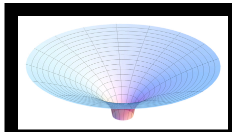
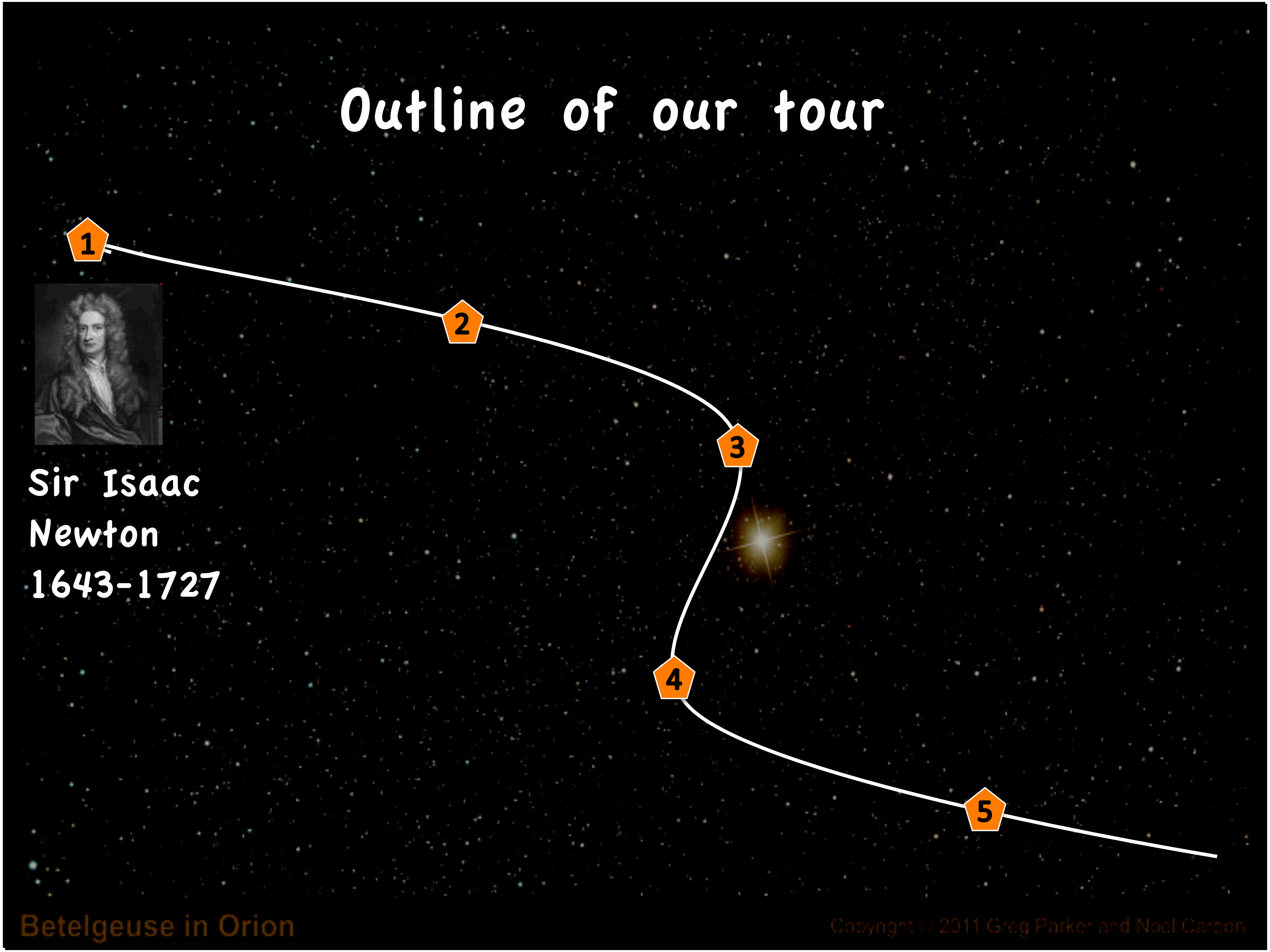


