25 easy pieces in math.statistics

1. Write expectation of a random variable (r.v.) Z $E_Z(Z)$ extensively a) for a discrete random variable and b) for a continuous random variable Z.
2. $Var(Z)$ can be written as $E(Y)$. What is Y?
3. Write $Var(Z)$ extensively for a) a discrete and b) for a continuous r.v.
4. What does the cumulative density function or (cumulative) distribution function (c.d.f.) tell you? $F_X(x) =$
5. X is a continuous r.v. How are the c.d.f $F_X(x)$ and the density $f_X(x)$ related?
6. $Cov(X, Y)$ can be written as $E(Z)$. What is Z?
7. Write $Cov(X, Y)$ extensively for X and Y a) a continuous and b) a discrete random variable
8. Express $E_{XY}(X \cdot Y)$ as a function of $Cov(X, Y)$
9. Write $E_{XY}(X \cdot Y)$ extensively for X and Y a) discrete and b) cont. r.v.s
10. $g(x)$ denotes a measurable function of the r.v. X (like e.g. $X^2, ln(X)$). Write extensively $E(g(x))$ for X continuous r.v.
11. X and Y are cont. r.v.s. $Z = g(X, Y)$ is a measurable function. Write extensively $E(g(X \cdot Y))$
12. X and Y are cont. r.v.s. What does the joint c.d.f $F_{XY}(x, y)$ tell you? Write the c.d.f extensively. What does the joint p.d.f. $f_{XY}(x, y)$ tell you? (discrete case)
13. How are $F_{XY}(x, y)$ and $f_{XY}(x, y)$ (joint density) related (X, and Y cont. r.v.s)
14. If X and Y independent
    $F_{XY}(x, y) =$
    $f_{XY}(x, y) =$
15. If X and Y independent
    $E_{XY}(X \cdot Y) =$
    $Cov(X \cdot Y) =$
16. If X and Y independent
    $E_{XY}(h(Y) \cdot g(Y)) =?$
17. $E_{XY}(X + Y) =$
    $E_{XYZ}(X + Y + Z) =$
    $Var(X + Y) =$
18. Write extensively for a) X and Y discrete r.v.s and b) X and Y cont. r.v.s
    $f_{X|Y}(X|Y = y)$
    $E_{X|Y}(X|Y = y)$
    $E_{X|Y}(X^2|Y = y)$

19. $E(a \cdot X) =$ a nonrandom scalar
    $Var(a \cdot X) =$

20. For $\underline{X} = (X_1, X_2, \cdot \cdot X_n)'$
    $E(\underline{X}) = \underline{\mu} \quad \underline{\mu} = ?$
    $Var(\underline{X}) = \Sigma \quad \Sigma = ?$

$$A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_n \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ a_{m1} & a_{m2} & \cdot & \cdot & amn \end{bmatrix}}_{\text{nonrandom matrix}}$$

$$\begin{aligned} \underline{Z} &= A \cdot \underline{X} \\ E(\underline{Z}) &= \\ Var(\underline{Z}) &= \end{aligned}$$

21. $Y = a + b \cdot X$
    $E(Y) =$
    $E(Y|X = x) =$

22. Given joint density $f_{XY}(x, y)$
    How do you get $f_X(x)$ and $f_Y(y)$?
    a) discrete r.v.s b) cont. r.v.s

23. under which circumstances can you get $f_{XY}(x, y)$ from $f_X(x)$ and $f_Y(y)$?

24. X and Y are jointly normally distributed
    $\begin{pmatrix} X \\ Y \end{pmatrix} \sim BVN(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho_{xy})$
    Relation of parameters and moments?
    $X \sim ?$
    $Y \sim ?$
    $X|Y = y$
    $Y|X = x$
    $E(X|Y = y) =$
    $Var(X|Y = y) =$

25. X and Y and Z are normally distributed
    $\mu = a \cdot X + b \cdot Y + c \cdot Z \sim ?$