

Are Spot and Forward Exchange Rates Cointegrated?

Seminar Paper

for the Seminar

Topics in International Economics and Finance

of Professor Dr. Joachim Grammig

Summer Term 2007

Gersmann, Anna

Schwärzlocherstr. 32

72070 Tübingen

International Economics

7. Fachsemester

CONTENTS

List of Tables	1
1. Introduction	1
2. Theoretical Framework	3
3. Framework for the Cointegration Relationship	6
4. The Data	9
5. Precondition I : Are all series I(1)?	9
6. Precondition II: Are the Forward Premiums I(0)?	13
7. So What About the Cointegration Relationships?	15
8. Conclusion	17
Appendix A. Data Source	18
References	19

LIST OF TABLES

1 Jarque-Bera Test Results	21
2 Ng-Perron Test Results - Spot and Forward Exchange Rates	22
3 ADF and DF_{GLS} Test Results	23
4 Ng-Perron Test Results - Forward Premiums	24
5 Ng-Perron Test Statistics - Subperiods	25
6 Johansen Test Statistics - Cointegration Rank	26
7 Results of the $\chi^2(4)$ Test for Joined Significance of the Forward Premiums Forming a Basis of the Cointegration Space	27

ARE SPOT AND FORWARD EXCHANGE RATES COINTEGRATED?

1. INTRODUCTION

During the past decades, there has been a large discussion on exchange rate behavior and on the Risk Neutral Market Efficiency Hypothesis (RNMEH). Simple regression analyses of this hypothesis, according to which the forward exchange rate is a natural forecast of the future spot exchange rate, lead to contradictory results and was mostly rejected (see e.g. Clarida and Taylor, 1996).

While early researchers often concluded that exchange rates follow a random walk (Hamilton 1990, p.710), later, different approaches were developed to improve forecasts of exchange rates. Baillie and Bollerslev (1989) and Clarida and Taylor (1996) put forward the hypothesis that, while the RNMEH does not hold, still information contained in forward exchange rates can help to improve forecasts of spot exchange rates. They find a cointegration relationship with the presumably stationary forward premiums constituting a basis of the cointegration space. Out-of-sample forecasts led to contradictory results across different studies. Whereas Clarida and Taylor conclude that, exploiting the cointegration relationship, out-of-sample forecasts have improved and outperform the simple random walk, other studies still find the random walk to be superior (e.g. Diebold, Gardeazabal and Yilmaz, 1994). Furthermore, Diebold et al. (1994) find that the cointegration relationship can only be confirmed if no trend is included.

There have been other approaches, though, that investigate and do not reject a "model of stochastic segmented trends". In these studies, a Markov regime switching model is used to uncover trend-stationarity of

exchange rates while allowing for structural breaks (Engel and Hamilton, 1990; Klaassen, 2005). Therefore, the absence of a trend is not unanimously agreed upon in econometric literature.

Apparently, not all of the models mentioned can represent the true data generating process. Exchange rate behavior cannot be well described by a random walk, a cointegrated system that delivers improved forecasts and a Markov regime switching model at the same time. Hence, further research is needed to reveal which of these models, if any, suits best.

This paper revisits the research conducted by Baillie and Bollerslev (1989) and Clarida and Taylor (1996) on a 1:1 cointegration relationship between spot and forward exchange rates for two reasons. Firstly, the graphical analysis of spot and forward exchange rates suggests a cointegration relationship between them as they apparently move closely together. Second, some of the abovementioned studies on cointegration suffer from weaknesses. Partly, sample periods of the data used are very short (e.g. five years in the study of Diebold et al., 1994). Also, unit-root tests are employed that are by now known to have low power against near-unit-root alternatives and with serial correlation present in the error terms which can be observed in exchange rate series (e.g. Clarida and Taylor, 1996).

Fortunately, time has moved on and larger samples can be obtained. Also, econometric methodology has advanced. Especially the new class of "efficient unit-root-tests" promises to deliver more reliable results against local-to-unity alternatives than earlier tests. If, under these improved circumstances, the 1:1 cointegration relationship between spot and forward exchange rates cannot be rejected either, support is provided for earlier findings in that direction. If it is rejected, though, further research is encouraged that uses different approaches such as the regime switching model proposed by Engel and Hamilton (1990).

The organization of this paper is as follows. In section 2 the RN-MEH will be presented and a summary of the related discussion in econometric literature is given. Section 3 explains the empirical framework for the testable hypothesis of 1:1 cointegration developed by Clarida and Taylor (1996). After section 4 described the data, section 5 will discuss the suitability of different unit-root-test used and present the empirical results. The results for tests on the behavior of forward premiums are presented and discussed in section 6, the results for tests on cointegration in section 7. Section 8 concludes. Tables are presented at the end of the paper.

2. THEORETICAL FRAMEWORK

The Covered Interest Parity (CIP), its uncovered counterpart (UIP) and the combination thereof in the Risk Neutral Market Efficiency Hypothesis (RNMEH) have widely been discussed and tested before. According to these theories and assuming risk neutrality and rational expectations, there will be no arbitrage opportunities when investing in foreign markets as corresponding interest rates and exchange rates would adjust according to laws of demand and supply (Sarno and Taylor, 2006). The UIP relates the expected spot exchange rate depreciation to the interest rate differential of the corresponding markets:

$$(2.1) \quad s_{t+k}^e - s_t = i_t - i_t^*$$

where s_t is the natural logarithm of the spot exchange rate (domestic to foreign) in time t , s_t^e denotes the rational expectation of the spot exchange rate in logarithmic form; i_t^* is the foreign and i_t the domestic interest rate.

