Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 5 — due 24.11. — T. Piecha

Exercise 1 (6 points)

Reduce the following terms to β-normal form:

(a) $(\lambda u.\, \mathbf{R}\, \underline{0}(\lambda uv.(\lambda xy.x)uv)u)\, \underline{1}$ (3 points)

(b) $(\lambda u.\, \mathbf{R}\, \underline{0}(\lambda uv.(\lambda xy.y)uv)u)\, \underline{1}$ (3 points)

where $\mathbf{R} :\simeq \mathbf{\Theta}(\lambda uxyz.\, \mathbf{D}x(y(\mathbf{V}z)(uxy(\mathbf{V}z)))z)$.

Remark: Reduce applications of **D** and **V** according to Lemma 1.29, cases 2 and 3.

Exercise 2 (8 points)

Construct combinators for the following (intuitively defined) recursive functions:

(a) (2 points)
$$mult(m,0) = 0$$
$$mult(m,n+1) = add(m,mult(m,n))$$

(b) (3 points)
$$fact(0) = 1$$
$$fact(n+1) = mult(n+1,fact(n))$$

(c) (3 points)
$$non(0) = 1$$
$$non(n+1) = 0$$

Remarks:

- First present definitional equations according to the schemata given in Definition 1.31.
- The function *add* has already been λ-defined in the lecture notes.

  You may also define auxiliary functions, if necessary.

Exercise 3 (6 points)

Evaluate the following terms stepwise to normal form:

(a) $\underline{2}\,\underline{3}$ (2 points)

(b) $\lambda x.\, \underline{2}(\underline{3}x)$ (2 points)

(c) $\lambda xy.(\underline{2}x)((\underline{3}x)y)$ (2 points)

Observing the behaviour of these terms provide combinators ***Add***, ***Mult*** and ***Exp*** for addition, multiplication and exponentiation on Church numerals without using the recursion combinator **R**.