From Newton to Einstein: A guided tour through space and time

with Carla Cederbaum

Betelgeuse in Orion



Outline of our tour
1
2
3
4
5
Sir Isaac
Newton
1643-1727
Betelgeuse in Orion
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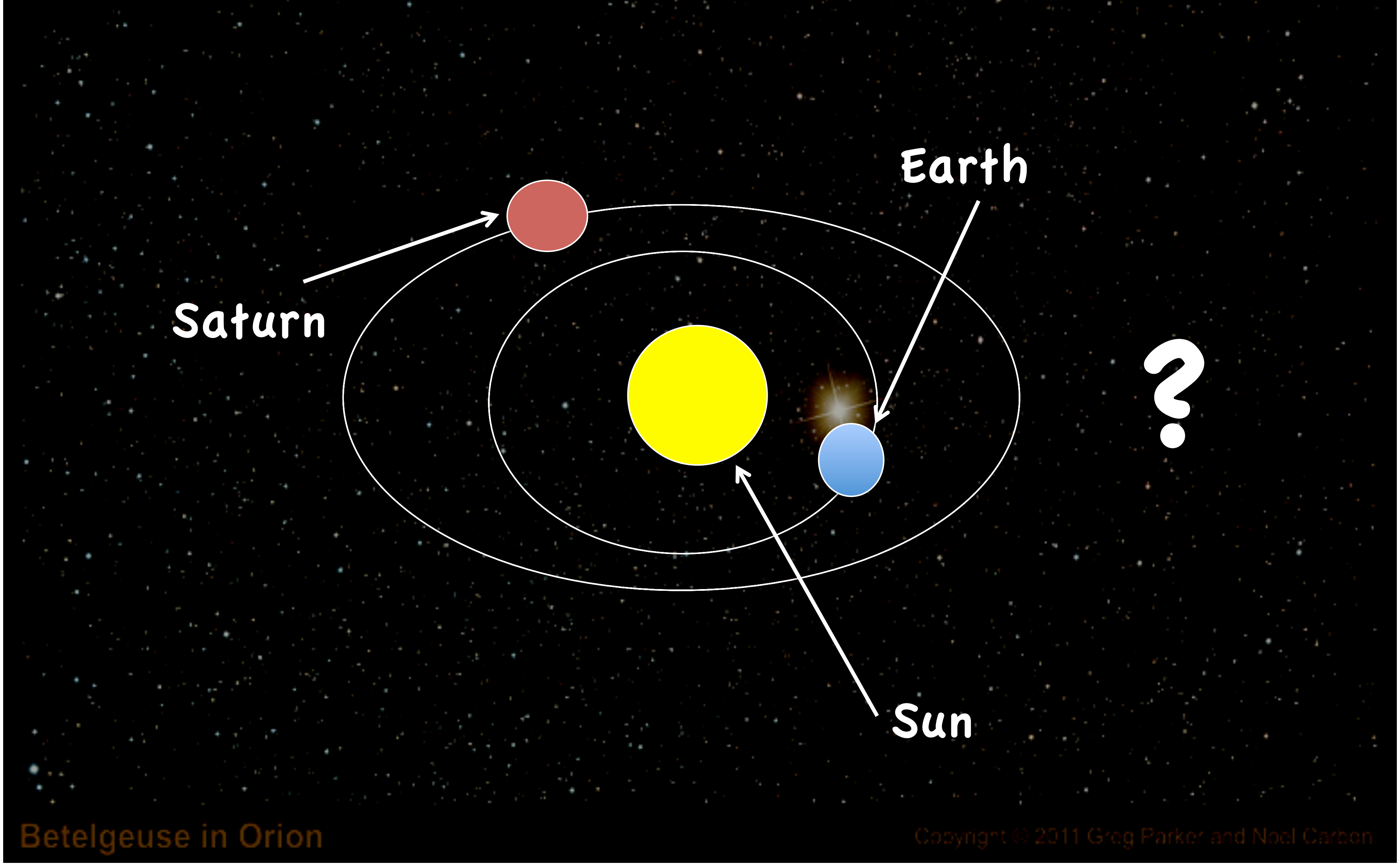
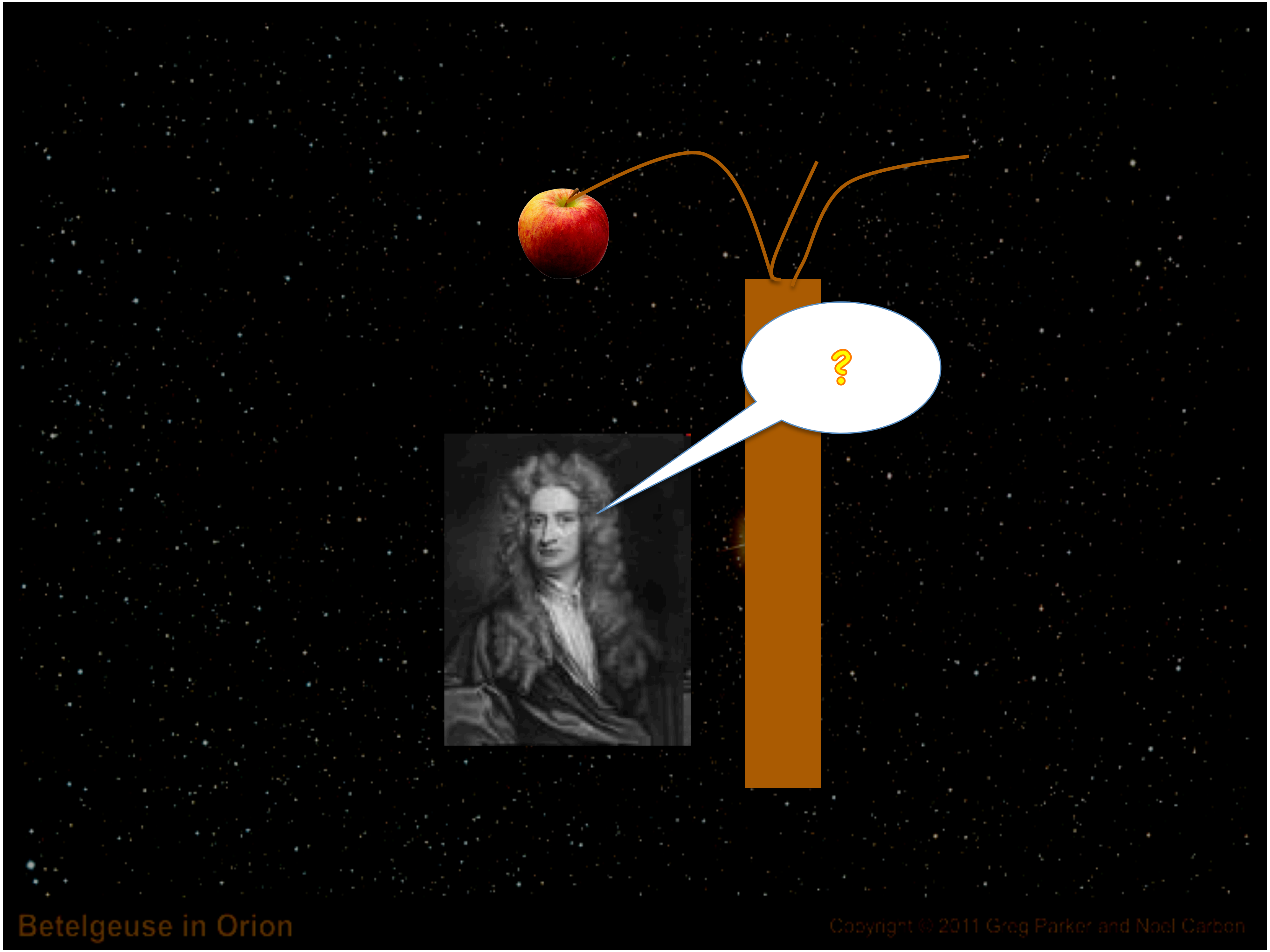
Why are the planets orbiting the sun?