The CIP instead relates the interest rate differential to the difference between forward rates and spot exchange rates:

$$(2.2) \quad f_t^k - s_t = i_t - i_t^*$$

where f_t^k denotes the k -period forward exchange rate contracted at time t and $f_t^k - s_t$ could be interpreted as forward premium .

If both of these equations hold, then also the combination thereof must hold, which leads to the above mentioned RNMEH:

$$(2.3) \quad s_{t+k}^e - s_t = f_t^k - s_t$$

This leads directly to the forward rate being regarded as the "natural forecast" of the future spot exchange rate:

$$(2.4) \quad s_{t+k}^e = f_t^k$$

While the CIP has been tested and found to hold—if not continuously—at least on average (Taylor, 1989), the UIP (Engel and Hamilton, 1990) as also the RNMEH have mostly been rejected in empirical studies whose authors applied various approaches.¹

One approach often used in empirical works simply regresses the rate of depreciation ($s_{t+k} - s_t$) onto the lagged forward premium ($f_t^k - s_t$), including an intercept (see e.g. Bilson 1981):

$$(2.5) \quad s_{t+k} - s_t = \alpha + \beta(f_t^k - s_t) + \epsilon_{t+k}$$

where ϵ is a disturbance term. β is expected to equal unity under the RNEMH.

¹Sarno and Taylor give a literature review on this topic and cite among others Froot and Thaler (1990), Hansen and Hodrick (1980) and McDonald and Taylor (1991).

Most estimations of equations like 2.5, though, find a slope parameter β rather close to minus than to plus unity,² and very small coefficients of determinations.³ Nonetheless the slope estimates are mostly significantly statistically different from zero.

Yet, the low R^2 should not lead to the conclusion that there is no or very little information in forward rates that accounts for future spot exchange rate depreciation. Some of the counter-theory results are certainly due to statistical problems which arise from testing an equation such as 2.5. First of all, it is difficult to distinguish the nonstationary exchange rate series from a simple random walk. However, if the random walk hypothesis was true, the changes in spot exchange rates should be purely random, which would—whether the RNMEH was true or not—lead to an expected $\beta = 0$, as well as to the problem of β being underidentified. That is since under the RNMEH *and* assuming s to follow a random walk $s_{t+k}^e = f_t^k = s_t$, and therefore $f_t^k - s_t$ would be close to zero.⁴

Combining the statistical significance of the estimated slope parameters for equations like 2.5 with the problematic of testing and interpreting such equations, Clarida and Taylor (1996) conclude that—even if the exact RNEMH does not hold—there is some information contained in forward rates relevant to future spot exchange rates. They develop an empirical framework that shows why it is plausible to assume

²Froot (1990) averages the estimate value of β , using the results of 75 published estimates and finds it to be -0.88.

³Clarida and Taylor (1996) give an overview over these studies. E.g. Bilson (1980) finds a $R^2 = 0.029$ and a t-ratio of -4.57, Fama (1984) finds R^2 between 0.00 and 0.04. Further examples can be reviewed in Cumby and Obstfeld (1984), Bekaert and Hodrick (1993).

⁴see Sarno and Taylor (2006, p.12 ff) for a more detailed discussion of this problematic.

a cointegration relationship between spot exchange rates and forward exchange rates.

3. FRAMEWORK FOR THE COINTEGRATION RELATIONSHIP

The cointegration framework developed by Clarida and Taylor (1996) is based on two assumptions, the first of which is that the spot exchange rate contains a unit-root and can be decomposed into a non-stationary and a stationary part (Beveridge-Nelson Decomposition):

$$(3.1) \quad s_t = m_t + q_t$$

where m_t denotes the unit-root part and q_t the stationary part of the process. For reasons of clarity and simplicity (but according to Clarida and Taylor without loss of generality) m_t is supposed to be a first-order autoregressive process:

$$(3.2) \quad m_t = \theta + m_{t-1} + e_t$$

where θ denotes a constant and e_t an error term.

Secondly, they assume the deviations from rational (mathematical) expectations γ_t^k to be well described by the difference between the forward rate f_t^k and the rational expectation of the spot exchange rate s_{t+k} :

$$(3.3) \quad \gamma_t = f_t^k - E(s_{t+k}|\Omega_t)$$

where $E(s_{t+k}|\Omega_t)$ denotes the mathematical expectation of s_t based on information available at time t (Ω_t). The whole cointegration framework hinges on the fact that deviations from rational expectations are stationary as will be shown below. By substituting 3.1 and 3.2 into $E(s_{t+k}|\Omega_t)$:

$$E(s_{t+k}|\Omega_t) = m_t + \theta k + E(q_{t+k})$$

and after rearranging 3.3:

$$f_t^k = \gamma_t^k + E(s_{t+k}|\Omega_t)$$

we get:

$$f_t^k = \gamma_t^k + m_t + \theta k + E(q_{t+k}|\Omega)$$

It is obvious that—based on the above mentioned two assumptions on the process of the spot exchange rate (3.1) and the deviation from rational expectations (3.3)—the forward rate f_t^k inherits the stochastic trend m_t from the spot exchange rate s_t via the rational expectation $E(s_{t+k}|\Omega_t)$. Therefore, subtracting s_t from f_t^k will eliminate this stochastic trend and—conditional on the fact that the deviation from rational expectations γ_t^k is stationary—the forward premium ($f_t^k - s_t$) will be stationary.

