6th set assignments Introductory Econometrics

Task 1

Consider the following assumptions:

1. linearity
2. rank condition: $K \times K$ matrix $E(\mathbf{x}_i \mathbf{x}_i') = \mathbf{\Sigma_{xx}}$ is nonsingular
3. predetermined regressors: $E(\mathbf{g}_i) = 0$ where $\mathbf{g}_i = \mathbf{x}_i \cdot \varepsilon_i$
4. $\mathbf{g}_i$ is a martingale difference sequence with finite second moments

i) Show, that under those assumptions, the OLS estimator is distributed asymptotically normal:

$$\sqrt{n}(\mathbf{b} - \boldsymbol{\beta}) \underset{d}{\rightarrow} N(0, \mathbf{\Sigma_{xx}}^{-1} E(\varepsilon_i^2 \mathbf{x}_i \mathbf{x}_i') \mathbf{\Sigma_{xx}}^{-1})$$

ii) Further, show that assumption 4 implies that the ε_i are serially uncorrelated or $E(\varepsilon_i \varepsilon_{i-j}) = 0$.

Task 2

Show, that the test statistic

$$t_k \equiv \frac{\sqrt{n}(b_k - \beta_k)}{\sqrt{[\widehat{Avar(b_k)}]}} \underset{d}{\rightarrow} N(0,1)$$

converges in distribution to a standard normal distribution. Note, that b_k is the k-th element of $\mathbf{b}$ and $Avar(b_k)$ is the (k,k) element of the $K \times K$ matrix $Avar(\mathbf{b})$. Use the facts, that $\sqrt{n}(b_k - \beta_k) \underset{d}{\rightarrow} N(0, Avar(b_k))$ and $\widehat{Avar(b)} \underset{p}{\rightarrow} Avar(b)$. Use Lemma 2.4(c) for argumentation.

Task 3

Show, that the test statistic

$$W \equiv (\mathbf{R}\sqrt{n}\mathbf{b} - \mathbf{r})'[\mathbf{R}\widehat{Avar(\mathbf{b})}\mathbf{R}']^{-1}(\mathbf{R}\sqrt{n}\mathbf{b} - \mathbf{r}) \underset{d}{\rightarrow} \chi^2(\#\mathbf{r})$$

converges in distribution to a Chi-square with $\#\mathbf{r}$ degrees of freedom. As a hint, rewrite the equation above as $W \equiv \mathbf{c}_n' \mathbf{Q}_n^{-1} \mathbf{c}_n$. Use Lemma 2.4(d) and the footnote on page 41 for argumentation.