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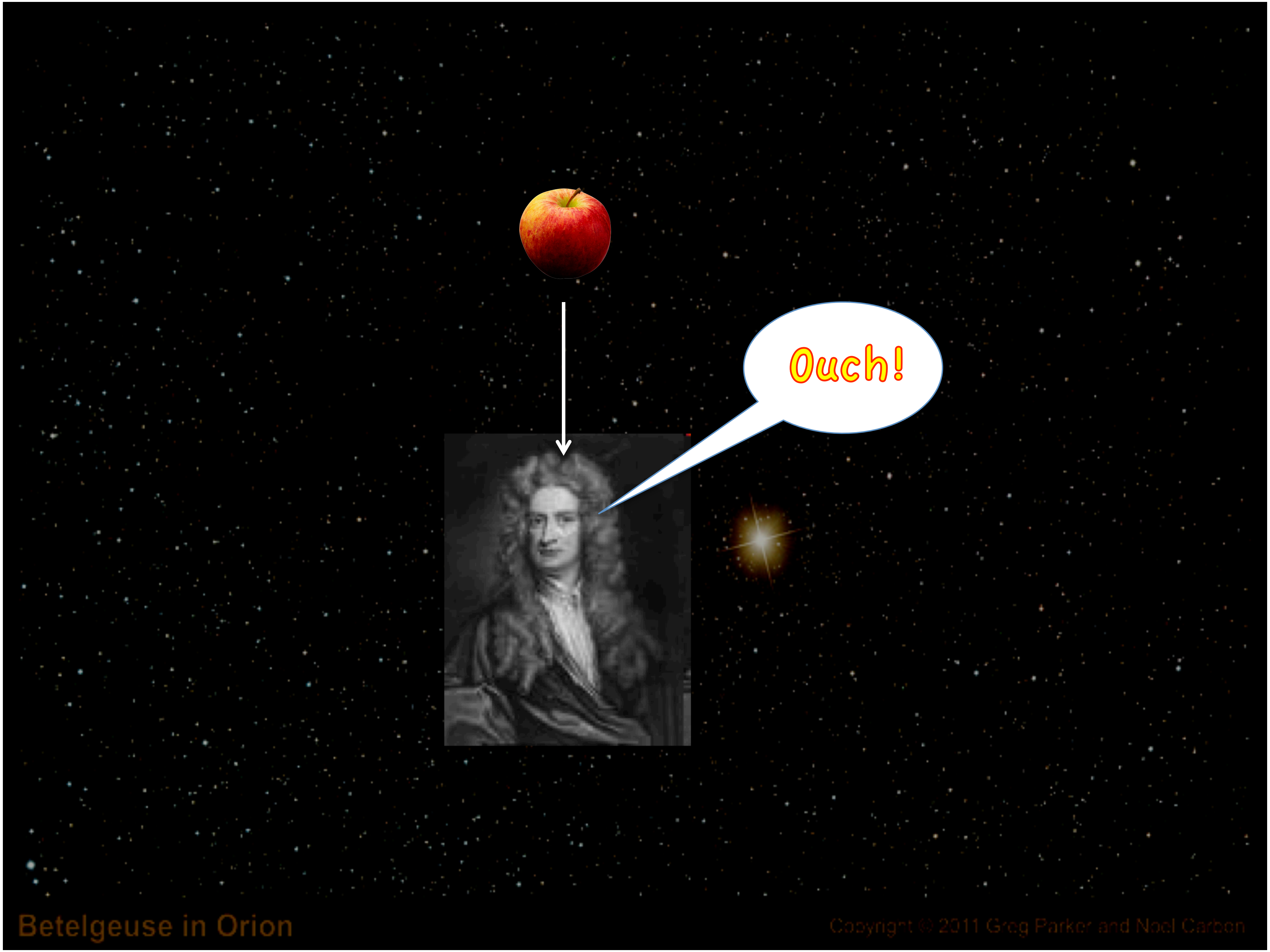
Ouch!
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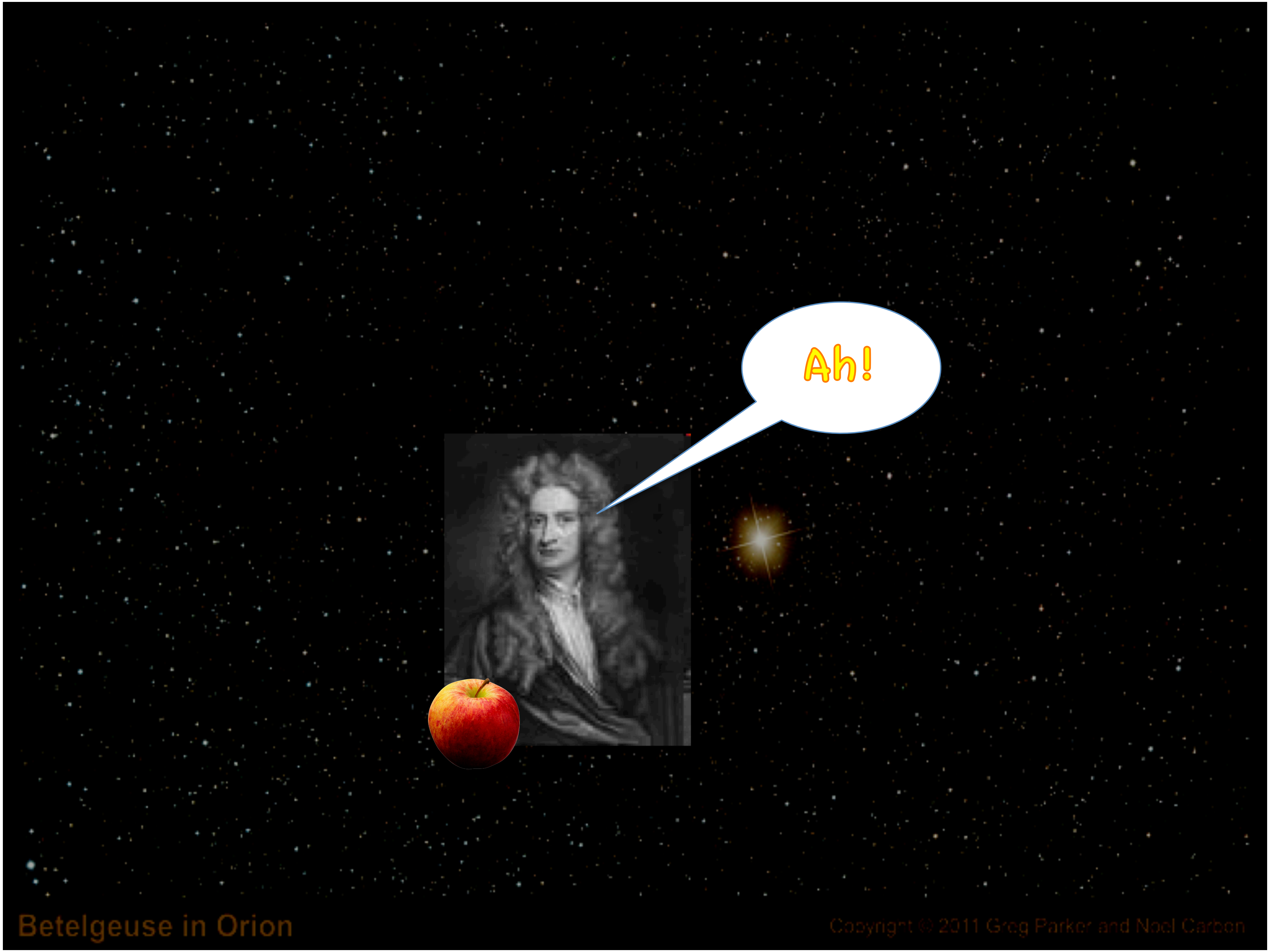
Ah!
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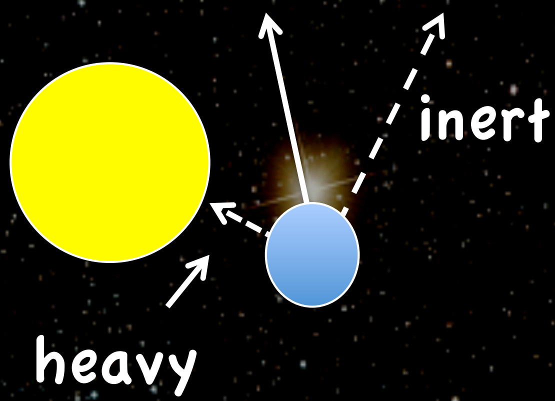
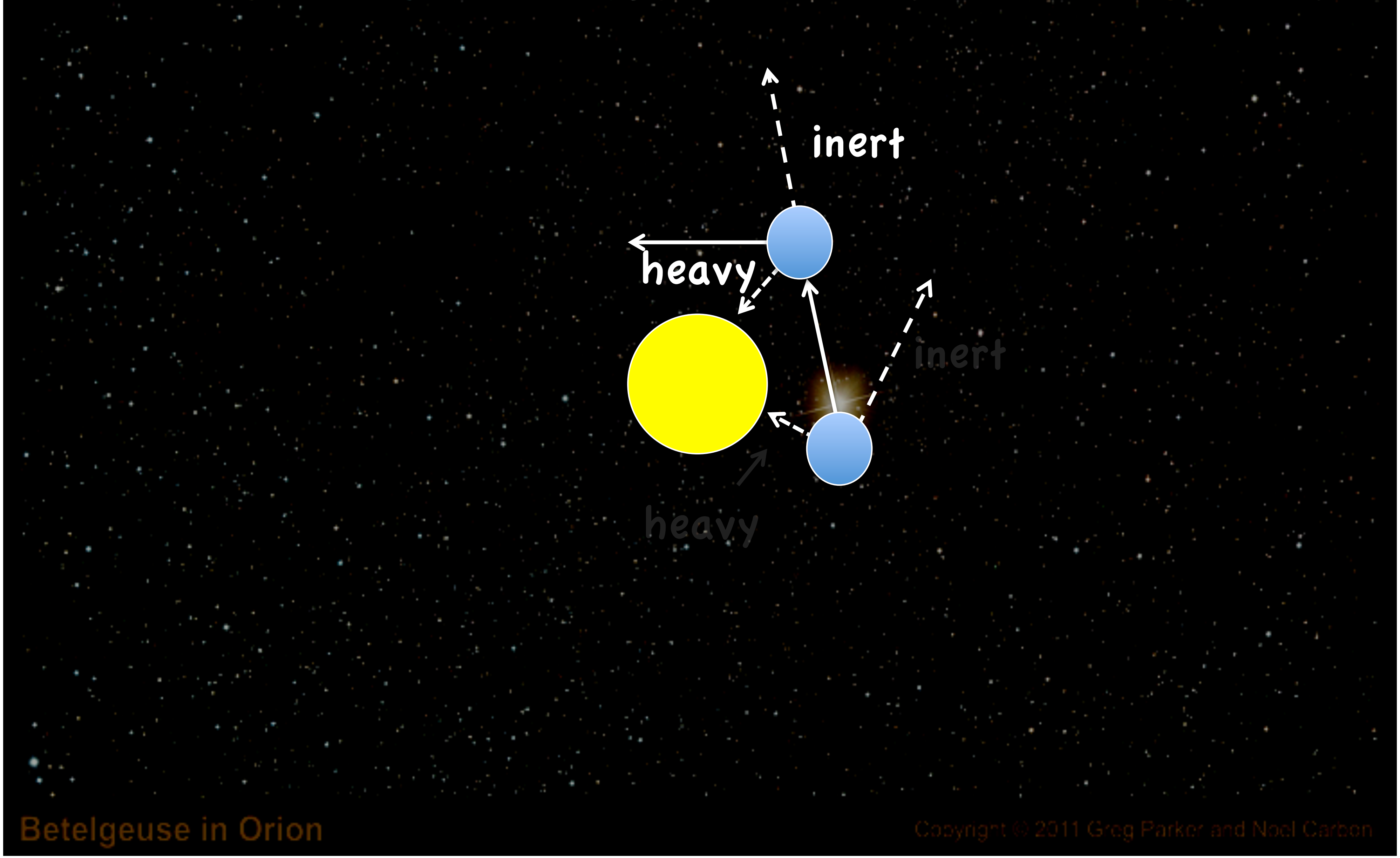
Why are the planets orbiting the sun?