As a cointegration relationship is defined by the existence of a cointegrating vector a so that $a'y_t \sim I(0)$ (Hamilton 1994, p.574), the spot and each forward exchange rate will be cointegrated with cointegrating vector $a' = [1, -1]$.

$$\begin{aligned} a'y_t = f_t^k - s_t &= \gamma_t^k + m_t + \theta k + E(q_{t+k}|\Omega_t) - m_t - q_t \\ (3.4) \quad \Rightarrow f_t^k - s_t &= \gamma_t^k + \theta k + E(q_{t+k} - q_t|\Omega_t) \end{aligned}$$

with $y_t = [f_t^k, s_t]'$.

Clarida and Taylor further generalize the concept to a system of one spot-exchange rate and j forward exchange rates. If equation 3.4 holds for f_t^k , where k is an unspecified time to maturity of a forward exchange rate contracted in time t , the equation must also hold for each of the j forward rates: $f_t^k, f_t^{k+1}, \dots, f_t^{k+j-1}$. Therefore, using a result obtained by Stock and Watson (1988), in a system of one spot and j forward exchange rates that share exactly one common stochastic trend (m_t) there will be exactly j linear independent cointegrating vectors. Equation 3.4 implies that the j forward premia will provide a basis for the cointegration space. This leads to the expectation of $[1, -1]$ to be a cointegrating vector for each of the j cointegration relationships in the system.

If the implications about the cointegration relationship between the spot and forward exchange rates hold, the system of one spot exchange rate and j forward exchange rates can according to the Granger representation theorem (Engle and Granger, 1987) be represented in a Vector Error Correction Model (VECM).

If a system of two random variables s_t and f_t is cointegrated it cannot be estimated as Vector Autoregression (VAR) in differences as the matrix polynomial of the associated moving average representation will be singular and the coefficient estimates of the regression will be downward biased (Hamilton 1994, p.573; Kirchgaessner and Wolters, 2006, p.181). In a cointegrated system it is decisive for each variable to know how distant it is from the others in order to adjust the distance in case it is too close or too far away from a certain long-term equilibrium. Hence it is quite intuitive that, when there is a long-term equilibrium relationship between a set of variables, not only the changes of each variable determine the change of the others. If the distance between them is too high at present, it may well be that the change in one variable will be positive while the change in the other will be negative, so that both move back towards equilibrium—one from below and one from above. In this case the algebraic signs differ although the series move together and first differences do not contain sufficient information to explain the behavior of the series. In the VECM the stationary long term relationship between the cointegrated variables - the error correction term - is included in the VAR in differences in order to correct the error in the system that would occur if the long term relationship between the variables was neglected.

So far, the cointegrating relationship, does not say anything about the *causal* relationship between the variables. Yet, in case the spot exchange rate s_t was Granger caused, i.e. it is influenced by variables other than its own history $(s_{t-1}, s_{t-2}, \dots)$, there can be information

found in the forward premiums valuable for forecasting spot exchange rates. In this case, forecasts will be expected to improve if the system is modelled as VECM.

4. THE DATA

For the empirical analysis, I use the logarithmic form of nominal exchange rate of the Japanese Yen against the U.S. Dollar as also four corresponding forward rates with following times to maturity: one month, three months, six months and one year, denoted f_t^1 , f_t^3 , f_t^6 and f_t^{12} . The data is of daily, weekly, monthly and quarterly frequency. Using various frequencies the differences in test outcomes between the more and less volatile data can be investigated and a broader basis for conclusions is provided. The data covers the period from 1.1.1984 until 30.3.2007, delivering 6065 observations.

5. PRECONDITION I : ARE ALL SERIES I(1)?

Data Properties and Unit-Root Tests. For a system of variables to be cointegrated, all variables have to be integrated of the same order (Engel and Granger, 1987). To investigate whether this condition holds for the system of one yen/dollar spot exchange rate and four corresponding forward exchange rates, first the descriptive statistics of the series were analysed. Based on these results, a suitable unit-root test was chosen and applied to all series in levels and in differences.

All series have a kurtosis above 4,8 and the skewness lies around 1,5. In comparison, a normal distribution has a kurtosis of three and a skewness of zero. Also, with a p-value of zero, the Jarque-Bera test statistic rejects that any of the investigated series ($s_t, f_t^1, f_t^3, f_t^6, f_t^{12}$) is normally distributed. Results are reported in Table 1.

Therefore, a unit-root test must be applied that is consistent for non-normal error terms. Comparing between the parametric approach of the Augmented-Dickey Fuller test (ADF) and the non-parametric

approach of the Phillips-Perron test (PP), the PP is preferred here. Though both of these conventionally used test statistics test the null hypothesis of a unit-root ($\rho = 1$) against the general alternative of stationarity ($\rho < 1$), there are substantial differences between them.

The ADF accounts for correlation in the error terms by adding lagged differences to the test equation. The test is very sensitive to changes in lag length and loses power when many lags are added. Similarly, it loses power in case autocorrelation is present in the error terms, but too few lags are added to account for it.

