservable differences in production efficiency (or firm heterogeneity). The  $\alpha_j$  are the output elasticities with respect to factor  $j$ . The sum  $\alpha_1 + \alpha_2 + \alpha_3 \equiv r$  is the degree of returns to scale (note, that  $r = 1$  would imply constant returns to scale). The cost function associated with such a production function is:

$$TC_i = r \cdot (A_i x_{i1}^{\alpha_1} x_{i2}^{\alpha_2} x_{i3}^{\alpha_3})^{-1/r} Q_i^{1/r} p_{i1}^{\alpha_1/r} p_{i2}^{\alpha_2/r} p_{i3}^{\alpha_3/r}$$

where  $TC_i$  are the total costs for firm  $i$ . Linearizing by taking logs yields:

$$\log(TC_i) = \mu_i + \frac{1}{r} \log(Q_i) + \frac{\alpha_1}{r} \log(p_{i1}) + \frac{\alpha_2}{r} \log(p_{i2}) + \frac{\alpha_3}{r} \log(p_{i3})$$

where  $\mu_i = \log[r \cdot (A_i x_{i1}^{\alpha_1} x_{i2}^{\alpha_2} x_{i3}^{\alpha_3})^{-1/r}]$ . Now, let  $\mu = E(\mu_i)$  and define  $\varepsilon_i \equiv \mu_i - \mu$  so that  $E(\varepsilon_i) = 0$ . This  $\varepsilon_i$  represents the industry's average efficiency relative to the firm's efficiency. We can write for the total cost equation:

$$\log(TC_i) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \varepsilon_i$$

where

$$\beta_1 = \mu, \beta_2 = \frac{1}{r}, \beta_3 = \frac{\alpha_1}{r}, \beta_4 = \frac{\alpha_2}{r}, \beta_5 = \frac{\alpha_3}{r}$$

- Download the Excel spreadsheet **nerlove.xls** and import the data in EViews.
- Create the necessary variables to estimate the cost equation with OLS.
- Estimate the cost equation and interpret the parameters.
- Discuss, whether the assumptions of the classic linear regression model are satisfied.
- Which restriction is implied by the notion that the degree of returns to scale is constant ( $\alpha_1 + \alpha_2 + \alpha_3 \equiv r$ )?
- Test with two alternative procedures if the above restriction can be rejected.  
Hint: The restricted model reads as:

$$\log\left(\frac{TC_i}{p_{i3}}\right) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log\left(\frac{p_{i1}}{p_{i3}}\right) + \beta_4 \log\left(\frac{p_{i2}}{p_{i3}}\right) + \varepsilon_i$$

- Reformulate the model to get the log of the average costs as the independent variable. Then, create the new independent variable and estimate this model. What happens to parameter  $\beta_2$  in this new setting? Discuss the difference in the two values for  $R^2$  obtained in the total cost and the average cost equation.
- Test the hypothesis that there exist constant returns to scale ( $H_0 : r = 1$ )
- Eventually, make a residual series from your restricted model estimated previously. Plot the residuals against  $\log(Q_i)$ . Are you disturbed after looking at the plot?

Data set description:

- TC  $\hat{=}$  total costs
- Q  $\hat{=}$  output
- PL  $\hat{=}$  wage rate (factor price for labor)
- PF  $\hat{=}$  factor price for fuel
- PF  $\hat{=}$  rental price for capital