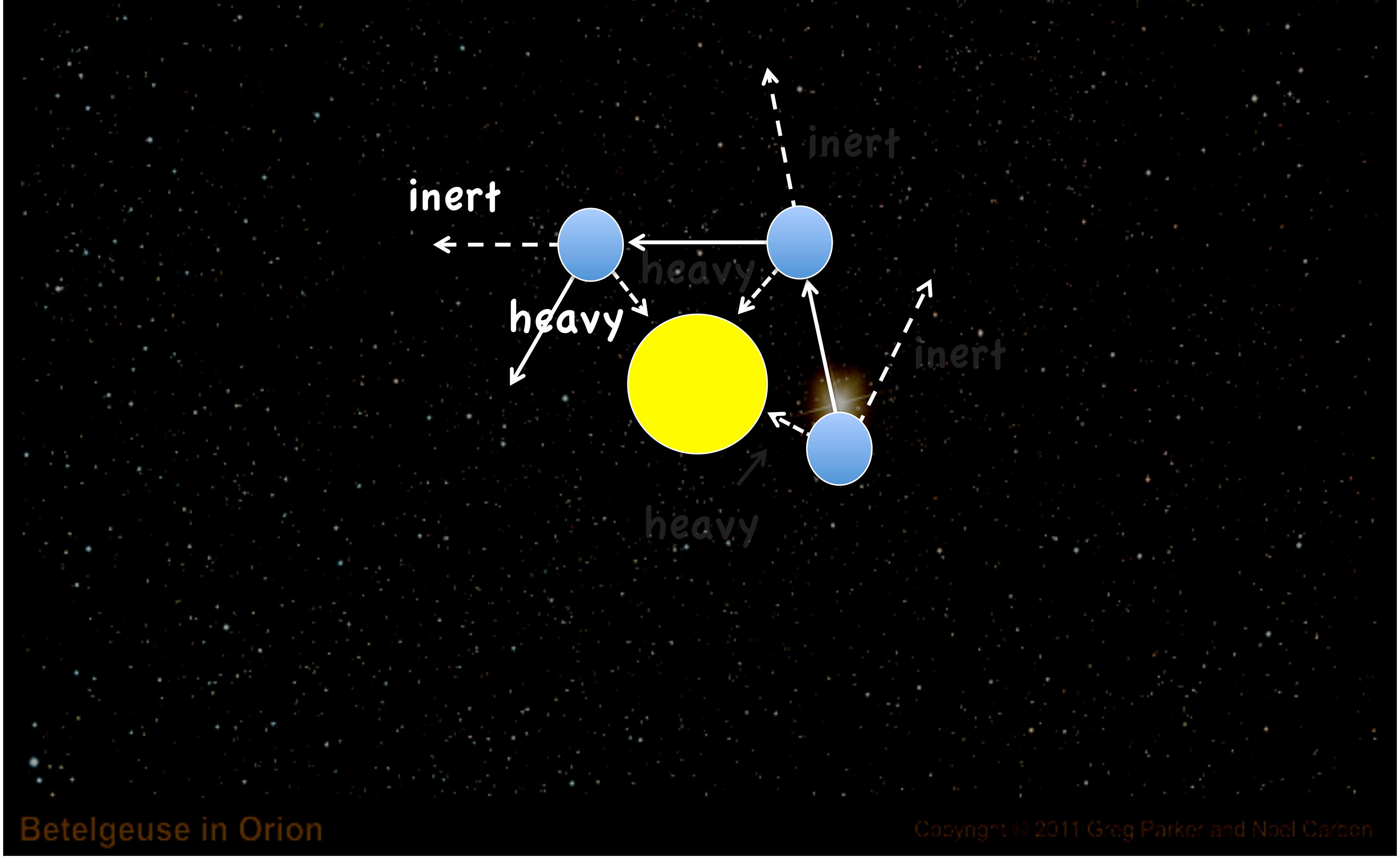


Betelgeuse in Orion

Why are the planets orbiting the sun?



Why are the planets orbiting the sun?



Newton's new math

- rate of change/derivative

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

- vectors:

velocity, acceleration, force

$\vec{v}$ $\vec{a}$ $\vec{F}$

Newton‘s law of gravity

m = mass of planet

M = mass of sun

G = gravitational constant

$\vec{r}$ = distance planet to sun

$$\vec{F} = -\frac{mMG\vec{r}}{r^3}$$

How do we measure mass?

mass

Betelgeuse in Orion

Outline of our tour

Siméon Denis
Poisson 1781-1840

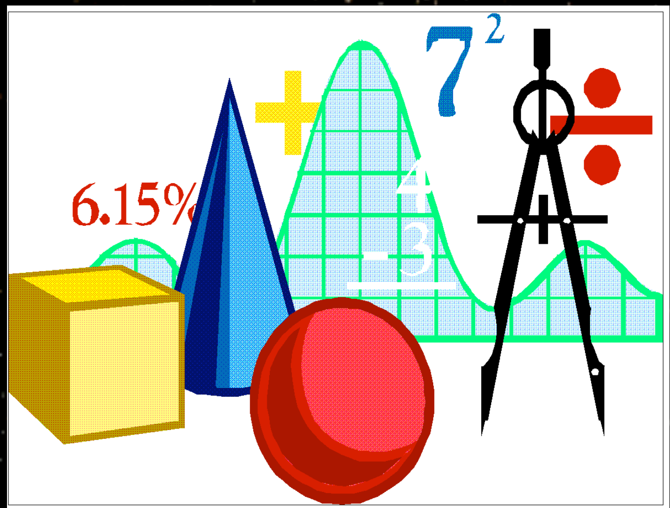
Sir Isaac
Newton
1643-1727

Pierre Simon
Laplace
1749-1827

1 2 3 4 5

Betelgeuse in Orion

Transform Newton's ideas into math!



Modeling gravitation with mathematics (vector calculus) allows to compute predictions and improve understanding!

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Vector calculus

Idea: generalize calculus to 3-dimensional space!

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\to \frac{\partial f}{\partial x}(x_0, y_0, z_0) = \lim_{x \to x_0} \frac{f(x, y_0, z_0) - f(x_0, y_0, z_0)}{x - x_0}$$

Newton's idea revisited

U = Newtonian potential of sun

G = gravitational constant

ρ = mass density =mass/volume

Δ = "differential operator"

$$\Delta U = 4\pi G\rho$$

Where is $\vec{F}$?

U = Newtonian potential of sun

m = mass of planet

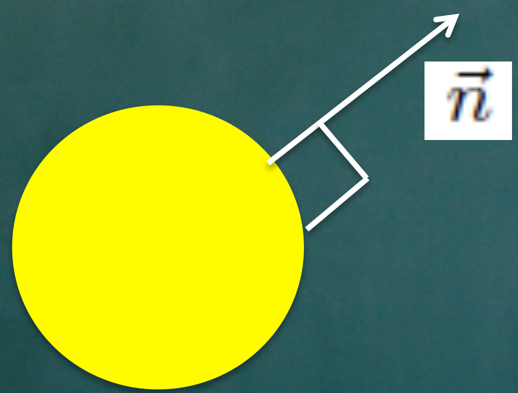
$\vec{\nabla}$ = a differential operator

$$\vec{F} = -m\vec{\nabla}U$$

What is now mass M?