The PP approach starts off from an AR(1) process. The test statistics are adjusted to autocorrelation in the error terms through a non-parametric estimate of the error variance. If autocorrelation is measured, the adjusted variance estimate s_{Tm}^2 is larger than the conventional estimate s^2 . The PP statistics are less sensitive to changes in lag length and therefore better suited for data with serially correlated error terms.

However, when the true parameter of the investigated model lies close to unity, both the PP and the ADF lack power.⁵ Yet, it is a stylized fact that exchange rates exhibit near-random-walk behavior. (Sarno and Taylor, 2006). Therefore a test with higher power against a local alternative to $\rho = 1$ would be more reliable here.

Ng and Perron (1996, 2001) have proposed two modifications of the PP to solve this problem. Firstly, they alter the PP statistics so that power to reject the null hypothesis of $\rho = 1$ is increased even in small samples. Secondly, they propose an autoregressive spectral density estimator instead of the error variance estimator proposed by Phillips and Perron. They argue that it works best to account for autocorrelation in the error terms (Maddala and Kim 1998, p.108ff).

⁵See Kirchgassner and Wolters (2006) for a more detailed discussion on the ADF and PP.

In contrast to the ADF and PP, the Ng-Perron test statistics use data that is GLS-demeaned or -detrended through a procedure proposed by Elliott, Rothenberg and Stock (1996), hereafter referred to as ERS. Those unit-root tests that use this procedure fall into the class referred to as "efficient unit root tests". In this class, the null hypothesis $\rho = 1$ is tested against a local-to-unity alternative $\rho = 1 + \frac{c}{n}$ with $c < 0$. Critical values are derived for $c=-7$ and $c=-13,5$ (Ng and Perron, 2001).

ERS also proposed, among others, a modified version of the ADF, the Dickey-Fuller GLS (DF_{GLS}) test. This test also has increased power against a local alternative. Still, the Ng-Perron tests are preferred in this study since its non-parametric approach is less sensitive to changes in lag length (Maddala and Kim 1998, p.109,113).

5.1. Empirical Results. The results of the Ng-Perron tests of the Yen-Dollar spot and forward exchange rates are reported in Table 2. According to the asymptotic critical values derived in simulations by Ng and Perron (2001) no series in levels can be rejected to contain a unit-root on any conventional significance level against the local alternative. For all differenced series, though, a unit-root can be rejected on the 1% significance level. The Schwarz Information Criterion (SIC) has been used for lag-length selection. Generally, the choice of lag length is a very sensitive issue. Different criteria have been proposed and modified and partly lead to contradictory results. The results obtained by the Ng-Perron test have therefore been tested to see whether they are robust to moderate changes in lag length, which they are.

Discussion of the Unit-Root Test Results. It should be noted though that there are some problems concerning the quarterly data. Firstly, the critical values reported by Ng and Perron are asymptotic, whereas the quarterly data only comprises 93 observations. Secondly,

the alternative is given by $\rho_1 = 1 + \frac{c}{n}$ which in this case equals $\rho_1 = 1 - \frac{7}{93} \approx 0,92$ — quite far away from unity for a local-to-unity analysis.⁶

To further increase the reliability of the conclusion (unit root present in levels) different unit root tests were conducted. Whereas the results of the DF_{ADF} were in line with the Ng-Perron Test, the ADF rejected the presence of a unit root in the quarterly data on a significance level of 1% when three lags were included (chosen on grounds of the SIC) against the alternative of a linear stationary process (i.e. $H_0 : \rho = 1$ and $H_1 : \rho < 1$). Results are presented in Table 3. The unit root could only *not* be rejected (on a 5% significance level) if one lag only was imposed—against the results of the SIC and a likelihood ratio test that was conducted to check for robustness of the lag length choice.⁷ Even though, for abovementioned reasons, I made the choice to rely on the Ng-Perron test rather than on the ADF, this result is worth mentioning. Firstly, the rejection of the unit root is so "strong" (on the 1% significance level). Secondly, the ADF is still frequently used in empirical work (Maddala and Kim, 1998). Furthermore, the lack of power leads to the expectation that the null hypothesis will be rejected too *seldomly*. The case found here is that, against this expectation, the null hypothesis actually *is* rejected.

It should also be considered, though, that exchange rates of high frequency exhibit high volatility. Therefore the aggregation over a whole quarter will, by smoothing the data, obscure much of the short-term movements in exchange rates. It could be concluded from these results

⁶In comparison, the daily data with 6065 observations is tested against the alternative of $\rho_1 = 1 - \frac{7}{6065} \approx 0,9988$. To reach the same alternative for the quarterly data, c must be set to 0,11. It should be taken into account that critical values become smaller in absolute value when c is smaller rendering rejection more likely.

⁷The *ADF* and *DF_{GLS}* have also been conducted for the daily, weekly and monthly data to see whether the results were in line with the Ng-Perron test, which they were. Results are also reported in Table 3.

that there is a long-term behavior of exchange rates which is different from its shorter-term behavior.⁸

In the following sections, for above mentioned reasons, I will rely on the results of the Ng-Perron test which does *not* reject the presence of a unit root in the levels of any investigated series. As it also *does* reject the presence of a unit-root in all differenced series, the yen/dollar spot and forward exchange rate series are assumed to be integrated of order one.

