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**PRICING FOREIGN EXCHANGE RATE RISK:
A GMM-BASED APPROACH**

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List of Symbols

Symbol	Description
p_t	asset price in period t
m_{t+1}	stochastic discount factor
	marginal rate of substitution
x_{t+1}	payoff in period $t + 1$
R_{t+1}	gross return in period $t + 1$
R_{t-1}^f	gross return on the risk-free rate
r_{t-1}^f	net return on the risk-free rate
β	subjective discount factor
$u(c_t)$	marginal utility of consumption in period t
ΔFX	change in the exchange rate
r^{DC}	net return in domestic currency
r^{FC}	net return in foreign currency
$r_{j,t}$	net excess return in domestic currency units on asset j
z_t	instrument variable
b	parameters to estimate
$g_T(b)$	vector of moment conditions
W	weighting matrix
I	identity matrix
L	number of countries included
n	number of assets included
i	currency i , either DEM, GBP or JPY
m	number of included assets
k	number of moment conditions
$\lambda_{i,t-1}$	price of foreign exchange risk, with $i = DEM, GBP, JPY$
$\lambda_{m,t-1}$	world market price of risk, risk aversion
$\lambda_{0,t-1}$	reflexion of the current level of the short interest rate relative to the current levels of the risk premia
M_t	marginal rate of substitution, pricing kernel
$r_{m,t}$	excess return on the world market portfolio
$r_{n,t}$	excess return on asset n
$r_{i,t}$	excess Eurocurrency rate i , excess return on a Eurocurrency deposit i

Symbol	Description
χ^2	Chi squared distribution
Z	instrument matrix
l	number of used instruments
ϕ_i	coefficient to estimate referring to country i
ϕ_m	coefficient to estimate referring to the world market
δ	coefficient to estimate
u_t	deviation from the MRS
ε_t	vector of residuals
re	matrix of excess returns
inst	matrix of instruments
moments	matrix of moment conditions
eps	matrix of residuals
\mathbf{h}_t	vector of transformed residuals
useries	results from elementwise vector multiplication

Abbreviations

GMM	Generalized Method of Moments
APT	Asset Pricing Theory
APM	Asset Pricing Model
PPP	Purchasing Power Parity
BPE	Basic Pricing Equation
CBM	Consumption Based Model
WLLN	Weak Law of Large Numbers
CAPM	Capital Asset Pricing Model

1 Introduction

Numerous recent studies witness an increasing degree of financial market integration since the end of the last century. For instance, the economists Maurice Obstfeld (1995) or Robert Flood and Andrew Rose (2003) provide convincing scientific evidence as to this widespread observation. Consequently, the investors' focus of interest, their concerns about adequate portfolio and risk allocation, has to cope with these developments. More and more countries enter the international asset markets' stage, however, these participants still compute their returns on the background of their respective national currencies. In other words, despite the more integrated character of financial markets, one domain is often vehemently defended: national currencies. These exercise a crucial impact on returns on distinct international investments.

In this regard, Bernard Dumas and Bruno Solnik address the question whether foreign exchange risk is priced in international equity and currency markets. They develop a conditional model and apply the econometric framework of the *Generalized Method of Moments (GMM)*. The underlying idea of "conditioning" information suggests that instruments introduce a certain, consciously selected choice of information into the estimation. Based on an optimization procedure, *GMM* picks parameters which best satisfy a set of *moment conditions*. This set of *moment conditions* can be interpreted as a set of imposed structural rules which has been derived from economic theory. Admittedly, the choice of instruments crucially impacts on the estimation's outcome and any subsequent inference we draw from the obtained parameters. Nevertheless, on account of the parsimonious character of the applied econometric method, Dumas and Solnik's model proves to be quite convenient.

This text is subdivided into three major parts: the theoretical background, the estimations and finally the obtained results. More precisely, it is organized as follows: initially, I will provide some theoretical details of *asset pricing theory (APT)* and thereby illustrate the motivation to construct such a model in an international context. By doing so, a discrimination between *classic* and *international asset pricing models (APM)* is presented. The econometric methodology, to wit the essentials of *GMM*, are briefly covered in the subsequent part. Turning to the estimation and my replication, the second major part, I describe the respectively used data and then continue with the issue of conditioning information. At length, I will elaborate on the central *moment conditions*. In the third part, the obtained results will be presented and in the end I will conclude. The question whether foreign exchange risk is priced in international financial markets will serve as a guide throughout this essay. Basically my estimation does not only replicate Solnik and Dumas's procedure, key results reveal the same conclusions: the *international conditional APM* cannot be rejected. Hence, besides a premium on world market risk, returns on international investments are remarkably determined by time-varying prices of foreign exchange risk.

2 Theoretical Background and Motivation

The purpose of this section is to set the stage for Solnik and Dumas' model among whose main ingredients count the *conditional moment restrictions*. These will be derived and explained in the following. If capital

markets were fully integrated, investors would be equally rewarded in real terms – entirely independent from their country of origin and the investments’ destination. In such a setting, the concept of *purchasing power parity (PPP)* would hold. This parity relation implies that exchange rate movements merely reflect national differences in inflation in order to grant identical real returns across countries. Yet, empirical evidence suggests that the *PPP* condition is far from being satisfied [SMcL 2003, 47,48]. Capital flows across borders can thus not even out real returns globally. In view of the different currencies in which investors compute their returns, an additional source of risk seemingly emerges. The flip side of the coin however implies, that returns which are denominated in different currencies can provide a hedge, depending on the composition of their respective portfolio. Thus, as already pointed out by Adler and Dumas (1983), the violation of *PPP* causes investors of distinct national backgrounds to form different, theoretically optimal portfolios [AD 1983, 929]. In their article, Bernard Dumas and Bruno Solnik abstract from local currency inflation risk and attribute changes in *PPP* deviations to real exchange rate changes [DS 1995, 448]. Nevertheless, we can infer that each investor assumes two distinct types of risk. First, the traditional market risk which is represented by an asset’s covariance with the world market portfolio. Second, a rather specific risk which is attributed to the covariance of the asset with exchange rates. Against the background of this rather intuitive idea, a formal derivation will be presented in the following.

2.1 Suitable Specifications of the *Basic Pricing Equation*

Starting from a quite general level, I will provide some basic details of APT. These will be of help to facilitate the understanding of Solnik and Dumas’ version of an international APM. John Cochrane [Coc 2005] contends that any asset price obeys the *basic pricing equation (BPE)*, which is depicted in (1).¹

$$p_t = E_t[m_{t+1}x_{t+1}] \quad (1)$$

Thereafter, the quoted asset price at point in time t equals the conditionally expected payoff in the subsequent period x_{t+1} , adjusted by the stochastic discount factor m_{t+1} . Due to the fact that gross returns R_{t+1} refer to the ratio of future payoffs x_{t+1} and current prices p_t , one easily obtains the return representation $E_t[m_{t+1}R_{t+1}] = 1$. The latter suggests that an investment of one currency unit today, will yield a return R_{t+1} in the subsequent period, with a discount factor balancing the relation. Conceptually, the *BPE* and its return representation rely on an optimization procedure. Within the framework of the *consumption based model (CBM)* the maximization of utility over time has to be performed, while taking conventional budget constraints into account. Thereafter, the stochastic discount factor m_{t+1} comprises the *marginal rate of substitution (MRS)* multiplied by the *subjective discount factor* β and can hence be stated as displayed in (2).²

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \quad (2)$$

¹Conditional Moments will be denoted as $E_t[\dots]$ or $Cov_t[\dots]$, serving as an equivalent of $E[\dots|\Omega_t]$ or $Cov[\dots|\Omega_t]$ which frequently occurs in the literature, as well.

²Cochrane utters that, for simplicity, m_{t+1} is often referred to as the *MRS*, only [Coc 2005, 7]

The *MRS* indicates to what extent a consumer is willing to sacrifice present consumption in order to reach a higher consumption level in the next period. Hence, a vague idea about today's and tomorrow's state of the economy implicitly enters the *BPE* in the guise of the expected level of future consumption. With reference to the international setting, the *MRS* is expressed as M_t . Dumas and Solnik pick up the expected return representation, apply it to a risk-free asset, yielding a gross return of $R_{t-1}^f = (1 + r_{t-1}^f)$, and thereby obtain equation (3). As the authors' portfolio incorporates a wider set of international investment opportunities, whose nominal returns are initially denominated in various foreign currencies, the ensuing changes will be detailed below.

$$E_{t-1}[M_t(1 + r_{t-1}^f)] = 1 \quad (3)$$

After John Cochrane, a *zero-cost portfolio* can be created by pursuing the following strategy [Coc 2005, 9]: at $(t - 1)$, we borrow one domestic currency unit, if necessary convert it into foreign currency and invest it into a risky asset j . At this point, foreign exchange³ dynamics come into play. Provided that exchange rates are stochastic, an appreciation of the foreign currency $\Delta FX > 0$ (corresponding to a depreciation of domestic currency) augments the foreign investment's reward. Accordingly, a depreciation of the foreign currency $\Delta FX < 0$ deteriorates the foreign investment's outcome. Dumas and Solnik's comprehensive notation already contains these exchange rate effects. One period later, in terms of domestic currency units, the foreign asset will yield a gross return of $R_{j,t} = (1 + r_{j,t}^{DC})$. To exhibit the additional currency component more explicitly, the return on the latter investment can be expressed as $R_{j,t} = (1 + r_{j,t}^{FC} + \Delta FX)$. Further, we have to bear the repayment of domestic debt in mind which, which amounts to $R_t^f = (1 + r_t^f)$. In total, the strategy offers an excess return of $(R_{j,t} - R_t^f) = (1 + r_{j,t})$ in period t and owes its name to the fact that no liquidity on behalf of the investor is required in period $t - 1$. The expected excess return representation of the *BPE*, as suggested by Solnik and Dumas, is given by equation (4). These specific formulations of the *BPE* will adopt central roles in our econometric model.

$$E_{t-1}[M_t(r_{j,t})] = 0 \quad (4)$$

These specific formulations of the *BPE* will adopt central roles in our econometric model.

2.2 The Marginal Rate of Substitution's meaning in the International Context

John Cochrane's interpretation of the *BPE* strongly relies on the inferences he draws from the *MRS*, particularly in the context of the *CBM*. As distinct from the *CBM*, *factor pricing models* just substitute marginal utility growth by a linear term to capture the *MRS* [Coc 2005, 149]. This is exactly what happens in the model at hand which will be described in this section. It is still suitable to allude once in a while to the neat reasoning provided by the *CBM* due to Cochranes concession that "all factor models are derived as specializations of the consumption-based model." [Coc 2005, 151].