M = mass of sun

$\vec{n}$ = normal vector to surface



What is now mass M?

Apply mathematical theorems (by Gauß and Stokes)

$$
\begin{aligned}
M &= \iiint_{\text{sun}} \rho \, dV \\
&= \dots \\
&= \iint_{\text{surface of sun}} \vec{\nabla} U \cdot \vec{n} \, dS
\end{aligned}
$$

Summary

New math allows to

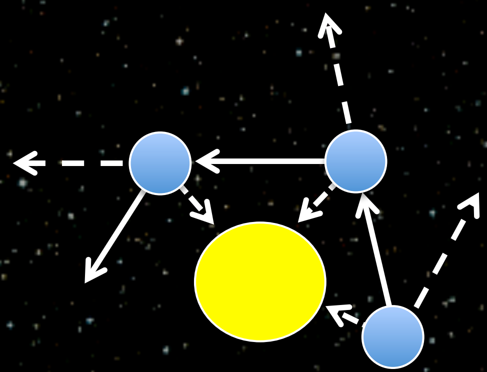
- write Newton's ideas as "differential equation" $\Delta U = 4\pi G\rho$

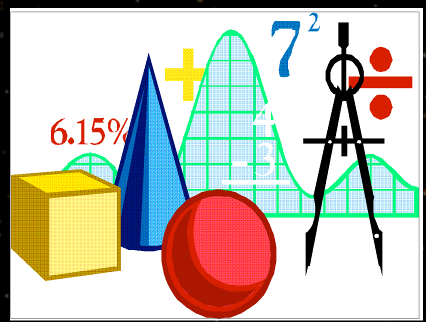
- express mass as an integral (using mathematical theorems)

$$M = \iint_{\text{surface of sun}} \vec{\nabla} U \cdot \vec{n}\, dS$$

Morale

- Use new math to "model" gravitation mathematically.
  - gives better methods for predictions
  - helps understand gravity better
- Newton's new physics inspired new math!





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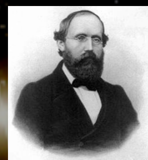
Outline of our tour

Siméon Denis Poisson 1781-1840

Sir Isaac Newton 1643-1727

Pierre Simon Laplace 1749-1827

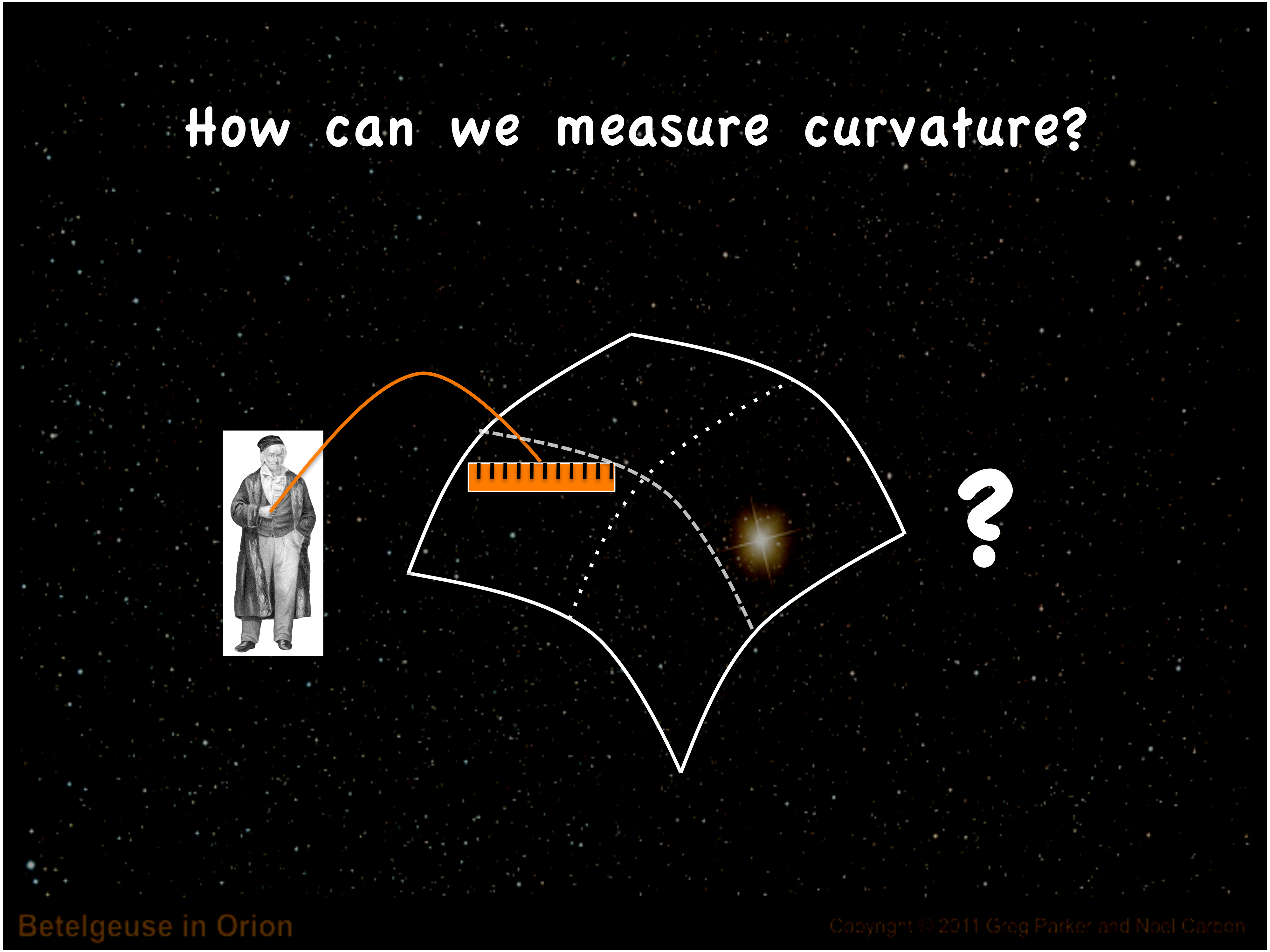
Carl Friedrich Gauß 1777-1855



Bernhard Riemann 1826-1866

1 2 3 4 5

Betelgeuse in Orion



How can we measure curvature?
?
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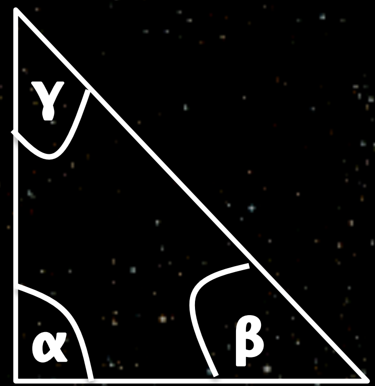
How can we measure curvature?



$\alpha+\beta+\gamma=180°$ $\alpha+\beta+\gamma\neq180°$

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Curvature is important for:

- Geodesy and Geography
- Astronomy
- Physics
- Engineering (wings of planes,…)
- Biology (surface of cells,…)
- Mathematics

-> differential geometry

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Differential Geometry

- studies curves and surfaces
- generalizes vector calculus
- allows rigorous definition of

curvature

(in terms of derivatives)

Curvature

- Curves can be curved.
- Surfaces can be curved.
- 3-dimensional space can also be curved!
- Can even think about higher dimensional (curved) space!!