6. PRECONDITION II: ARE THE FORWARD PREMIUMS I(0)?

Empirical Evidence. As described in section 3 the second precondition (after establishing the nonrejection of the spot and forward exchange rates to be I(1)) for this particular cointegration framework with j cointegrating vectors [1,-1] is that the forward premiums $f_t^k - s_t$ are stationary. Remember that this result stems from equation 3.4:

$$a'y = f_t^k - s_t = \gamma_t^k + \theta k + E(q_{t+k} - q_t | \Omega_t)$$

Again, the Ng-Perron test was used to test the null hypothesis of a unit-root process ($H_0 : \rho = 1$) against a local alternative ($H_1 : \rho = 1 + c/n, c = -7$). An intercept was included, but no trend. The SIC was used for lag length selection.

The Ng-Perron test statistics reported in Table 4 indicate different behaviors when volatility of the data changes. In cases of lower frequency (monthly and quarterly) the presence of a unit-root was mostly

⁸Actually one could also consider the short-term volatility to obscure a less volatile—and maybe more predictable—process in the data of very low frequency. As mentioned above, there have been studies searching for "long swings" in the data (Engel and Hamilton 1990, Klaassen 2005) that reject the random walk in spot exchange rates in favor of a Markov regime switching model.

rejected, if only on the 10% significance level. For the quarterly three-month and six-month forward premiums the unit-root could even be rejected on a 5% significance level.

The results differ when the data is of higher frequency (i.e. daily and weekly). Here, the unit-root cannot be rejected on any conventional significance level with one exception. The one-month forward premium was rejected to contain a unit-root on the 1% and 10% significance level for the daily and weekly data respectively.

Discussion of the Forward Premium Behavior. These results are contradictory to those obtained by Clarida and Taylor (1996), who used weekly data from 1977 to 1990 and rejected the unit-root (using the ADF) for any forward premium. In fact, the behavior of the Dollar-Yen forward premiums has changed over different periods in time. In my data set, which covers the years between 1984 and 2007, test results change quite considerably when only subperiods (1984-1990, 1990-2000, 2000-2007) are investigated. Even though, in my data set, the unit-root can not be rejected for the weekly data (except for the one-month forward premium), the differences in the test-statistics are quite substantial. Values come quite close to (but still lie a little above) the 10% critical value for the first period (1984-1990) whereas evidence is very strong in favor of the unit-root during the second period.

Apparently, the forward-premium behavior has changed over the past decades. Maybe speculators risk to contract forward rates which diverge substantially and unexpectedly from mathematical expectations of spot exchange rates. Or, spot exchange rates became more volatile, rendering the formulation of expectations more difficult. But whatever the reasons, even though evidence for stationarity of daily and weekly data has been found during the seventies and eighties, afterwards, covering a larger timespan and therefore a larger sample, unit-root behavior of forward-premiums cannot be rejected.

Following these results, for daily and weekly data, a 1:1 cointegration relationship between spot and forward exchange rates as was proposed by Clarida and Taylor (1996) cannot be confirmed.

The question that remains to be answered is whether the system of $j + 1$ forward exchange rates and spot exchange rates are cointegrated with j forward premiums constituting the basis for the cointegration space if the data is aggregated over a month or a quarter as in these cases short-term volatility is reduced and a long-term equilibrium relationship might be revealed.

7. SO WHAT ABOUT THE COINTEGRATION RELATIONSHIPS?

Johansen tests to determine the cointegration rank, the number of linearly independent cointegrating vectors, for the system of one spot and four forward exchange rates lead to the results presented in Table 6. The λ_{max} -statistic analyses whether the cointegration rank is *exactly* r , the trace-statistic whether the cointegration rank is *at most* r in a system of k variables with $r < k$.

Of interest are the results obtained for the monthly and quarterly data,⁹ which are quite ambiguous. For the monthly data, the λ_{max} statistic does not reject the cointegration rank to equal two on the 5% significance level, whereas the trace-statistic cannot reject the null hypothesis of the rank being smaller or equal to three. Results for the quarterly data are similarly unclear. The λ_{max} -statistic does neither reject the null hypothesis of two nor of three cointegrating vectors, while the trace-statistic indicates that there is no cointegration relationship at all, i.e. it indicates $r = 5$. The rejection of the null hypothesis of $r \leq 4$ is contradictory to the assumption of a stochastic trend in the

⁹As expected on grounds of unit-root tests conducted in section 6, the λ_{max} -statistic rejects the null hypothesis of $r = 4$ on the 5% and 10% significance level for the data with daily and weekly frequency.

system.¹⁰ On a 10% significance level, the same contradictory result can be observed in the monthly data.

A likelihood-ratio test comparing restrictions on the cointegrating vector with the unrestricted alternative leads to the same results. It strongly rejects the null hypothesis of the four forward-premiums $(f_t^1 - s_t, f_t^3 - s_t, f_t^6 - s_t, f_t^{12} - s_t)$ forming a basis of the cointegration space. The likelihood-ratio statistic is asymptotically χ^2 distributed (Johansen, 1988). Results are presented in Table 7.

Overall, the Johansen λ_{max} -, trace- and likelihood ratio statistic reject the hypothesis developed in section 3, that in a system of one spot and four forward exchange rates the four forward premiums form the basis of the cointegration space, on a 5% significance level. The evidence opposes even the existence of four linear independent cointegration vectors in general.