To come to terms with the international setting of their model, Dumas and Solnik suggest their version of

³In compliance with Dumas and Solnik, direct quotation is used in this text. Thereafter, the exchange rate states the amount of domestic currency units which is required to purchase one foreign currency unit [SMcL 2003, 4, 741].

the *MRS* by equation (5).

$$M_t = \frac{[1 - \lambda_{0,t-1} - \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} - \lambda_{m,t-1} r_{m,t}]}{1 + r_{t-1}^f} \quad (5)$$

Recurring to the *CBM* for a short moment, it would yield an intuitive interpretation of this *MRS*: in equilibrium, the *MRS* equals the relative price of consumption today and tomorrow⁴. Indeed, the right-hand side features time-varying $\lambda_{i,t-1}$ and $\lambda_{m,t-1}$, which stand for the market prices of risk due to currency i and due to the world market portfolio m [DS 1995, 448, 449]. The numerator includes $r_{n+i,t}$ and $r_{m,t}$ which denote the return on these currencies⁵ and the world market portfolio m , while $\lambda_{0,t-1}$ appears in order to satisfy equation (3). John Cochrane develops his *BPE* on the basis of a single asset in which the consumer can decide to invest. For this reason, the price of this very asset would have to adjust in order to satisfy the optimality condition, provided that only homogenous investors exist, with a predetermined level of consumption. Cochrane emphasizes the validity of the following equation to arrive at the optimal portfolio choice [Coc 2005, 5].

$$p_{t-1} u'(c_{t-1}) = E_{t-1} [\beta u'(c_t) x_t] \quad (6)$$

Hence, the marginal loss from purchasing the asset, that is, the fictional “price” one has to pay in terms of foregone period $t - 1$ utility is displayed on the left-hand side. It has to equal the conditionally expected marginal gain, as stated by the right-hand side. A short transformation yields the more comprehensive form of the *BPE*, thus $p_{t-1} = E_{t-1} [\beta \frac{u'(c_t)}{u'(c_{t-1})} x_t]$. From the viewpoint of macroeconomic theory, (6) alludes to a standard, intertemporal *Euler* condition [Mark 2001, 84]. In this line of argument and with reference to the risk-free rate, transforming the *BPE* into its return representation suggests that $E_{t-1} [m_t] = \frac{1}{R_{t-1}^f}$. The macro interpretation is relatively straightforward and will provide helpful insights into the international version of the *MRS* as depicted in (6). Hence the left-hand side of the latter equation, $E_{t-1} [m_t]$, requires the *MRS* between $t - 1$ and t consumption to amount to $\frac{1}{R_{t-1}^f}$. This indicates the relative price of period t consumption in terms of period $t - 1$ consumption.

At this point, we can project insights gained from the macro interpretation on the international *MRS*. The analogy to (5) hints at the fact that the numerator of the relative price of consumption would decline under the premise that all $\lambda_{i,t-1}$ and $\lambda_{m,t-1}$ were positive. However, as suggested by Adler and Dumas, different national currency denominations might provide a hedging opportunity and thus, investors do not require to be “compensated” for bearing additional risk. Instead, they pay a certain price to obtain the hedge. Hence $\lambda_{i,t-1}$ ⁶ can be negative, as well, depending on the respective setting. To recall, Solnik and Dumas appoint $\lambda_{i,t-1}$ to refer to the *world prices of exchange rate risk* and $\lambda_{m,t-1}$ to represent the *world price of market risk* [DS 1995, 448]. These risks are sold and bought on world financial markets. In our case, they determine the inherent character of asset j . The price of market risk $\lambda_{m,t-1}$ serves as a mirror image of the expected compensation which is required by an investor who adopts the risk. To illustrate the underlying reasoning as to $\lambda_{i,t-1}$, a comparison between a domestic and foreign investment into the respective risk-free assets is

⁴The macroeconomic explanation is provided below

⁵One may imagine the return on a Eurocurrency deposit

⁶The index i refers to different currencies, such as DEM, GBP and JPY in our case.

drawn. Based on equation (5), the MRS and hence the relative price of period t to period $t - 1$ consumption falls [rises] if the foreign currency is considered risky [provides a hedge]. This derives from the fact, that an investor has to be rewarded for bearing additional risk [pays to obtain the hedge], thus $\lambda_{i,t-1}$ is positive [negative]. Apart from the currency risks, comparative statics suggest that an increase in the price of world market risk ($\lambda_{m,t-1}$) would require a lower equilibrium MRS. In the next section, expected returns and their relation to the prices of risk will be considered.

2.3 Differentiation of the *Expected Excess Return* in the Classic and the International APM

The most fundamental question from the investor's viewpoint arises with respect to expected excess return of an asset, especially in the international context. A short sketch will overview some selected, core contributions of how the pricing of international assets evolved in the literature. The choice is based on the extent to which these models might facilitate the understanding of Dumas and Solnik's approach.

For a short moment, I will abstract from the international setting to introduce the *Capital Asset Pricing Model (CAPM)* in its expected return-beta version [Coc 2005, 152]. The depiction is reduced to most crucial details, as papers frequently just refer to this model, which is attributable to Sharpe (1964) and Lintner (1965) [Coc 2005].

$$E[R_i] = R^f + \beta_i(E(R_m) - R^f) \quad \text{with} \quad \beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \quad (7)$$

Thus, the expected return on asset i is composed by two elements: a risk-free and a risky part. The risky component contains the excess return of the market portfolio over the risk-free rate, denoted as $(E(R^m) - R^r)$. Indeed, the contribution of this risky part to the return of asset i is weighted by a factor β , which in turn consists out of the ratio of the asset's covariance with the market return relative the variance of the market return.

Returning to the international level, the CAPM serves as a basis for many approaches. Adler and Dumas (1983) bring forward a "multi-beta CAPM" [AD 1983, 950] which results from their utility maximization against an international background. Hence, they obtain a CAPM which incorporates several covariances terms. The latter refer to the covariances between the asset and the considered countries' inflation rate apart from its covariance with the market. In defiance of their consideration of various risk premia, their procedure differs significantly from the one presented by Solnik and Dumas. In 1991, Campbell Harvey bases his model on a *conditional* version of the CAPM. He makes an attempt to explain the cross-sectional variation in different countries' expected returns by their distinct degree of risk exposure. The purpose of his paper is to refine the *conditional CAPM* in an international setting. To achieve this aim, he applies the same econometric technique which will be elaborated on below, to wit (*GMM*). In Harvey's study, he defines *country risk* as the conditional covariance of the country's excess return $r_{j,t}$ to the world market portfolio's excess return $r_{m,t}$ [Har 1991, 111,112]. Thereafter, the conditionally expected excess return $E_{t-1}[r_{j,t}]$ on asset j is supposed to vary proportionally to the covariance. In this sense, the *world price of covariance risk*, that is, the expected reward received by the investor for bearing one unit of covariance risk, assumes the role of the proportionality factor. Equation (8) features this proportionality factor as the ratio of conditionally

expected world market excess returns $E_{t-1}[r_{m,t}]$ to its conditionally expected variance.

$$E_{t-1}[r_{j,t}] = \frac{E_{t-1}[r_{m,t}]}{\text{Var}_{t-1}[r_{m,t}]} \text{Cov}_{t-1}[r_{j,t}, r_{m,t}] \quad (8)$$

Despite the fact that Dumas and Solnik will pick up some essential details of Harvey's model, its major shortcoming looms large: the author fails to distinguish between the two separate risk premia. In other words, Harvey applies the *classic conditional APM* to the international context and thereby cannot provide a distinction between the currency risk and the world market price of risk. Admittedly, Harvey's coefficient of the covariance, termed *world price of covariance risk* will shape up in Dumas and Solnik's model as $\lambda_{m,t}$. For this reason, we can infer that his specification lacks the currency component. To complete the international model and hence to capture the currency premium as well, an additional term is required.

Solnik and Dumas (1995) choose the "multi-beta'CAPM" [AD 1983, 950] as a point of departure. Yet, their main contribution as to the discussion of *international APMs* rely on two aspects. First, by admitting the *prices of risk*, $\lambda_{i,t-1}$ and $\lambda_{m,t-1}$, to vary over time. Second, by basing their approach on *conditional* first moments as can be ascertained in (9). This equation describes $E_{T-1}[r_{j,t}]$, that is, the conditional return on a portfolio or asset j in excess of the risk-free rate of the currency in which returns are measured. Being a core assumption, it needs to be stated, that the model is set up against the background that $m = n + L + 1$ risky assets beyond our base-currency deposit exist. At length, n refers to a variety of equities or portfolios, L indicates the number of nonmeasurement-currency-deposits. Finally, the world-market portfolio enters as the last asset ⁷.

$$E_{T-1}[r_{j,t}] = \sum_{i=1}^L \lambda_{i,t-1} \text{Cov}_{t-1}[r_{jt}, r_{n+i,t}] + \lambda_{m,t-1} \text{Cov}_{t-1}[r_{jt}, r_{m,t}] \quad (9)$$

Hereafter, the conditional excess return on j is feeded by two sources. On the one hand, we can identify various currency risks which enter in the guise of the sum over all "world prices of exchange rate risk" [DS 1995, 448], represented by $\lambda_{i,t-1}$. These are multiplied by the covariance of the respective currency and the asset j . On the other hand, world market risk undergoes the analogous treatment. As proposed by the authors, the "world price of market risk" [ibid.] $\lambda_{m,t-1}$, weights the covariance between j and the world market portfolio.

At this point, the distinction between the two sources of risk comes to the fore: currency risk and world market risk. To compensate for these, two distinct premia are required as already stated above⁸. Comparative statics suggest that an increase in the price of world market risk $\lambda_{m,t-1}$ will augment the excess return. As to the price of foreign exchange risk $\lambda_{i,t-1}$, we have to differentiate: if currency i was considered risky [as a hedge] and thus implies a positive [negative] price, the excess return increases [declines] if the price raises in absolute terms. It deserves to be mentioned, that an investment into a foreign currency deposit, risk-less at first sight, actually contains a exchange rate risk, which may be disadvantageous [advantageous]. Thus, we should not succumb the fallacy to think about a Eurocurrency deposit as entirely independent from risk .

⁷The formal derivation of equation (9) is presented in the annex.

⁸To clarify the terminology, "Risk premia are equal to market prices of risk times the ex ante measures of risk (covariances with exchange rates and with the market portfolio)" [DS 1995, 465]

The main goal of Solnik and Dumas' article is to differentiate between the *Classic APM* and the *International APM*. They intend to provide evidence about the exchange rate risk being priced in international asset markets. The discrimination between the *classic* and the *international* version is tested by for instance expressing one model as the reduced form of the other. Put differently, equation (9) represents the *international APM*, however, setting all $\lambda_{i,t-1}$ equal to zero does the trick. We thereby obtain equation (10) as the *classic APM*.

$$E_{T-1}[r_{j,t}] = \lambda_{m,t-1} \text{Cov}[r_{jt}, r_{m,t} | \Omega_{t-1}] \quad (10)$$

This specification looks quit familiar once recalling Harveys approach. Indeed, it is the limiting case provided that the price of all currency risk is zero or the covariance itself is not substantial. Thus, the reward is limited to the compensation of world market risk. Dumas and Solnik state that the *classic APM* might also apply to a situation in which investors immediately exchange returns into consumption [DS 1995, 448]. Further, it is of importance to note that, contrasting with Adler and Dumas (1983), here, local currency inflation risk is neglected and we concentrate on deviations in the real exchange rate. At this stage, the APT part apparently ends. However, theory unambiguously enters into the econometric setup of Dumas and Solnik's model and therefore, complementary remarks will occur in the subsequent sections.