Outline of our tour

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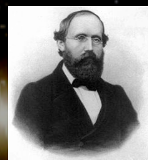
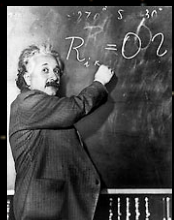


Bernhard Riemann 1826-1866



Albert Einstein 1879-1955

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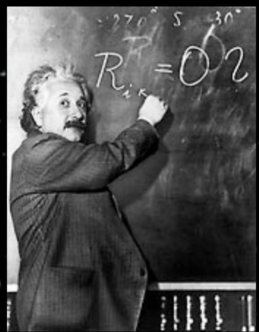
Why are the planets orbiting the sun?



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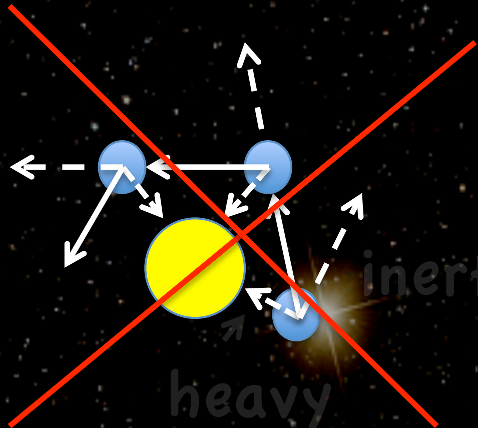

!

Conflicts with observations and electrodynamics!

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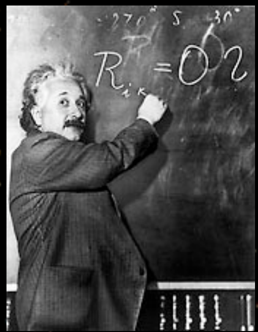
General Relativity

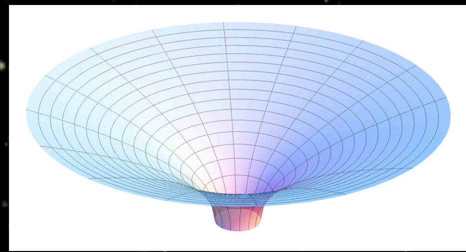


: gravitation = curvature !

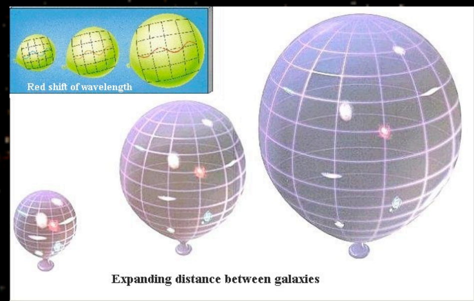
Modeling gravitation with mathematics (differential geometry) allows to compute predictions and improve understanding!

Math allows to make predictions like

- Black holes:



- Expansion of universe:



- Gravitational waves?

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Einstein‘s theory

- is called “general relativity”
- uses ideas from differential geometry like curvature
- describes gravitational effects by a differential equation

General relativity

Main equation:

$$\mathrm{Ric} - \frac{1}{2}\,\mathrm{g}\,R = \frac{8\pi G}{c^4}\,\mathrm{T}$$

c = speed of light
R, Ric: measure curvature
g: measures distance
T: describes matter

Describes the world

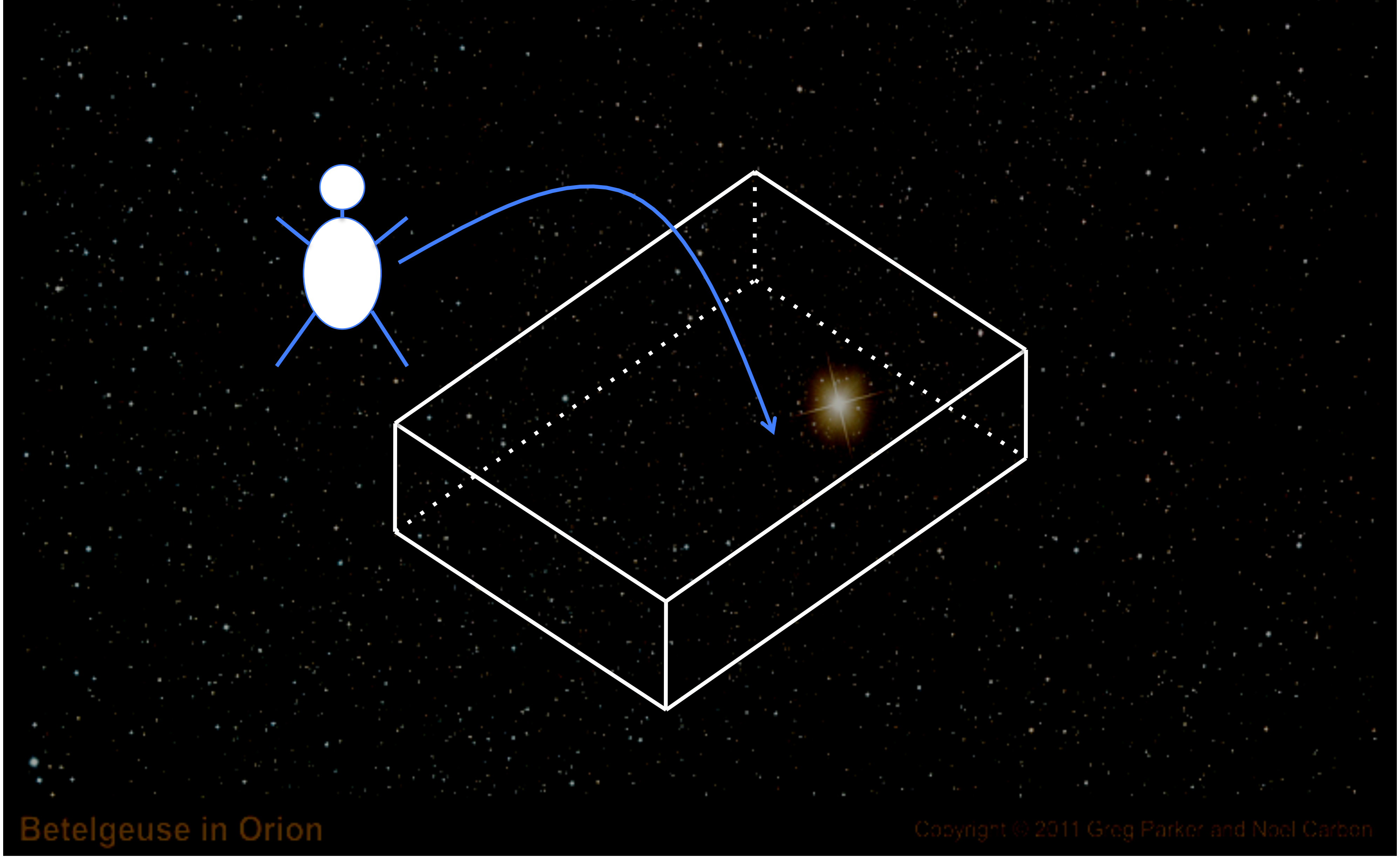
Einstein's theory is consistent with many measurements:

- bending of light
- gravitational red shift
- …

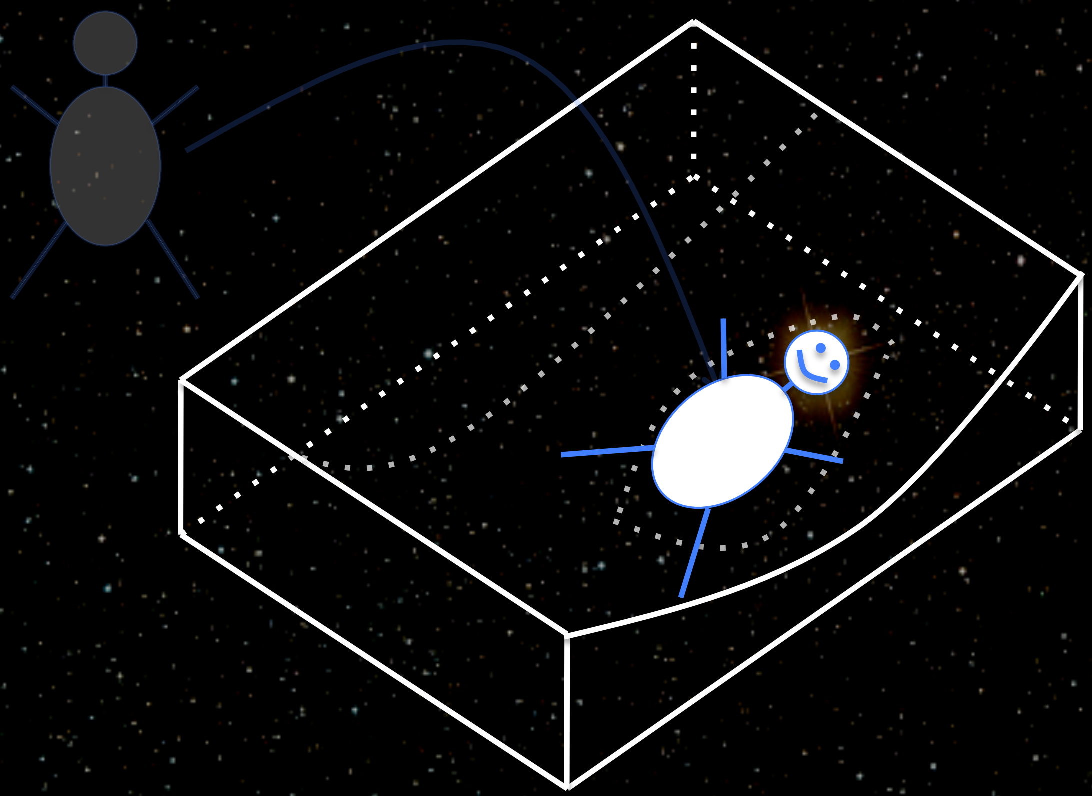
Applications

- General Positioning System
- satellites
- space travel

General relativity in every day life:

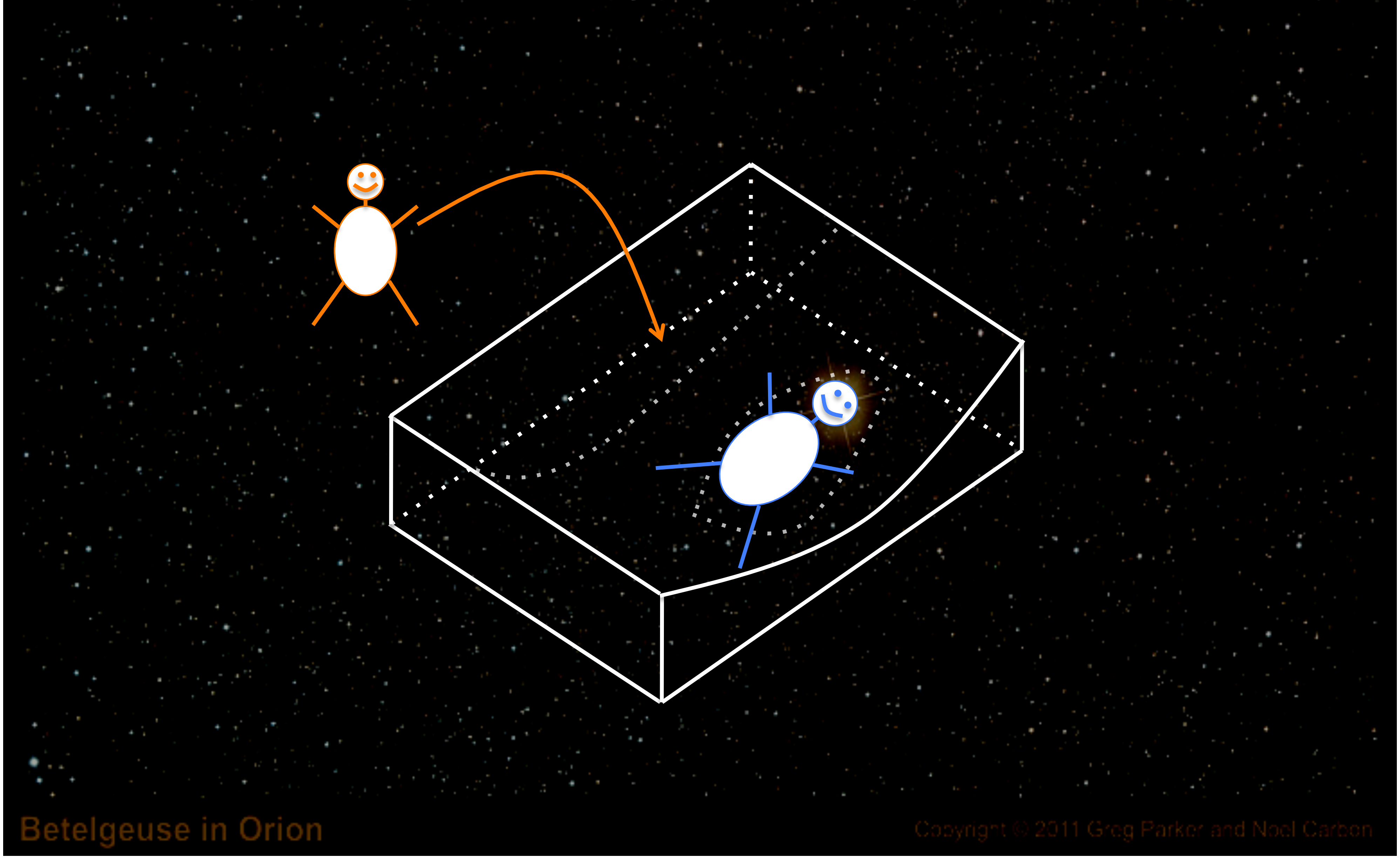


General relativity in every day life: matter curves space-time

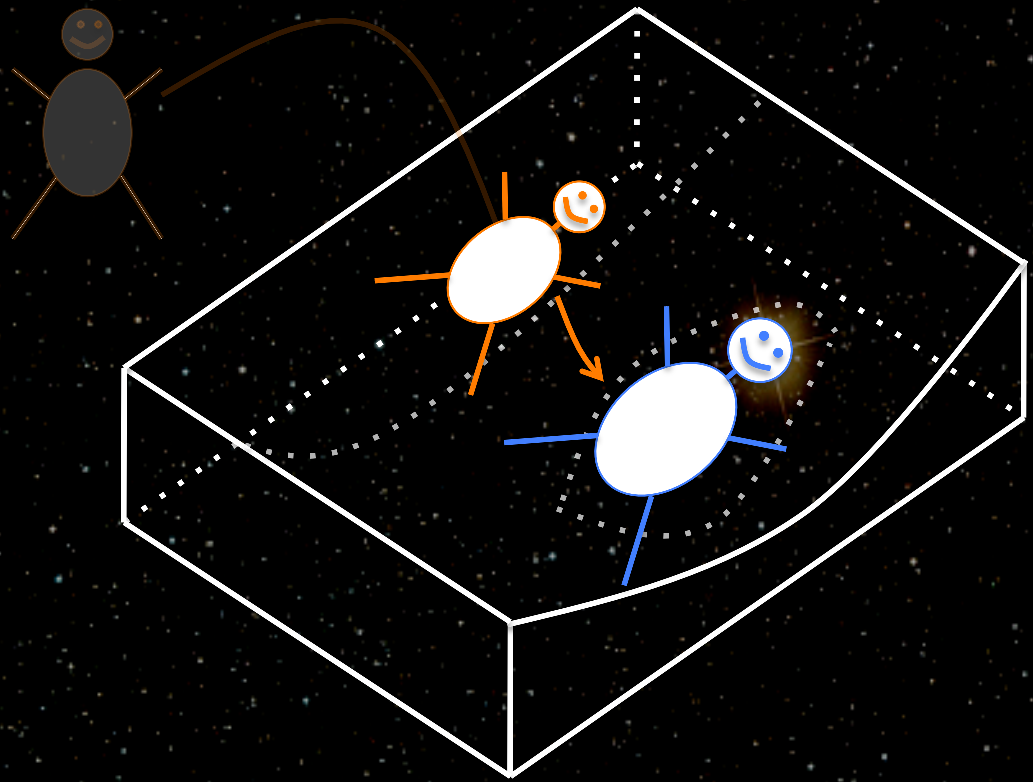


Betelgeuse in Orion

General relativity in every day life:



General relativity in every day life: curvature influences movement



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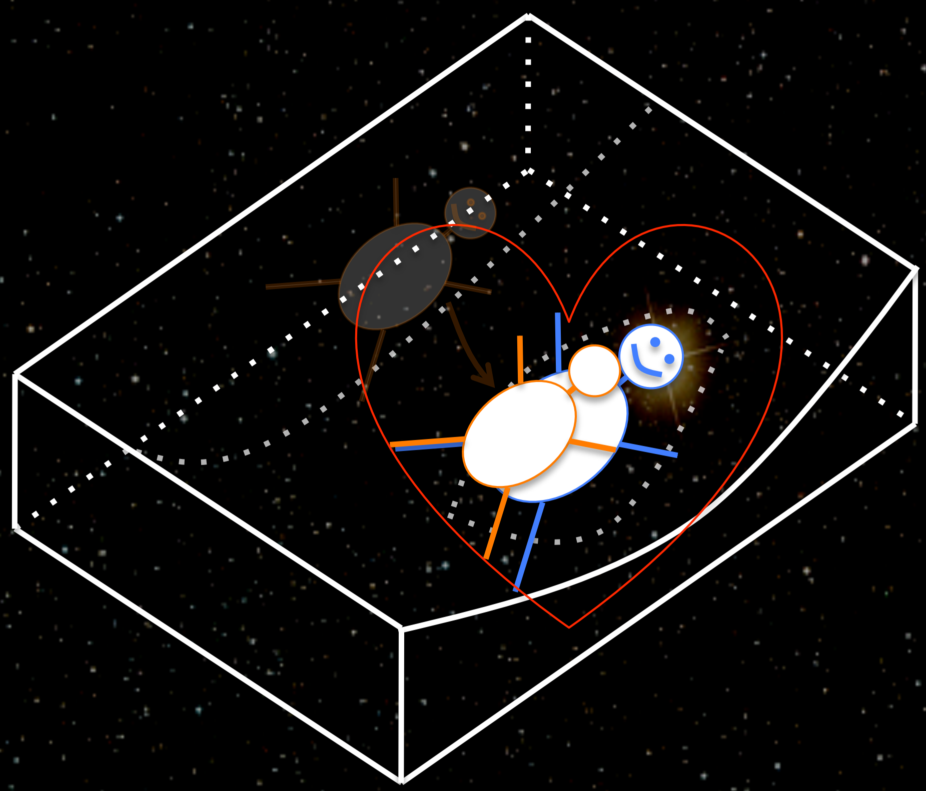
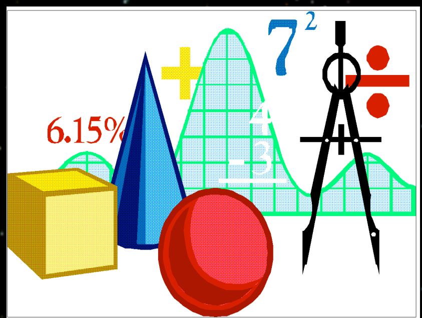
General relativity in every day life:

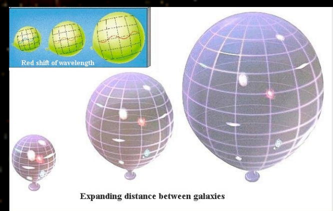


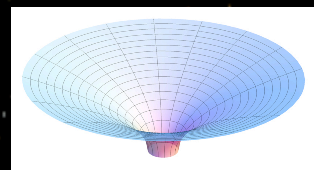
Betelgeuse in Orion

Morale

- Again: Use math to model gravitation.
  - gives better methods for predictions
  - helps understand gravity better
- Gauß/Riemann's new math allows to predict new phyics!







Outline of our tour

1

Sir Isaac Newton 1643-1727

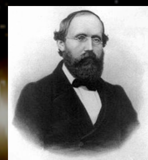
2

Siméon Denis Poisson 1781-1840

Pierre Simon Laplace 1749-1827

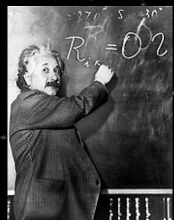
3

Carl Friedrich Gauß 1777-1855



Bernhard Riemann 1826-1866

4



Albert Einstein 1879-1955

5

Jürgen Ehlers 1929-2008

today

Betelgeuse in Orion

Can we forget about Newton?

Naive Idea: Yes!
Einstein's general relativity
is much better
(in predicting experiments)
And much more beautiful!

But: also more difficult!

Betelgeuse in Orion

Can we forget about Newton?

Better: Reconcile the theories:
Think of Newton's theory
as an approximation to Einstein's?

Also:
Try to learn from Newton's theory
how to interpret relativistic notions!

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Example:
What is mass in general relativity?

Negative mass?

Many different definitions

At infinity?

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My thesis: What is a good definition of relativistic mass?

Step 1: differential geometry
+ vector calculus
= new formula for mass

Step 2: use Newtonian limit by to compare new definition with Newtonian mass

Betelgeuse in Orion

Mass in general relativity

new formula for mass

(analogy to Newtonian formula):

$$M = \iint_{\text{surface of sun}} \vec{\nabla} U \cdot \vec{n} \, dS$$

U, $\vec{n}$, dS , $\vec{\nabla}$ constructed from geometry

Newtonian limit

Newton's theory: c=infinite

Einstein's theory: c=300.000km/s

Newtonian limit:

take c to infinity

My thesis: When is relativistic mass approximatively Newtonian mass?

Result: When a star or galaxy does not move

0 m/h

then its relativistic mass is approximately equal to its Newtonian mass.

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My theorem

Theorem 6.4.1 (Newtonian Limit of Mass Theorem). *Let* $\mathcal{F}(\lambda) := (\mathbb{R} \times E^3, s^{\alpha\beta}(\lambda), t_{\alpha\beta}(\lambda), \Gamma^{\mu}_{\alpha\beta}(\lambda), T^{\alpha\beta}(\lambda), \lambda)$ *be a family of static isolated ends in frame theory parametrized by* $\lambda \in (0, \varepsilon)$ *for some* $\varepsilon > 0$ *and let* $\mathcal{F}(0) := (\mathbb{R} \times E^3, s^{\alpha\beta}(0), t_{\alpha\beta}(0), \Gamma^{\mu}_{\alpha\beta}(0), T^{\alpha\beta}(0), 0)$ *be a static isolated system of FT with global Cartesian coordinates* $(x^k(0))$*. Assume that there exist global asymptotically flat systems of coordinates* $(x^k(\lambda))$ *for* $\mathcal{F}(\lambda)$ *converging to* $(x^k(0))$ *uniformly on* M^3 *as* $\lambda \to 0$*. Let* ${}^3g_{ij}(\lambda)$, $\gamma_{ij}(\lambda)$, $\gamma_{ij}(0)$, $U(\lambda)$, *and* $U(0)$ *denote the physical and pseudo-Newtonian metrics and potentials of* $\mathcal{F}(\lambda)$ *and* $\mathcal{F}(0)$*, respectively. Then*

$$m_{ADM}({}^3g(\lambda)) = m_{PNFT}(\gamma(\lambda), U(\lambda)) \to m_{PNFT}(\gamma(0), U(0)) = m_N(U(0))$$

as $\lambda \to 0$.

What is the Newtonian Limit?

See movie

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