It should be considered, though, that the Johansen tests suffer from some weaknesses. Among other problems, they are very sensitive to changes in lag length and provide spurious results in case the variables are not I(1) but are difficult to distinguish from an I(1) process. Maddala and Kim argue that residual based tests are more robust and should be used instead. The efficient-unit-root tests would be suitable for this data set. Yet, critical values would have to be derived for the residual-based cointegration test as the residuals used in these tests are estimated errors (Maddala and Kim 1998, p.220-22).

¹⁰Kirchgaessner and Wolters (2006, p.196) explain that "as a general rule it holds that for a cointegration rank r with $0 < r < k$ the system of k variables contains $k - r$ common stochastic trends and r linear independent cointegration vectors." They also discuss the result of $r = k$ to be contradictory to the assumption of a unit-root present in the system.

8. CONCLUSION

In this paper a larger sample than in previous studies and efficient unit-root tests have been used to analyse the behavior of yen/dollar spot and forward exchange rates. The hypothesis that four forward premiums provide a basis of the cointegration space in a system of the yen/dollar spot exchange rate and four corresponding forward exchange rates has been rejected. Whereby the evidence for a unit-root present in daily and weekly forward premiums was quite strong, the evidence for a unit-root in monthly and quarterly data was weaker. Still, the Johansen tests for the cointegrating rank reject the existence of four linear independent cointegrating vectors for all investigated levels of frequency.

Future research should apply more powerful cointegration tests to confirm the results obtained by the Johansen test in section 7. Also, the behavior of quarterly forward premiums should be studied more closely. Here, more structure might be found as suggested by the rejection of unit-root (on a 10% significance level).

Also, research investigating the one-month forward premiums might be constructive. For these evidence was found against the unit-root even for daily and monthly frequencies and it is quite plausible to assume that forward premiums behave differently whether they stretch over a horizon of one, three, six or twelve months.

APPENDIX A. DATA SOURCE

As starting point for the analysis I used the non-logarithmic spot and forward exchange rates series of daily frequency obtained from datastream (series: BBJPYSP, BBJPY1F, BBJPY3F, BBJPY6F, BB-JPYYF).

REFERENCES

- Bekaert, G., and R.J. Hodrick**, 'On Biases in the Measurement of Foreign Exchange Risk Premiums', *Journal of International Money and Finance*, 12, 115-38 (1993)
- Bilson, John F.O.**, 'The Speculative Efficiency Hypothesis', *Journal of Business*, 54 (Oct.1981), 435-51
- Baillie, Richard T. and Tim Bollerslev** 'Common Stochastic Trends in a System of Exchange Rates', *The Journal of Finance*, 44 (Mar. 1989), 167-181
- Clarida, Richard H., and Mark P. Taylor**, 'The Term Structure of Forward Exchange Premiums and the Forecastability of Spot Exchange Rates: Correcting the Errors', *The Review of Economics and Statistics*, 79 (Aug. 1997), 354-61
- Cumby, R.E., and M. Obstfeld**, 'International Interest Rate and Price Level Linkages under Flexible Exchange Rates: A Review of Recent Evidence', in J.F.O. Bilson and R.C. Marston (eds.): *Exchange Rate Theory and Practice, National Bureau of Economics Research Conference Report*, Chicago: University of Chicago Press, 121-51 (1983)
- Diebold, Francis X.; Gardeazabal, Javier and Kamil Yilmaz** 'On Cointegration and Exchange Rate Dynamics', *The Journal of Finance*, 49 (Jun. 1994), 727-35
- Engle, R. and C.W.J. Granger** 'Cointegration and error correction: representations, estimation and testing', *Econometrica*, 55 (1987), 252-76
- Engel, Charles, and James D. Hamilton**, 'Long Swings in the Dollar: Are They in the Data and Do Markets Know It?', *The American Economic Review*, 80 (Sep. 1990), 689-713
- Elliott, G., T.J. Rothenberg and J.H. Stock** 'Efficient Tests for an Autoregressive Unit Root', *Econometrica*, 64, (1996), 813-836
- Fama, E.F.**, 'Forward and Spot Exchange Rates', *Journal of Monetary Economics*, 14 (1984), 319-38
- Froot, Kenneth A.**, 'Short Rates and Expected Asset Returns', *NBER Working Paper*, 3247 (Jan. 1990)
- Froot, Kenneth A., and Richard H. Thaler**, 'Anomalies: Foreign Exchange', *Journal of Finance*, 47 (1992), 1461-84
- Hamilton, James D.**, *Time Series Analysis*, Princeton University Press, Princeton, US (1994)