2.4 The Generalized Method of Moments (GMM)

The purpose of this section is to briefly review the statistical method of *GMM*, proposed by Dumas and Solnik to estimate the parameters. Hereafter, at the outset stands a set of k model-based moment equations, specifying a relationship which is presumably valid in the population. Economic theory suggests these relationships which in the presented case rely on the *BPE*. Thus, the *BPE* as depicted in its expected return (3) and excess return representation (4), play – with inserted (*MRS*) as specified in 2 – a decisive role. The latter are called *conditional moment restrictions* or just *moment conditions* according to the context [Coc 2005, 190]. They comprise data as well as the parameters which we intend to estimate, sensibly specified in compliance with theory. The notion "conditional" is crucial since it reminds us of the fact that expectations in (4) crucially depend on information available at $t - 1$. These need to be incorporated, thus, we have to handle the transition from *conditional* to *unconditional* moments. Cochrane tackles the problem by means of the following devices [Coc 2005]. Applying the *law of iterated expectations* to "condition down" is the most simple solution, but not always appropriate on its own. Therefore, he suggests to let "scaled payoffs" enter the model by multiplication of the *BPE* with an instrument z_t . He then pretends dealing with the *unconditional* version. The procedure can be justified by reading the changed equation as a *managed portfolio* which mimics an investor's decision to allocate more or less money to an investment opportunity - based on some signal captured by z_t . The presentation of his third, indeed the most subtle solution is deferred to the concrete estimation as it is an integral part of Solnik and Dumas procedure. It is associated with the problem to cope with time-varying parameters in a *factor pricing model* and still bears strong analogy to the previous suggestion.

Once having overcome the last difficulty, we have to anchor the theoretical concept of *moment conditions*

in the real world. Put differently, the transition from sample means – which originate from a small selected part of reality – to population moments has to be justified. The *weak law of large numbers (WLLN)* performs this function, as thereafter sample moments converge in probability to population moments. [Hay 2000, 95]. Now, $g_T(b)$ denotes a vector of $(k \times 1)$ *sample moment conditions* which in turn may also be defined as *sample means* of the pricing errors u_t . The parameters to estimate enter the equation in the guise of b . *GMM* sets up a quadratic form $g_T(b)'Wg_T(b)$ and minimizes the ensuing scalar. The $(k \times k)$ matrix W is required to be a symmetric and positive definite weighting matrix. Initially, we use the identity matrix I . Thus, the first stage estimate of vector b results from minimizing the sum of squared average pricing errors while attributing equal weight to each asset within the sample moment conditions. As pointed out by Cochrane, the estimate \hat{b}_1 is *consistent* and *asymptotically normal* [Coc 2005, 191]. Further refinement is achieved by using the optimal weighting matrix in a second-step estimation which is based on the variances and covariances of the pricing errors across assets from the first. Hence, one obtains a second-stage estimate \hat{b}_2 which is *consistent*, *asymptotically normal* and *efficient*. The property of *efficiency* ensues from the particular weighting matrix \hat{S} which down-weights similar assets, exhibiting a high correlation of pricing errors.

Requirements, Advantages and further Advantageous Properties The application of the *WLLN* requires our data to be *stationary*, implying that its joint distribution – of for instance z_t and z_{t-j} – merely depends on j , but not on t [Hay 2000, 98]. *GMM* features a great advantage, namely that "the estimator is specified without benefit of any distributional assumption" [Gre 2000, 448]. The reasoning behind *GMM* proposes, that all assets are priced simultaneously in an attempt to minimize the mean squared error. Departing from a situation in which the number of parameters to estimate exceeds the number k of moment conditions, we can test whether the overidentifying restrictions hold true by using a *J-statistic*. At this point, Cochrane's assertion displays the *GMM*'s advantages: the underlying χ^2 *distribution* of the *J-statistic* as well as the *Standard Wald test* help to verify the joint significance of all coefficients. However, one caveat turns out to be worth mentioning: the comparison of alternative model specifications, as featured by Dumas and Solnik, can only be drawn on the basis of a commonly used weighting matrix⁹. Apart from this, we abstain from frequently required assumptions of variables being *homoskedastic*, *i.i.d.* or *normally distributed* [Coc 2005, 199]. Further details follow below in the more practical part of this text. This next section provides an illustrating, complementary application of the formal background presented up to this point.

3 Replicating Dumas and Solnik's Estimation

At the core of Dumas and Solnik's article lies the question of whether exchange rate risk is priced in international asset markets. To test this hypothesis, the authors differentiate between four specifications of their model: characteristic of the *international* type are the *world prices of exchange rate risk* $\lambda_{i,t-1}$ whereas its counterpart, the *classic* model, markedly sets these to zero. Each type can further assume a *conditional* and an *unconditional* format. Considering the *unconditional* version, we obtain *time-invariant* λ_0 and prices

⁹Later, I will return to this so-called D-statistic.

of risk, λ_m and λ_i . These immediately derive from estimating the model via *GMM*. As opposed to this, in case of the *conditional* version, $\lambda_{0,t-1}$, $\lambda_{m,t-1}$ and $\lambda_{i,t-1}$ vary over time and result from linear combinations of instruments with estimated coefficients. My replication is reduced to testing the core hypothesis with regard to the distinct four specifications. Calculations are performed in *GAUSS*. Supplementary tests, as performed by Dumas and Solnik, will be covered, provided that they are closely related to the main issue. The core message of the paper can be summed by the fact that, indeed, currency risk premia are significant in international asset markets. My results support this finding as well. The underlying data is presented in the next section. Subsequently, further elaboration on the concepts, the exact statement of the employed *moment conditions*, further details about the procedure and the comparison between my and Solnik and Dumas' results is presented.

3.1 Underlying Data and Descriptive Statistics

At this stage, I will describe the data which enters the estimation. As it is an attempt to retrace Dumas and Solnik's model, only fundamental discrepancies as to the choice of variables will be pointed at¹⁰. In the end, I will briefly discuss core data issues which are of importance for the inference we aim to draw from our estimations.

Solnik and Dumas dispose of data ranging from March 1970 to December 1991 which amounts to effectively 262 observations. Admittedly, our sample starts later, to wit in August 1978 on account of availability reasons. It covers a time span up to February 2007 and therefore furnishes us with 343 return observations. Monthly excess returns of eight assets in logarithms can be subdivided into two groups: equity and currency. To obtain excess returns, one subtracts from the respective market's return (expressed in USD) the Eurodollar one-month risk-free rate. Equity measures refer to *Morgan Stanley country indices (MSCI)* quoted in Germany, the UK, Japan and the US. Correspondingly, excess currency returns originate from Eurocurrency one-month interest rates on Eurocurrency deposits. These are taken for the Deutsche Mark (DEM), the Pound Sterling (GBP) and the Japanese Yen (JPY). The latter are compounded by exchange rate changes with respect to the USD. As the final asset, the world equity return is captured by the MSCI world, undergoing the same treatment as previous equity measures. In compliance with Dumas and Solnik's procedure, I express 1% as 0,01 within the framework of return data.

Despite the fact that the detailed explanation of the instruments' relevance for the idea of conditioning information is deferred until a later section, their data will be introduced at this point. Excess returns are computed in the same fashion as already shown. Thereafter, besides a constant and a January dummy, this group of annual data contains the following lagged logarithms: the equity world market's excess returns,¹¹ and the US bond yield which is computed as the difference between the 10year treasury rate and the one-month Eurodollar rate. Hence, in contrast to Dumas and Solnik, their US dividend yield is now replaced by a US default premium, namely the spread between Baa-rated and Aaa-rated corporate bonds. Finally,

¹⁰Descriptive statistics and further detailed results can be found in the appendix

¹¹I.e. monthly MSCI world returns, as cited above, are merely annualized. In the context of instruments, the annualized lagged excess return will from here on be refer to as $r_m(-1)$.

the Eurodollar rate itself serves as an instrument. To grant comparability with Solnik and Dumas as far as possible, in the context of instruments, 1% is cited as 1.

On account of the fact that the samples capture divergent historical periods of turmoil on financial markets and switches in exchange rate regimes, small deviations in mean returns and standard deviations of the data series can easily be justified. Descriptive statistics hence show that conclusions from the replication rely on a relatively solid basis. However, discrepancies may ensue from this data issue.

One major concern refers to the stationarity of the series which is required by a correct application of *GMM*. One may appease this by hinting at the use of excess returns ¹². Actually, unit root tests might more forcefully prove stationarity. Yet, Solnik and Dumas present autocorrelations and contend that serial dependence in the instruments vanishes over a two-year horizon [DS 1995, 453].

As to the model's conclusion, the appropriate choice of instruments plays a crucial role. We aim for suitable instruments, featuring some "predictive ability" [DS 1995, 454] of foreign market returns. Solnik and Dumas substantiate their decision by performing OLS regressions and various tests. The reasoning behind hints at their assumption that the selected US variables mirror the US business cycle. This, in turn, is related to the business cycle of other countries. Thus, the selection of US data represents a legitimate set of instruments.

3.2 Conditioning Information and Instrumental Variables

To translate the idea of conditioning information into common language, it might be regarded as the econometricians decision about which little knowledge packages of reality he likes to incorporate into his model. In our case, he attempts to imitate the reasoning of an investor. That is, the econometrician intends to feed his theoretical model with this very set of insights and background information as he suspects it to captures some fundamental ingredients of prices of risk.

The major contribution of Solnik and Dumas, by means of which their model can be distinguished from previous approaches, comes to the fore once considering their handling of time-varying *prices of currency* and *world market risk* ($\lambda_{i,t-1}$ and $\lambda_{m,t-1}$). Put differently, the conditioning of information – as well as the usage of instruments – take center stage in order to show that currency risk is priced in international asset markets. To achieve this aim, $\lambda_{i,t-1}$ and $\lambda_{m,t-1}$ have to be further specified with the help of instruments as described by equation (11).

$$\lambda_{0,t-1} = -\mathbf{Z}_{t-1}\delta \quad \lambda_{i,t-1} = \mathbf{Z}_{t-1}\phi_i \quad \lambda_{m,t-1} = \mathbf{Z}_{t-1}\phi_m \quad (11)$$

Thereafter, \mathbf{Z}_{t-1} is the ($l \times 1$) vector of instrumental variables which is combined with time t data in the estimation, bearing in mind the necessary lag. The entire set of instruments would be captured by a ($T \times l$) matrix \mathbf{Z} with l indicating the number of instruments (six in our case). The task of *GMM* in the *international conditional* specification remains to determine the coefficient vectors ϕ_i , δ_i and ϕ_m . Of course, the latter are

¹²This procedure might cause multicollinearity among the variables. Yet, central results remain unaffected as purported by Dumas and Solnik [DS 1995, 453].

not time-varying. Rather, the product of ϕ_i ¹³ with the instrument matrix Z yields the series of *prices of currency risk*. Analogous reasoning applies to ϕ_m and the *world market risk*. Remarkably, the ensuing λ_t are time varying and may alternatively be interpreted as rewards per unit of risk for exchange rate risk and market risk aversion, respectively. Vector λ_0 warrants the validity of (3). According to Solnik and Dumas it can be seen as a "pure reflexion of the current level of the short rate of interest $[r_{t-1}^f]$ compared to the current levels of the risk premia" [DS 1995, 449].

