Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 4 — due 17.11. — T. Piecha

Exercise 1 (7 points)
A term M is called *minimal* with respect to β-reduction iff for all terms N: If $M \triangleright_\beta N$, then $M \equiv_\alpha N$. Show that

(a) all β-normal forms are minimal; (6 points)

(b) not all minimal terms are β-normal forms. (1 point)

Exercise 2 (9 points)
Find λ-terms **B**, **W**, **X** and **Z** such that the following equalities hold:

(a) $\mathbf{B}xyz =_\beta x(yz)$

(b) $\mathbf{W}xy =_\beta xyy$

(c) $\mathbf{X}xy =_\beta \mathbf{X}yx$

(d) $\mathbf{Z}x =_\beta y\mathbf{Z}$

Show that your λ-terms have the desired behaviour by reducing the following terms:

- $\mathbf{B}MNO$ (2 points)
- $\mathbf{W}MN$ (2 points)
- $\mathbf{X}MN$ (2 points)
- $\mathbf{Z}MNO$ (2 points)

Could **Z** be a combinator? (1 point)

Exercise 3 (4 points)
Show that the fixed-point combinators

$$\mathbf{\Theta} :\simeq (\lambda zx.x(zzx))(\lambda zx.x(zzx))$$

and

$$\mathbf{\Upsilon} :\simeq \lambda x.(\lambda y.x(yy))(\lambda y.x(yy))$$

have no β-normal forms. (Show in each case that there exists a non-terminating leftmost or quasi-leftmost reduction series.)