- Hansen, Lars P., and Robert J. Hodrick**, 'Forward Exchange Rates as Optimal Predictors of Future Spot Exchange Rates', *Journal of Political Economy*, 88 (Oct.1980), 829-52
- Johansen, S.**, 'Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, 12 (1988), 231-254
- Kirchgässner, Gebhard und Jürgen Wolters**, *Einführung in die moderne Zeitreihenanalyse*, Vahlen, München (2006)
- Klaassen, Frank**, 'Long Swings in Exchange Rates: Are They Really in the Data?', *Journal of Business & Economic Statistics*, 23 (Jan. 2005)
- MacKinnon, James G.** 'Numerical Distribution Functions for Unit Root and Cointegration Tests', *Journal of Applied Economics*, 11 (Nov.-Dec. 1996)
- Maddala, Gangadharrao S. and In-Moo Kim**, Unit Roots, Cointegration and Structural Change, Cambridge University Press, Cambridge (1998)
- Ng, Serena and Pierre Perron**, 'Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power', *Econometrica*, 69 (Nov.2001), 1519-54
———, 'Useful Modifications to Some Unit Root Tests with Dependent Errors and Their Local Asymptotic Properties', *Review of Economic Studies*, 63, 435-65
- Osterwald-Lenum, Michael**, 'A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank test Statistics', *Oxford Bulletin of Economics and Statistics*, 54, (1993), 461-72
- Sarno, Lucio, and Mark P. Taylor**, *The economics of exchange rates*, Cambridge University Press, Cambridge, UK, 4th edition (2006)
- Stock, James and Mark Watson** 'Testing for Common Trends', *Journal of the American Statistical Association*, 83 (1988)
- Taylor, Mark P.**, 'Covered Interest Arbitrage and Market Turbulence', *Economic Journal*, 99 (June 1989), 376-91

TABLE 1. Jarque-Bera Test Results

	Daily	Weekly	Monthly	Quarterly	
y_t	Jarque-Bera	Jarque-Bera	Jarque-Bera	Jarque-Bera	p-value
s_t	3228,63	647.60	150,41	51,25	0,00
f_t^1	3185,05	638.81	148,37	50,56	0,00
f_t^3	3077,85	617.35	143,38	48,85	0,00
f_t^6	2903,12	582.30	135,26	46,05	0,00
f_t^{12}	2557,46	513.00	119,09	40,52	0,00

The null hypothesis is tested that each series is normally distributed. The Jarque-Bera test statistic follows a χ^2 distribution with 2 degrees of freedom. s_t denotes the logarithmic form of the yen/dollar spot exchange rate, f_t^1 , f_t^3 , f_t^6 , f_t^{12} are the corresponding 1-month, 3-month, 6-month and 12-month forward exchange rates.

TABLE 2. Ng-Perron Test Results - Spot and Forward Exchange Rates

	Daily			Weekly			Monthly			Quarterly		
	MZ_ρ	MZ_t	$\hat{1}$	MZ_ρ	MZ_t	$\hat{1}$	MZ_ρ	MZ_t	$\hat{1}$	MZ_ρ	MZ_t	$\hat{1}$
y_t												
s_t	0,23	0,28	0	0,13	0,13	1	-0,14	-0,12	1	-1,4	-0,72	3
f_t^1	0,23	0,29	0	0,12	0,13	1	-0,14	-0,11	1	-1,4	-0,71	3
f_t^3	0,24	0,29	0	0,13	0,14	1	-0,14	-0,12	1	-1,42	-0,72	3
f_t^6	0,24	0,3	0	0,14	0,15	1	-0,14	-0,11	1	-1,47	-0,73	3
f_t^{12}	0,23	0,28	0	0,14	0,15	1	-0,13	-0,11	1	-1,55	-0,76	3
Δs_t	-37,37***	-4,32***	17	-154,39***	-8,79***	10	-156,80***	-8,84***	1	-14,56***	-2,69***	2
Δf_t^1	-36,65***	-4,27***	17	-158,01***	-8,88***	10	-157,57***	-8,86***	1	-14,75***	-2,71***	2
Δf_t^3	-52,62***	-5,12***	14	-250,77***	-11,19***	7	-157,56***	-8,86***	0	-14,87***	-2,72***	2
Δf_t^6	-53,30***	-5,16***	14	-243,38***	-11,03***	7	-167,31***	-9,14***	0	-14,93***	-2,72***	2
Δf_t^{12}	-68,88***	-5,87***	12	-233,62***	-10,80***	7	-161,38***	-8,97***	0	-14,82***	-2,71***	2

*** denotes rejection on the 1% significance level. Critical values are given by Ng and Perron (2001). They are -13,8000, -8,1 and -5,7 for the 1%, 5% and 10% significance level of the MZ_ρ -statistic and -2,58, -1,98 and -1,62 for the MZ_t -statistic. c is set to -7. The data is GLS demeaned. The Schwarz Information Criterion is used for lag-length selection. $\hat{1}$ denotes the estimated lag length. and are the versions of the Phillips-Perron test statistics Z_ρ and Z_t modified by Ng and Perron. s_t denotes the logarithmic form of the yen/dollar spot exchange rate, $f_t^1, f_t^3, f_t^6, f_t^{12}$ are the corresponding 1-month, 3-month, 6-month and 12-month forward exchange rates.