The procedure mimics Cochrane's third suggestion to solve the problem in the context of conditioning information. Hence, the "dependence of parameters ... on variables in the time-t information set" [Coc 2005, 144] is explicitly modeled. To some extent, the set of factors in our *international APM* is expanded. A facilitated description of the proceedings might suggest that we actually shift the estimation's mechanism to a remote spot. At first sight, we obtain time-varying λ_i , λ_m and λ_0 , the *prices of risk*. However, behind the curtain, the whole apparatus of time invariant estimates ϕ_i , δ and ϕ_m paired with the instrument matrix Z shape them. Once again it turns out that, on behalf of the model's sensibility as regards the choice of instrumental variables, caution has to be exercised. In the case at hand, the US dividend and bond yield, the annual return on world equity, the eurodollar rate, the constant and the January dummy have been decided on and distinctly impact on our resulting prices of risk.

3.3 Implementing Conditional Moment Restrictions into GAUSS

The implementation of Solnik and Dumas model into *Gauss* requires the statement of the *moment restrictions* in the *international* and *classic* specification, while each of these occurs in a *conditional* and *unconditional* version. First, I will describe the *international conditional* variant in detail, as it turns out to be the most sophisticated model. This procedure will facilitate the depiction of the remaining three types which can be interpreted as reduced versions of the most complex one.

The International Conditional Specification To recall, the *international conditional* version of our *APM* comprises all time-varying prices of risk $\lambda_{i,t-1}$, $\lambda_{m,t-1}$ and $\lambda_{0,t-1}$, hence the attribute *international*. The λ s are in turn shaped in compliance with (11) to construct their linear relationship to the instruments. Thus, the attribute *conditional* occurs. In the case at hand, we are challenged to estimate 30 parameters. This number ensues from the five λ s ($\lambda_{DEM,t-1}$, $\lambda_{GBP,t-1}$ and $\lambda_{JPY,t-1}$ referring to various currency prices of risk, $\lambda_{m,t-1}$ capturing the world market risk and $\lambda_{0,t-1}$, described below). Each single λ contains six (referring to the number of instruments l) unknown parameters which turns out clearly by hinting at (11). These unknowns are indeed determined by the underlying parameter vectors ϕ_{DEM} , ϕ_{GBP} , ϕ_{JPY} , ϕ_m and δ . Our estimation relies on 54 *moment conditions*. Namely, the number of instruments ($l = 6$) multiplied by the number of considered assets plus one ($m+1$) [DS 1995, 451]. The latter statement leads us to a more detailed consideration which provides insights for instance into the asserted number of *moment conditions*.

¹³The index i refers to the three returns on currencies considered: ϕ_{DEM} , ϕ_{GBP} and ϕ_{JPY}

The set of *conditional moment restrictions* is build up by two parts: one single equation which will be presented below as (14) and a vector of m equations as alleged by (15). To start with the single equation, it relies on the expected return representation (3) which is self-evidently expressed as a conditional expectation. Once we abstract from the conditional expectation operator in (3) and switch terms around, we can define u_t as the innovation in the *MRS* [DS 1995, 450]. The outcome is exhibited in (12).

$$u_t = 1 - M_t(1 + r_{t-1}^f) \quad (12)$$

Algebraic transformation and the insertion of the *MRS* from (5) leads to a more extensive version as depicted in (13).

$$u_t = -Z_{t-1}\delta + Z_{t-1}\phi_{DEM}r_{DEM,t} + Z_{t-1}\phi_{GBP}r_{GBP,t} + Z_{t-1}\phi_{JPY}r_{JPY,t} + Z_{t-1}\phi_m r_{m,t} \quad (13)$$

By definition, the conditional expectation of u_t , while conditioning refers to information available at point in time $t - 1$, has to equal zero as displayed in (14). This equation will become the first element of the *conditional moment restriction* and stands for the "plus one" as mentioned above.

$$E_{t-1}[u_{j,t}] = 0 \quad (14)$$

It is worth annotating that the latter equations feature scalar values as they all refer to one particular point in time. The temporal aspect will come into play during the optimization procedure.

To deduce the second component of the *conditional moment restrictions*, the implication of the *BPE* in expected excess return representation is useful. For a single asset j , it is of help to define $(h_{j,t} = r_{j,t} - r_{j,t}u_t)$ which can be rewritten as $(h_{j,t} = r_{j,t}(1 - u_t))$. In compliance with (12) the latter can be changed and after taking conditional expectations again, one arrives at $E_{t-1}(h_{j,t}) = E_{t-1}(r_{j,t}M_t(1 + r_{t-1}^f))$. The right-hand side product is composed by three factors, with merely the third being known at point in time $(t - 1)$. Thus, the *BPE* as presented in (4) allows to state that $(E_{t-1}(h_{j,t}) = 0)$. As a matter of fact, this assertion has to holds true for all assets j , which enables us to form an $(mx1)$ vector out of the entire set of assets. Equation (15) picks up this reasoning. Again, a short remark shall clarify that this vector distinctly refers to one single point in time.

$$E_{t-1}[\mathbf{h}_t] = 0 \quad (15)$$

At this stage we dispose of all necessary ingredients to build up the *conditional moment restrictions*. The conditional expectation of the innovation to the *MRS* as depicted in (14) adopts the first position in a new $((m + 1)x1)$ vector of residuals, called ε_t . Remaining positions are filled with the $(mx1)$ vector stated in (15) to end up with $(\varepsilon_t = (u_t, \mathbf{h}_t))$. It is possible to combine the statements expressed in (14) and (15) to infer that $E[\varepsilon_t | \mathbf{Z}_{t-1}] = 0$. Hereafter, it is suggested that the conditional expectation operator (E_{t-1}) ¹⁴ can be replaced by explicitly conditioning on information contained in vector \mathbf{Z}_{t-1} . Put differently, as explained in the previous section, the econometrician chooses an extract of real information available at time $(t - 1)$, trying to mimic the investor. This attempt assumes a tangible format in the formulation of the instrument vector \mathbf{Z}_{t-1} . Recurring to econometric rules, we can use Hayashi's established criteria for a proper set of

¹⁴Solnik and Dumas use the notation $(E(\dots|\Omega_{t-1}))$ instead of the shorter but equivalent (E_{t-1}) .

instruments which requires that an instrument has to be *orthogonal* to the error term [Hay 2000, 191,200]. Moreover, at this point, Cochranes suggested solutions to the "conditioning down" problem come to the fore. To estimate the time-varying λ_{DEM} , λ_{GBP} , λ_{JPY} , λ_m and λ_0 , based on time-invariant ϕ_{DEM} , ϕ_{GBP} , ϕ_{JPY} , ϕ_m and δ , we rely on his third suggestion. To recall, this solution deals with an expanded set of factors. After Cochrane, forming a *linear discount factor model* leads to an interpretation as *scaled factors*. Simplifying the matter: this is, what happens behind the surface. Yet, he proposes to handle *scaled returns* as payoffs to *managed portfolios* [Coc 2005, 144]. For this reason, the instrument vector \mathbf{Z}_{t-1} appears noticeably another time, in line with the *orthogonality condition* and this time outside ε_t . It turns out that we can follow Cochrane's advice and legitimately transformed the *conditional international* specification into an apparently (unconditional) model. This is exhibited in (16) due to the fact that the information we condition on implicitly enters the equation via \mathbf{Z}_{t-1} [Coc 2005, 132].

$$E[\varepsilon_t \mathbf{Z}_{t-1}] = 0 \quad (16)$$

The transition from conditional to unconditional moments relies on his proposal to "incorporate conditioning information while still looking at unconditional moments instead of conditional moments" [Coc 2005, 134]. Under the terms of the *WLLN* it is possible to consistently estimate population moments, which allows us to state equation (16) as (17) with \mathbf{Z} featuring a $(l \times (T - 1))$ matrix and ε representing a $((T - 1) \times (m + 1))$ matrix ¹⁵.

$$\mathbf{Z} \varepsilon = 0 \quad (17)$$

The last equation (17) actually occurs in a rather general format. It offers an ample scope to cover all four specifications due to the fact that the changing characters are hidden in the details behind. Now, these variants will be presented.

Implementation of the International Conditional Model in GAUSS Due to the fact that detailed optimization procedures based on the concept of *GMM* are performed by a GAUSS tool, our remaining task is to furnish the program with the *moment conditions* in matrix notation. Dumas and Solnik resign from providing the more sophisticated depictions. Thus, I will bridge this gap and sketch the implementation.

First of all, we will create a return matrix (`ret`) which consists out of m ($= 8$) columns, featuring the time series of excess returns on the n ($= 4$) equity assets, the L ($= 3$) Eurocurrencies and world equity. Further, an instrument matrix (`inst`) is set up, whose l ($= 6$) columns represent the six above mentioned time series of instruments. To account for the conditioning of information, and the thereby introduced timelag between returns and instruments, the first row of (`re`) ¹⁶ and the last row of (`inst`)¹⁷ are cut off. We aim for the estimation of $\overrightarrow{\phi_{DEM}}$, $\overrightarrow{\phi_{GBP}}$, $\overrightarrow{\phi_{JPY}}$, $\overrightarrow{\phi_m}$ and $\overrightarrow{\delta}$ which are $(l \times 1)$ vectors. The $\overrightarrow{\lambda}$ s are constructed by multiplication of the respective $\overrightarrow{\phi}$ with (`inst`) to obtain them as $(T - 1)$ vectors. Based on the extensive depiction

¹⁵Indeed, we lose one observation by using one time lag in (\mathbf{Z}) as proposed in 16.

¹⁶which transforms to a $[(T - 1) \times (m)]$ matrix

¹⁷now being of the dimension $[(T - 1) \times (l)]$

of the *MRS*'s deviation as stated in (13), the time-series vector (`useries`) results from elementwise vector multiplication as shown in (18).

$$(\text{useries}) = \overrightarrow{\lambda_{o,t-1}} + \overrightarrow{\lambda_{DEM}}.\text{re}(\cdot, 5) + \overrightarrow{\lambda_{GBP}}.\text{re}(\cdot, 6) + \overrightarrow{\lambda_{JPY}}.\text{re}(\cdot, 7) + \overrightarrow{\lambda_m}.\text{re}(\cdot, 8) \quad (18)$$

Consequently, $h_{j,t}$ ¹⁸ is now calculated by $\mathbf{h} = \text{re} - \text{re}.\text{useries}$. The residual matrix `eps` of the dimension $[(T-1) \times (m+1)]$ combines vector `useries` and matrix `h`. Once again, elementwise multiplication is required to set up the *moment restrictions* as depicted in (19). This allusion to equation (17) represents a $[(T-1) \times ((m+1)l)]$ matrix, in our context $[(T-1) \times (9 \times 6)]$, and thus justifies the contention about 54 moment restrictions. It ensues from the elementwise multiplication of each column of matrix `eps` with the instrument matrix `inst`. Due to the fact that `eps` is made up of $(m+1)$ columns, we perform the column with `inst` multiplication nine times, while each computation results in a $((T-1) \times 6)$ matrix according to the six instruments.

$$(\text{moments}) = \text{eps}[:, 1].\text{zet} \sim \text{eps}[:, 2].\text{zet} \sim \dots \sim \text{eps}[:, 9].\text{zet} \quad (19)$$

GAUSS Implementation of the Other Specifications Briefly, the three other variants will be described. With regard to the *international unconditional* version, instruments do not play a role. Time-invariant λ_{DEM} , λ_{GBP} , λ_{JPY} , λ_m and λ_0 , occur as the five coefficients which shall be estimated. We can abstract from the `moments` matrix and instead hand `eps` over to `GAUSS`. Besides, in this sense, reducing the dimension of `re` is not necessary. The five estimates rely on nine moment conditions as suggested by the $[Tx(m+1)]$ dimension of `eps` in this case.

The *classic conditional* model bears stronger analogy to the *international conditional* version, it only disregards currency returns. Hence, we reduce our focus to the estimation of $(lx1)$ vectors $\overrightarrow{\phi_m}$ and $\overrightarrow{\delta}$. Time-varying $\overrightarrow{\lambda_m}$ and $\overrightarrow{\lambda_0}$ are obtained while we go through the same steps as above. Hence, $[(m+1)l]$ or = 54 *moment conditions* deliver twelve estimates.

Finally, the most simple *classic unconditional* version refers to the *international unconditional* specification. We follow a similar procedure with the exception of currencies considerations. Put differently, we estimate time-invariant λ_m and λ_0 on the basis of $(m+1) = 9$ *moment conditions* suggested by `eps`.

In the last section, I will present the results and provide an outline of about further tests performed by Solnik and Dumas.

4 Estimation Results and Further Tests

In the following, the outcome of Dumas and Solnik's, as well my own estimation of the four specifications will be presented and to some extent a comparison will be drawn. As a matter of fact, I will focus on most central results and refrain from more detailed interpretations of the estimated individual coefficients. This procedure can be justified by alluding to the fact that, in both estimations, conducted t-statistics of individual

¹⁸which now transforms to a $[(T-1) \times (m)]$ matrix `h`"

coefficients deliver relatively poor results. Instead, I want to adopt a rather holistic perspective and test one model specification against the other.

4.1 Conditional Models are Supportive of Pricing Risk

Dumas and Solnik come to the conclusion that exchange rate risk is indeed priced on international currency markets and thus permit the existence of time-varying λ s [DS 1995, 459]. This inference is based on the fact that the *international conditional APM* cannot be rejected. In econometric terms, the J-statistic's p-value amounts to 0,2276, indicating that the null hypothesis – which contends that the overidentifying restrictions hold true – cannot be rejected. Moreover, several time-invariant coefficients ϕ turn out to be significant¹⁹. My own estimation confirms this conclusion by featuring a p-value of 0,15966. The various world prices' of risk behavior can be visualized. To facilitate the comparison, deviations from the estimated *unconditional* values are plotted, neglecting the seasonal pattern caused by the January dummy. Dumas and Solnik contend that prices of exchange risk, $\widehat{\lambda}_i$, exhibit a more volatile pattern than $\widehat{\lambda}_m$. From the surface, my estimations confirm this statement. In the authors' depiction, world market price of risk, $\widehat{\lambda}_m$, assumes positive and negative values, whereas my graph features only positive values. This discrepancy can derive from two sources. First, the way we compute these values relies on subtracting the estimated *unconditional* value. If the latter is quite small, the positive values may result. Second, as already ascertained, the periods which are captured by the respective estimation overlap only in part and give rise to discrepancies due to essential historical events²⁰. The question arises, whether the obtained pattern can be interpreted. Intuition suggests that market risk premia are positive, as they depict the an investor's expected compensation, or return, for assuming risk in general. As to premia for currency risk, the a priori premise of a positive sign can be deceptive and hence not necessarily appropriate. On the one hand, they reward the investor for bearing this additional risk. On the other hand, the aspect of hedging against cannot be neglected. Consequently, premia on foreign exchange risk may be positive and negative. Based on these considerations, the depiction of "excess" $\widehat{\lambda}_m$ and $\widehat{\lambda}_i$ from my estimation turns out to be consistent with theory. Finally, from my own attempt, it is possible to conclude that the price of Japanese currency risk exhibits the greatest variations. As regards the *classic conditional APM*²¹, Dumas and Solnik reject the null of correctly stated overidentifying restrictions on account of a J-statistic's p-value of 0,0053 [DS 1995, 461]. Again, my estimation leads to the same inference by yielding a p-value of 0,0096913. If we delete the currencies from the return data, the resulting model bears resemblance to Harvey's approach which has been cited above.

The architecture of the *conditional* models allows for a comparison between the *classic* and the *international* model. Based on the fact that these are nested, a so-called "D-test" created by Newey and West, and suggested by Cochrane [Coc 2005, 206] can be performed. Actually, this test tears the significance of the exchange rate pricing into question, by proposing the *classic* version as the null hypothesis, and the *interna-*

¹⁹All results and graphs concerning the *international conditional model* of Solnik and Dumas, as well as of my own estimation, are presented in the Appendix "*International Conditional Model*"

²⁰A third possibility might hint at some error in the calculation of the model, which hopefully is not true

²¹Results are featured in the Annex "*Classic Conditional Model*"

tional specification as its alternative. The underlying idea suggests to estimate the *classic conditional* model, as the restricted model, however, using the weighting matrix S which originates from the estimation of the *international conditional* model [DS 1995, 460]. Due to this test, the authors once again reject the "hypothesis of zero price on exchange rate risk in the conditional version of the international APM" [DS 1995, 460]. Thus, their result proves to be sustainable. Furthermore, they conduct tests to determine which assets are responsible for the rejection of the *classic* APM against the background of a *conditional* model. The authors find that eurocurrency deposit rates of the deutsche mark and the Japanese yen contribute chiefly to the rejection of the *classic conditional* model [DS 1995, 462]. Finally, they run a test whose null contends the time invariance of λ_i and λ_m are time-invariant. Small P-values lead to a rejection of the hypothesis [DS 1995, 465] and thus confirm previous results.

4.2 Unconditional Models

Concerning the *unconditional* specifications of the *APM*, the following results are obtained²². Solnik and Dumas reject the *international unconditional* model on a conventional significance level of 5percent due to a p-value of 0,049. Yet, this value is close to the threshold. My estimation arrives at a p-value of 0,9807463 and thereby I cannot reject the null hypothesis that the model is correct. The comparison of these results hints at the most pronounced discrepancies between the two attempts. Facing the *classic unconditional* version, the results do not contradict each other: Solnik and Dumas cannot reject the correctness of overidentifying restrictions with regard to a p-value of 0,161, whereas my estimation features a p-value of 0,7068.

One has to concede that the rejection of the *unconditional international* version in my attempt would have reinforced the validity of the *conditional international* APM, which can be appointed to be the protagonist of Solnik and Dumas' paper. Nevertheless, in the previous section it has not been rejected and therefore we can attribute some credibility to the *international conditional* version.

4.3 Supplementary Robustness Tests

Generally, risk premia are constituted by the product of market prices of risk and ex ante measures of risk, that is, an asset's covariance with the market portfolio or exchange rates. The applied methodology of *GMM* did not demand for the concrete formulation of second moments. So far, this appeared to be advantageous, however, it implies that we cannot gain an impression about the potential size of the risk premia. It is of interest, to draw a comparison between a linear statistical model, which assumes these second moments to be constant, and the *unconditional intern.* and *classic* model which does not impose any restrictions on them. Dumas and Solnik find that the models reveal similar estimates. Put differently, the premise about second moments' behavior does not exercise a substantial impact on the results [DS 1995, 467].

Most central concerns as to the correctness of the econometric methodology deal with the question, whether the requirements of *GMM* are met. For instance, the incorporated data has to be stationary. In this context, the Eurodollar rate can render our results quite vulnerable. Yet, Dumas and Solnik estimate the model

²²These can be verified in the Annex "*Unconditional Models*"

based on first differences and reach the same conclusions [DS 1995, 467]. Further, other variables enter as excess returns and can thus negate the problem. Serial Dependence in the sample moments might become a contentious issue, as well. The authors, however, reject this concern as well as the matter of the finite sample size or the choice of the measurement currency and other variables. Basic inferences turn out to be robust. Besides, Dumas and Solnik find that international stock and foreign exchange markets reveal a certain degree of integration against the background of the *intern. conditional* APM [DS 1995, 467]. In the end, they test an *intertemporal* against the *international* APM and conclude that exchange risk premia and intertemporal risk premia are equivalent.

5 Conclusion

In their article, Solnik and Dumas intend to throw some light on the question whether *exchange rate risk* is priced in international financial markets. The authors provide convincing evidence in favor of this hypothesis by developing an econometric model based on the *Generalized Method of Moments*. Thus, while following this procedure, the replication of their model benefits from its parsimonious character. Thereafter, we can refrain from a detailed specification of rate of returns' second moments and focus on the estimation of an investor's *MRS*. Further, first moments are not explicitly estimated. These facilitating properties derive from the fact that less parameters need to be estimated which in turn increases the power of the test.

Nevertheless, the flip side of the coin implies that we cannot gain insights into the relative size of the foreign exchange risk premia in relation to the common premium for bearing market covariance risk. Solnik and Dumas' main contribution relies on a better understanding of the conditionally expected returns, with the foreign exchange-risk premia featuring an integral, significant part.

Their core results could be reconstructed and basically come to the same conclusions. Yet, due to the fact that the data's sample periods overlap only to a certain extent, their different caption of highly influential historical events on world financial markets might explain the small discrepancies mentioned in the text. Another perspective might indicate that, indeed, the pricing of foreign-exchange risk premia turns out to play a significant role on the background of various circumstances. Put differently, the main results of Solnik and Dumas' paper prove to be time-invariant.