TABLE 3. ADF and $DFGLS$ Test Results

y_t	Daily		Weekly		Monthly		Quarterly	
	t_{ADF}	t_{DFGLS}	t_{ADF}	t_{DFGLS}	t_{ADF}	t_{DFGLS}	t_{ADF}	t_{DFGLS}
s_t	-2,54	0,29	-2,62	-0,11	-2,59	0,13	-3,51**	-0,82
f_t^1	-2,53	0,29	-2,61	-0,10	-2,58	0,13	-3,50**	-0,81
f_t^3	-2,52	0,29	-2,60	-0,11	-2,57	0,14	-3,48**	-0,90
f_t^6	-2,50	0,30	-2,58	-0,11	-2,55	0,15	-3,45**	-0,83
f_t^{12}	-2,46	0,28	-2,55	-0,10	-2,52	0,15	-3,38**	-0,85
Δs_t	-75,09***	-10,45***	-11,84***	-11,84***	-26,05***	-5,55***	-3,99***	-3,98***
Δf_t^1	-76,67***	-10,31***	-11,88***	-11,88***	-25,98***	-5,60***	-4,00***	-4,01***
Δf_t^3	-76,22***	-12,65***	-11,80***	-11,80***	-26,02***	-7,18***	-4,00***	-4,02***
Δf_t^6	-75,97***	-13,33***	-11,79***	-11,80***	-26,09***	-7,19***	-4,00***	-4,01***
Δf_t^{12}	-77,39***	-15,55***	-11,78***	-11,80***	-26,16***	-7,15***	-3,98***	-3,91***

, * denotes rejection of the unit-root on the 5% and 1% significance level respectively. t_{ADF} is the t-statistic of the ADF, t_{DF-GLS} is the t-statistic of the $DFGLS$. Critical values are given by MacKinnon (1996). Lag length is selected using the Schwarz Information Criterion.

TABLE 4. Ng-Perron Test Results - Forward Premiums

y_t	Daily			Weekly			Monthly			Quarterly		
	MZ_ρ	MZ_t	$\hat{\imath}$	MZ_ρ	MZ_t	$\hat{\imath}$	MZ_ρ	MZ_t	$\hat{\imath}$	MZ_ρ	MZ_t	$\hat{\imath}$
$f_t^1 - s_t$	-21,03***	-3,23***	18	-6,50*	-1,79*	8	-5,89*	-1,7*	2	-5,82*	-1,68*	0
$f_t^3 - s_t$	-4,79	-1,52	12	-3,93	-1,38	8	-5,82*	-1,68*	1	-11,50**	-2,38**	2
$f_t^6 - s_t$	-4,11	-1,41	6	-3,29	-1,26	8	-6,41*	-1,77*	1	-8,33**	-2,03**	1
$f_t^{12} - s_t$	-4,04	-1,41	5	-3,88	-1,38	8	-7,27*	-1,90*	1	-7,79*	-1,97*	5

*, **, *** denote rejection of the unit-root on the 10%, 5% and 1% significance level respectively. Critical values are given by Ng and Perron (2001). c is set to -7 . Lag length is selected by the Schwarz Information Criterion.

TABLE 5. Ng-Perron Test Statistics - Subperiods

y_t	First Period		Second Period		Third Period	
	MZ_ρ	MZ_t	MZ_ρ	MZ_t	MZ_ρ	MZ_t
$f_t^1 - s_t$	-5,61	-1,48	-0,74	-0,34	-0,60	-0,51
$f_t^3 - s_t$	-2,86	-0,89	0,73	0,78	-0,34	-0,38
$f_t^6 - s_t$	-1,43	-0,51	0,93	1,03	-0,35	-0,37
$f_t^{12} - s_t$	-4,53	-1,19	0,98	1,16	-0,10	-0,15

Critical values are given by Ng and Perron (2001). They are -13,8000, -8,1 and -5,7 for the 1%, 5% and 10% significance level of the MZ_ρ -statistic and -2,58, -1,98 and -1,62 for the MZ_t -statistic. SIC was used for lag-length selection and c is set to -7 as throughout this study.

TABLE 6. Johansen Test Statistics - Cointegration Rank

H_0	Daily			Weekly			Monthly			Quarterly		
	λ_i	λ_{max}	Trace	λ_i	λ_{max}	Trace	λ_i	λ_{max}	Trace	λ_i	λ_{max}	Trace
$H_0 : r = 0$	0,28	1953,20**	4268,58**	0,29	406,59**	724,18**	0,41	144,7**	289,12**	0,38	43,30**	118,18**
$H_0 : r \leq 1$	0,26	1840,36**	2315,38**	0,18	246,49**	317,59**	0,27	87,37**	144,36**	0,29	30,99**	74,88**
$H_0 : r \leq 2$	0,07	452,07**	475,02**	0,04	51,48**	71,10**	0,12	33,80*	57,01**	0,19	18,83	43,89**
$H_0 : r \leq 3$	0,00	14,83*	22,95**	0,01	11,41	19,62*	0,05	15,10*	23,21*	0,15	13,80	25,06**
$H_0 : r \leq 4$	0,00	8,12*	8,12*	0,01	8,22*	8,22*	0,03	8,12*	8,11*	0,12	11,26**	11,26**

*, ** denote *rejection* of the H_0 on the 10% and 5% significance level respectively. λ_i denotes the i^{th} eigenvalue after all k eigenvalues were ordered from large to small where k is the number of variables in the system (here: $k=5$). One lag in first differences was included. Critical values are given by Osterwald-Lenum (1992, Table 1).

TABLE 7. Results of the $\chi^2(4)$ Test for Joined Significance of the Forward Premiums Forming a Basis of the Cointegration Space

	$\chi^2(4)$	p-Value
Monthly	95,72***	0,00
Quarterly	21,35***	0,00

*** denotes *rejection* of the null hypothesis that four forward premiums constitute the basis for the cointegration space of a system of one spot exchange rate and four forward exchange rates. The degrees of freedom result from the number of restrictions imposed (4).