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Appendix

Derive the expected return representation (4) from the return representation (3)

Given by [DS 1995, 449]

$$E_{t-1}[M_t(1 + r_{t-1}^f)] = 1 \quad (3)$$

with (r_{t-1}^f) being non-stochastic, we obtain

$$(1 + r_{t-1}^f)E_{t-1}[M_t] = 1$$

change to

$$E_{t-1}[M_t] = \frac{1}{1+r_{t-1}^f} \quad (3a)$$

or equivalently

$$E_{t-1}[M_t] = \frac{1}{R_{t-1}^f} \quad (3a)$$

Basic pricing equation in returns representation as presented by Cochrane [Coc 2005, 14]

$$E_{t-1}[M_t R_t] = 1$$

adding and subtracting $E_{t-1}[M_t R_{t-1}^f]$ on the left-hand side yields

$$E_{t-1}[M_t R_{t-1}^f] - E_{t-1}[M_t R_{t-1}^f] + E_t[M_t R_t] = 1$$

tear into the brackets

$$E_{t-1}[M_t R_{t-1}^f] + E_{t-1}[M_t (R_t - R_{t-1}^f)] = 1$$

change to

$$E_{t-1}[M_t (R_t - R_{t-1}^f)] = 1 - E_{t-1}[M_t R_{t-1}^f]$$

with R_{t-1}^f being non-stochastic

$$E_{t-1}[M_t (R_t - R_{t-1}^f)] = 1 - R_{t-1}^f E_{t-1}[M_t]$$

according to (3a)

$$E_{t-1}[M_t (R_t - R_{t-1}^f)] = 1 - \frac{R_{t-1}^f}{R_{t-1}^f}$$

using excess return notation

$$E_{t-1}[M_t (r_{j,t})] = 1 - 1$$

thus

$$E_t[M_t (r_{j,t})] = 0 \quad (4)$$

Deduce the Expected Excess Return

As shown above

$$E_{t-1}[M_t(r_{j,t})] = 0 \quad (4)$$

insertion of the *MRS* from equation (5) yields

$$E_{t-1}\left[\frac{[1 - \lambda_{0,t-1} - \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} - \lambda_{m,t-1} r_{m,t}]}{1 + r_{t-1}^f}(r_{j,t})\right] = 0$$

with $(1 + r_{t-1}^f)$ being non-stochastic

$$E_{t-1}[(1 - \lambda_{0,t-1} - \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} - \lambda_{m,t-1} r_{m,t})(r_{j,t})] = 0$$

split

$$E_{t-1}[(r_{j,t}) - (r_{j,t})\lambda_{0,t-1} - (r_{j,t})\sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t} - (r_{j,t})\lambda_{m,t-1} r_{m,t}] = 0$$

transform

$$E_{t-1}[(r_{j,t}) - (r_{j,t})\lambda_{0,t-1} - \sum_{i=1}^L \lambda_{i,t-1} r_{n+i,t}(r_{j,t}) - \lambda_{m,t-1} r_{m,t}(r_{j,t})] = 0$$

change to

$$E_{t-1}[(r_{j,t})] - \lambda_{0,t-1} E_{t-1}[(r_{j,t})] - \sum_{i=1}^L \lambda_{i,t-1} E_{t-1}[(r_{n+i,t})(r_{j,t})] - \lambda_{m,t-1} E_{t-1}[(r_{m,t})(r_{j,t})] = 0$$

apply $E(XY) = E(X)E(Y) + Cov(X, Y)$

$$E_{t-1}[(r_{j,t})] - \lambda_{0,t-1} E_{t-1}[(r_{j,t})] - \sum_{i=1}^L \lambda_{i,t-1} [E_{t-1}[r_{n+i,t}]E_{t-1}[(r_{j,t})] - Cov_{t-1}[(r_{n+i,t}), (r_{j,t})]] - \lambda_{m,t-1} [E_{t-1}[r_{m,t}]E_{t-1}[(r_{j,t})] - Cov_{t-1}[(r_{m,t}), (r_{j,t})]] = 0$$

change ordering

$$E_{t-1}[(r_{j,t})] * [1 - \lambda_{0,t-1} - \sum_{i=1}^L \lambda_{i,t-1} [E_{t-1}[r_{n+i,t}]] - \lambda_{m,t-1} [E_{t-1}[r_{m,t}]]] \\ = \sum_{i=1}^L \lambda_{i,t-1} Cov_{t-1}[(r_{n+i,t}), (r_{j,t})] + \lambda_{m,t-1} Cov_{t-1}[(r_{m,t}), (r_{j,t})]$$

due to (3) the left-hand side = $E_{t-1}[(r_{j,t})]$

$$E_{t-1}[(r_{j,t})] = \sum_{i=1}^L \lambda_{i,t-1} Cov_{t-1}[(r_{n+i,t}), (r_{j,t})] + \lambda_{m,t-1} Cov_{t-1}[(r_{m,t}), (r_{j,t})]$$

Which yields the expected excess return (9)

Annex: Descriptive Statistics

Source: Solnik and Dumas 1995

(numbers in brackets refer to t-statistics)

Table I

Summary Statistics

Excess rates of return on assets are coded as 0.01 for a 1 percent rate of return per month. The instrumental variables that are yields or rates of return (except for the lagged world index rate of return) are coded as 1 for 1 percent per year. $r_m(-1)$ is the monthly rate of return on the world stock market lagged by one month. *USbondy-E\$* is the yield on an index of U.S. bond prices in excess of the Eurodollar deposit rate. *USDivy-E\$* is the dividend yield on the U.S. stock index in excess of the Eurodollar deposit rate. *Euro\$* is the one-month Eurodollar deposit rate. *JanD* is a dummy variable for the month of January.

Number of Observations = 262 (March 1970–December 1991)								
Panel A: Excess Returns								
	Mean/Month			Std. Dev./Month				
German equity	0.0051			0.0623				
U.K. equity	0.0066			0.0775				
Japanese equity	0.0090			0.0659				
U.S. equity	0.0025			0.0468				
German deutsche mark	0.0017			0.0349				
British pound	0.0017			0.0318				
Japanese yen	0.0027			0.0332				
World equity	0.0032			0.0436				
Panel B: Instruments								
	Mean	St. Dev.		Pairwise Correlations				
$r_m(-1)$	0.0361	0.5214	1.0	0.27	0.26	-0.25		
<i>USbondy-E\$</i>	0.1545	2.1343		1.0	0.78	-0.74		
<i>USDivy-E\$</i>	-4.7019	2.6823			1.0	-0.97		
<i>Euro\$</i>	8.9063	3.2765				1.0		
Panel C: Instrument Auto-Correlations								
	rho1	rho2	rho3	rho4	rho8	rho12	rho24	rho36
$r_m(-1)$	0.11	-0.03	0.03	-0.01	-0.03	0.05	0.00	0.03
<i>USbondy-E\$</i>	0.91	0.82	0.76	0.70	0.60	0.47	0.06	-0.23
<i>USDivy-E\$</i>	0.93	0.85	0.79	0.73	0.62	0.48	0.05	-0.24
<i>Euro\$</i>	0.95	0.89	0.85	0.80	0.71	0.59	0.21	0.05

Source: own data (computed as above)

Equity

Mean Returns

		Germany	UK	Japan	U.S.
per month	T=(1975-2007)	0,004189496	0,00711345	0,00294451	0,00478629
	T=(1975-1991)	0,003310537	0,00899119	0,00716797	0,0040114
Annualized	T=(1975-2007)	0,050273946	0,0853614	0,03533418	0,05743551
	T=(1975-1991)	0,03972644	0,10789428	0,08601558	0,04813683

Equity Standard Deviation

	Germany	UK	Japan	U.S.
per month				
T=(1975-2007)	0,061496011	0,0612724	0,06332512	0,04273761
T=(1975-1991)	0,064366188	0,07553164	0,06559044	0,04564906

Currency Mean Returns

	Germany	UK	Japan	U.S.
per month				
T=(1975-2007)	-8,52222E-05	0,00147374	-0,00130608	0,00448088
annualized				
T=(1975-1991)	0,000443625	0,00111426	-0,00018962	0,00465517
T=(1975-2007)	0,001022667	0,01768487	-0,015673	0,05377061
T=(1975-1991)	0,005323498	0,01337108	-0,00227544	0,05586209

Currency Standard Deviation

	Germany	UK	Japan	U.S.
per month				
T=(1975-2007)	0,032071779	0,03071965	0,03562302	0,04074929
T=(1975-1991)	0,034716479	0,03414358	0,03788869	0,04315707

Instruments Mean

	$r_m(-1)$	USbony-E\$	UScorpy-E\$	Eurodollar Rate
annualized				
T=(1975-2007)	0,05338303	0,94725636	1,08831169	6,74076961
T=(1975-1991)	0,05902882	0,53976404	1,33837438	9,07230493

Instruments Standard Deviation

	$r_m(-1)$	USbony-E\$	UScorpy-E\$	Eurodollar Rate
annualized				
T=(1975-2007)	0,48957018	1,74527966	0,43559527	3,64567318
T=(1975-1991)	0,51719743	1,92670147	0,43851439	3,33860852

Annex: International Conditional Model

Estimation Results

Source: Solnik and Dumas 1995

(numbers in brackets refer to t-statistics)

	δ	δ_{DEM}	Φ_{GBP}	Φ_{JPY}	Φ_m
Constant	0.1003 (0.38)	22.4735 (1.53)	19.3737 (0.83)	18.2531 (1.01)	-18.5363 (-2.42)
$r_m(-1)$	-0.0991 (-0.07)	-371.31 (-3.27)	180.101 (2.00)	100.625 (1.30)	31.6038 (1.00)
<i>JanD</i>	1.3406 (2.47)	-60.460 (-3.20)	40.6945 (1.83)	0.8343 (0.03)	22.8588 (2.66)
<i>USbony-E\$</i>	-0.0358 (-0.93)	-1.7018 (-0.44)	-7.9977 (-2.00)	3.4880 (1.28)	1.9627 (1.68)
<i>USDivy-E\$</i>	-0.0197 (-0.23)	0.4544 (0.09)	5.6074 (1.17)	-3.7700 (-1.04)	10.1277 (4.53)
<i>Euro\$</i>	0.0195 (0.30)	-1.6611 (-0.50)	0.5133 (0.12)	-3.6828 (-1.21)	7.6852 (4.52)

* Chi-square: 28.8039; right-tail *p*-value: 0.2276; degrees of freedom: 24.

Source: own estimation

	δ	Φ_{DEM}	Φ_{GBP}	Φ_{JPY}	Φ_m
Constant	0,05611348	-0,01998451	0,16368757	-0,02811351	0,11539194
(t-stats)	0,26954729	0,22882608	0,33296186	0,67301201	0,47889541
rm(-1) $\Delta \ln(\text{MSCIworld})$	-0,00736719	21,5698066	3,45411252	-18,0782993	-4,57471096
(t-stats)	0,27781654	0,55370205	0,18678876	0,42318181	0,56976465
JanD	-0,48397457	-2,63074111	-1,41656781	0,36038726	-8,02022446
(t-stats)	0,01820119	0,57504895	0,03155676	0,0166177	0,19863292
US Govern. Bond Yields-E\$	1,65183309	-1,03546889	0,88681581	-14,0002537	-10,1121745
(t-stats)	0,17775345	0,05641287	0,16169313	0,51859856	2,19709929
US Corp. Bond Yield -E\$	16,1142012	2,12507905	11,3495639	-0,34513546	4,93260496
(t-stats)	0,36829464	0,45801268	0,84510116	0,11784464	0,41549953
Euro\$	5,3496198	-1,5631268	-1,65230472	8,00968962	-1,53059152
(t-stats)	1,00318678	0,08046944	0,70364267	0,72842704	1,28917806

J-Statistic P-Value

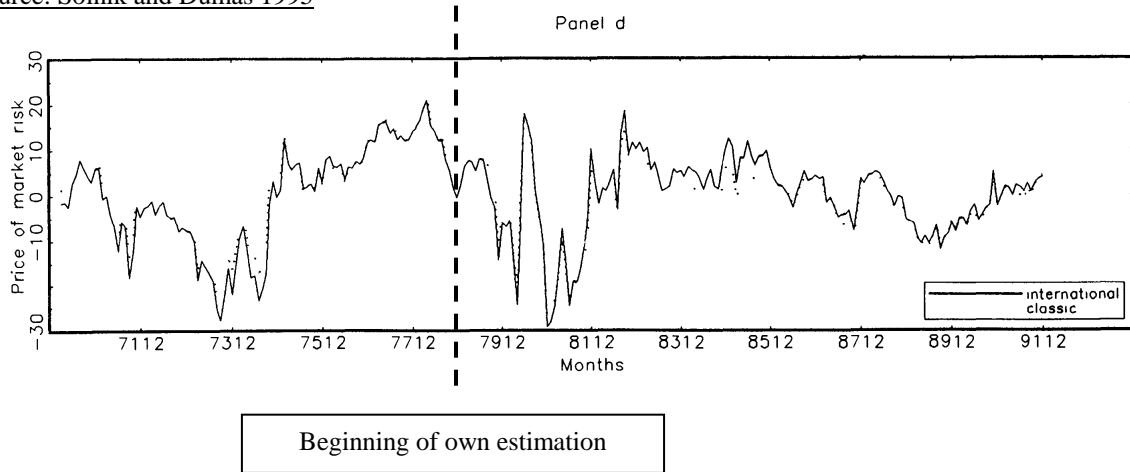
30,7992279 0,159656386

Number of Observations: 342
 Number of Equations 54
 Number of Parameters 30

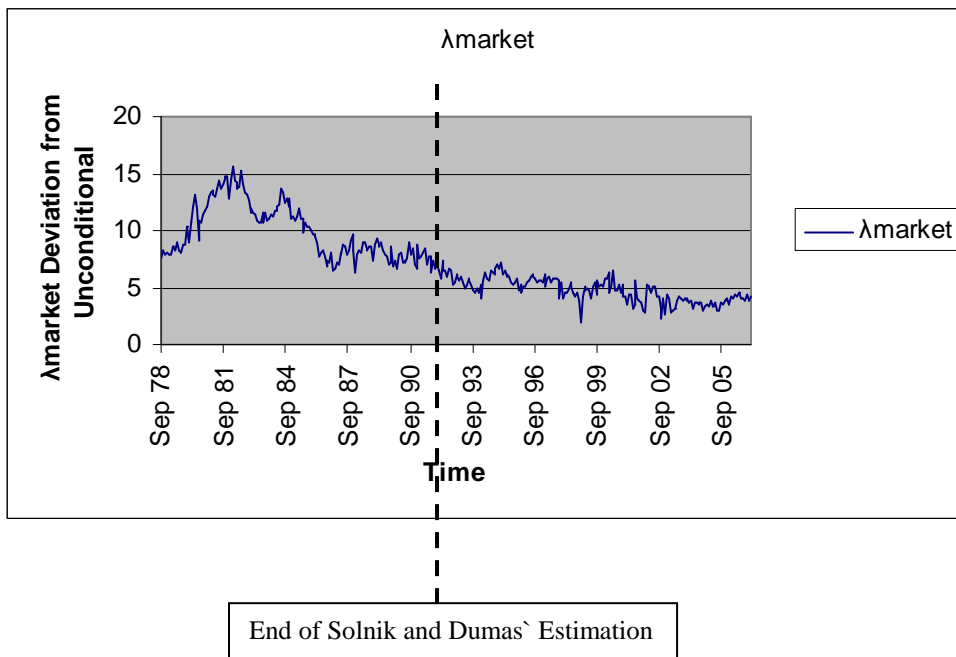
Plotting Time-Varying Lambda: World Market Price of Risk: λ_m

These Panels show time series of the estimated prices of risk as linear functions of the instrumental variables. Shown are deviations from their unconditional values, relying on the unconditional means and the seasonal effect of the January dummy.

Source: Solnik and Dumas 1995

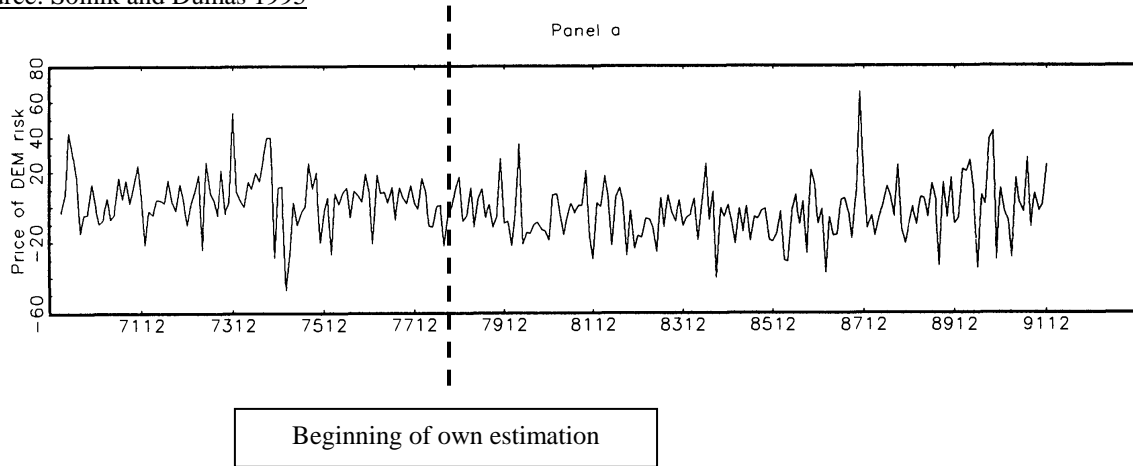


Source: own estimation

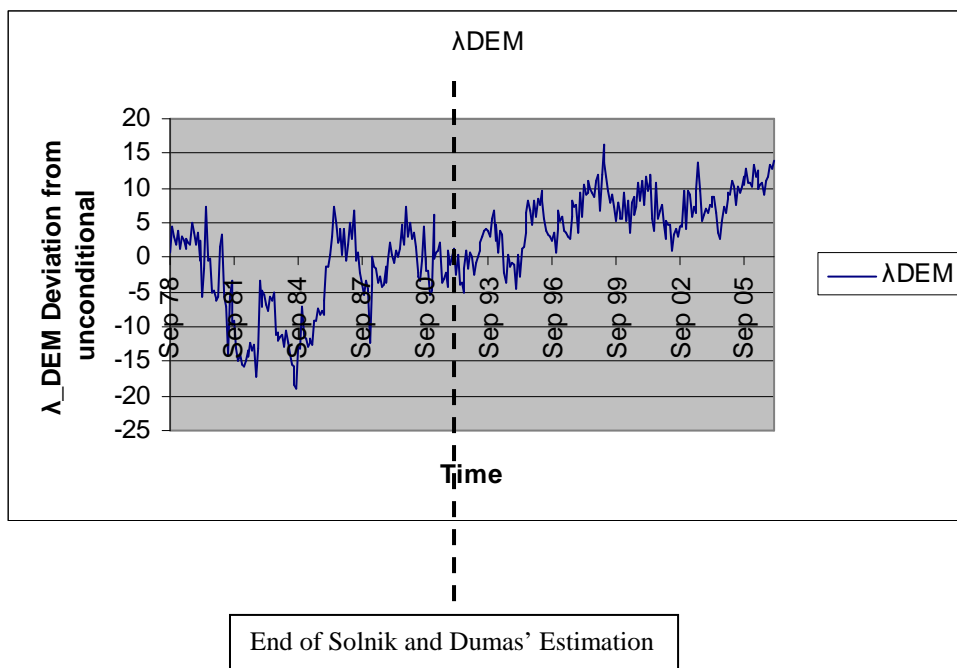


Plotting Time-Varying Lambda: DEM Price of Risk: λ_{DEM}

Source: Solnik and Dumas 1995

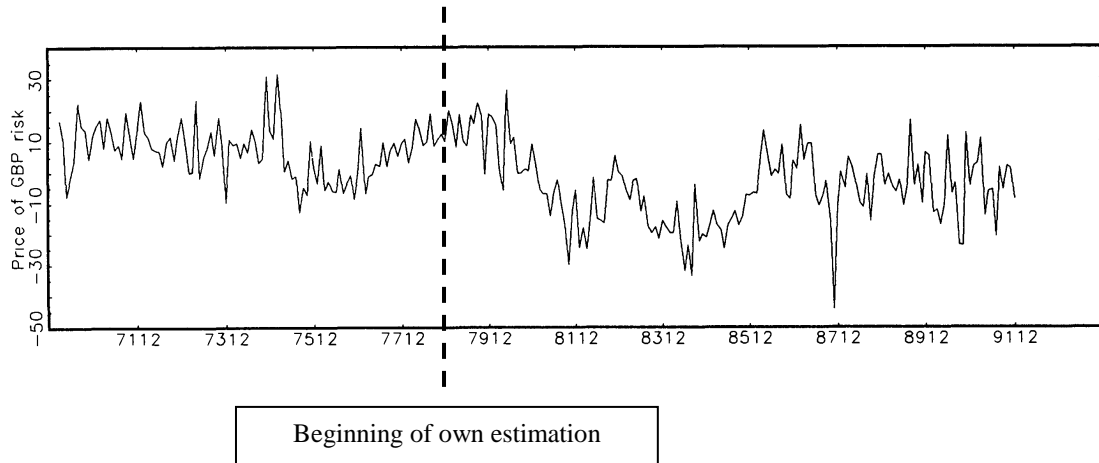


Source: own estimation

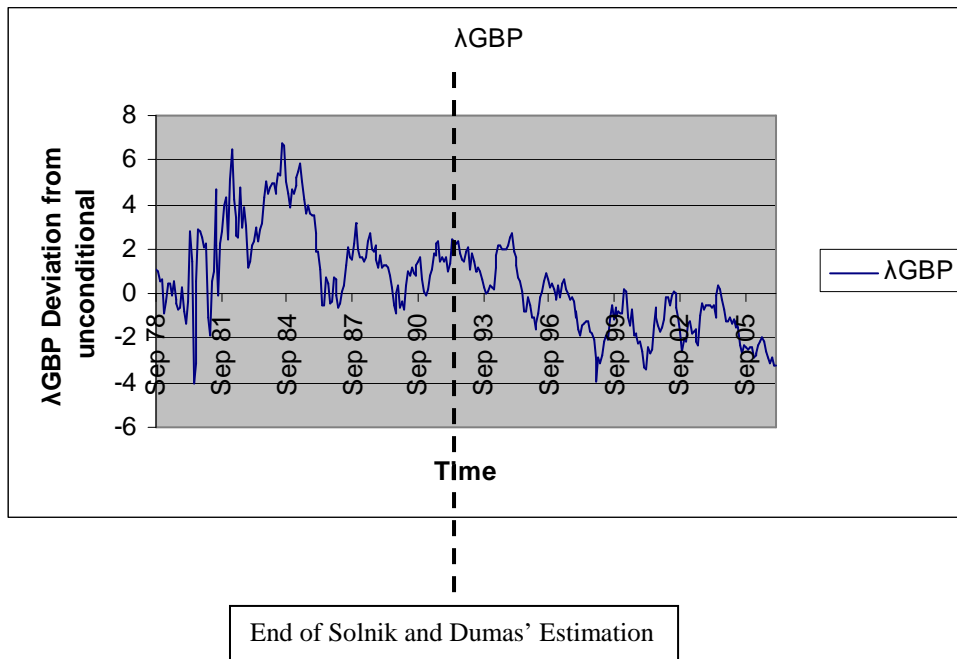


Plotting Time-Varying Lambda: GBP Price of Risk: λ_{GBP}

Source: Solnik and Dumas 1995

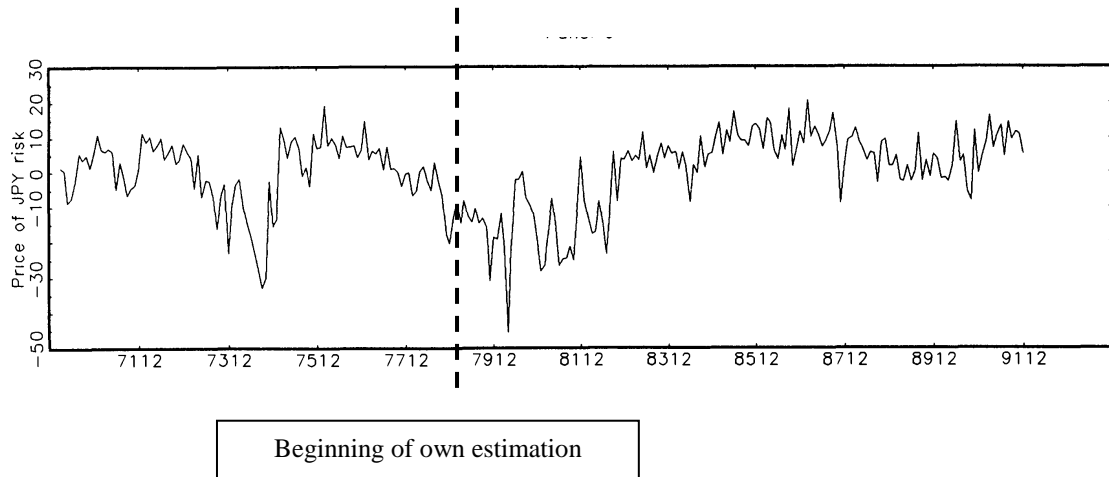


Source: own estimation

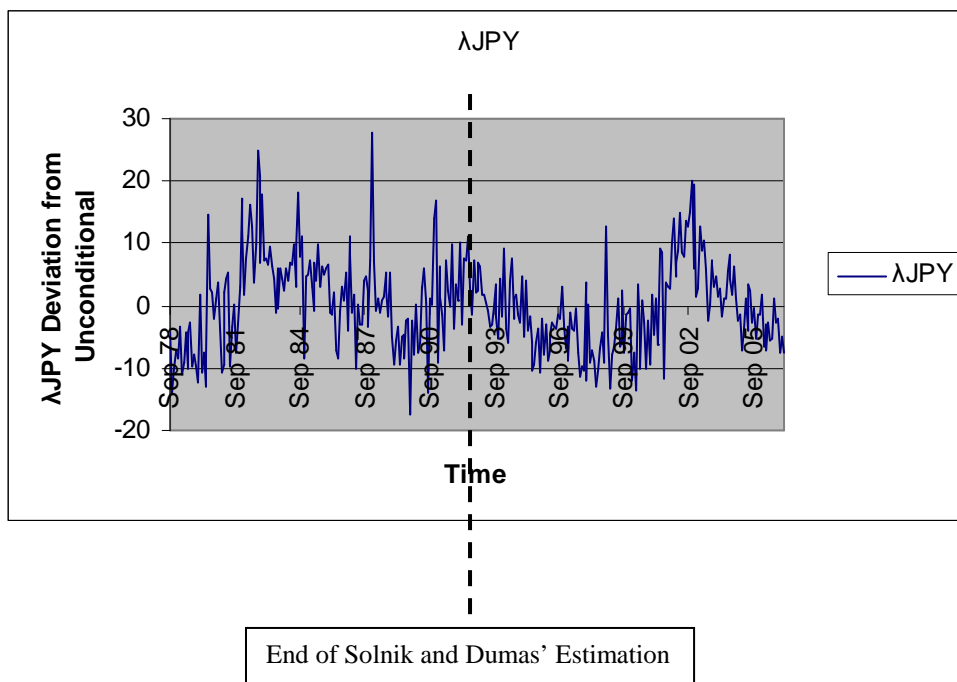


Plotting Time-Varying Lambda: JPY Price of Risk: λ_{JPY}

Source: Solnik and Dumas 1995



Source: own estimation



Annex: Classic Conditional Model

Estimation Results

Source: Solnik and Dumas 1995

(numbers in brackets refer to t-statistics)

	δ	Φ_m
Constant	-0.0637 (-0.61)	-9.6176 (-1.64)
$r_m(-1)$	0.5866 (1.08)	24.6887 (0.99)
<i>JanD</i>	0.1264 (1.03)	0.0802 (0.03)
<i>USbony-E\$</i>	-0.0425 (-2.36)	0.0442 (0.05)
<i>USDivy-E\$</i>	0.1090 (2.03)	11.1430 (6.12)
<i>Euro\$</i>	0.0792 (2.06)	7.5439 (5.45)

* Chi-square: 69.1189; right-tail *p*-value: 0.0053; degrees of freedom: 42.

Source: own estimation

	δ	Φ_m
Constant	0,0356494	0,67428358
(t-stats)	0,26489786	0,02978408
rm(-1) $\Delta \ln(\text{MSCIworld})$	0,0890375	8,28712967
(t-stats)	0,66808583	0,86371022
JanD	0,00048762	4,78511926
(t-stats)	0,00221444	0,19512232
US Govern. Bond Yields-E\$	0,0146128	3,43734682
(t-stats)	-0,43065799	0,81653811
US Corp. Bond Yield -E\$	-0,09997445	-14,214172
(t-stats)	-0,43065799	-0,79894558
Euro\$	0,00449264	1,8962171
(t-stats)	0,23033181	0,76716312

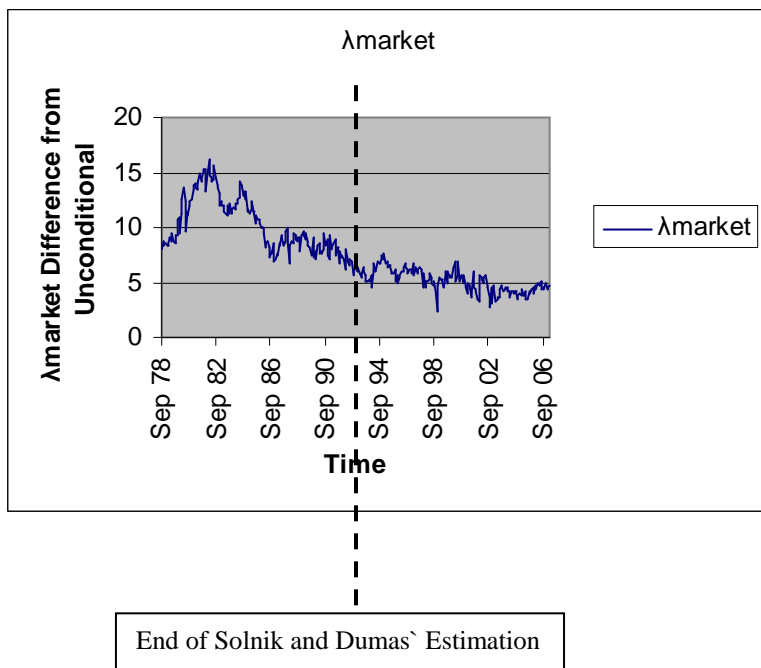
J-Statistic P-Value
66,3514787 0,00969127

Number of Observations: 342
 Number of Equations 54
 Number of Parameters 12
 Degrees of Freedom 42

Plotting Time-Varying Lambda: World Market Price of Risk: λ_m

This Panel shows time series of the estimated prices of risk as linear functions of the instrumental variables. Shown are deviations from their unconditional values, relying on the unconditional means, and the seasonal effect of the January dummy.

Source: own estimation



Annex: Unconditional Models

Estimation Results: International APM

Source: Solnik and Dumas 1995

(numbers in brackets refer to t-statistics)

Panel B: International APM (Number of Factors = 4)

λ_0	λ_{DEM}	λ_{GBP}	λ_{JPY}	λ_m
0.0099	-0.3607	0.3417	2.2238	1.1334
(0.82)	(-0.14)	(0.13)	(0.91)	(0.74)

Chi-square: 9.54

Right-tail p -value: 0.049

Degrees of freedom: 4

Source: own estimation

	λ_0	λ_{DEM}	λ_{GBP}	λ_{JPY}	λ_{market}
coefficient	0,02369219	-2,16461341	4,44701286	-3,07331322	2,63338634
(t-statistic)	1,298829374	-0,5826538	1,258001466	-1,13477849	1,558794579

J-Statistic P-Value

0,420712 0,9807463

Number of Observations: 343

Number of Equations 9

Number of Parameters 5

Degrees of Freedom 4

Estimation Results: Classic APM

Source: Solnik and Dumas 1995

(numbers in brackets refer to t-statistics)

Panel A: Classic APM (Number of Factors = 1)	
λ_0	λ_m
0.0055 (0.59)	1.7032 (1.16)

Chi-square: 10.53
Right tail p -value: 0.161
Degrees of freedom: 7

Source: own estimation

	λ_0	λ_{market}
Coeff	0,00884805	2,16612064
(p-value)	0,409214002	0,140708464

J-Statistic **P-Value**
4,61492578 0,70683772

–
Number of Observations: 343
Number of Equations 9
Number of Parameters 2
Degrees of Freedom 7