



Problem Statement

- Consistent particle tracking requires conforming, mass-conservative velocity fields.
- This is not guaranteed by the standard Finite Element Method (FEM).

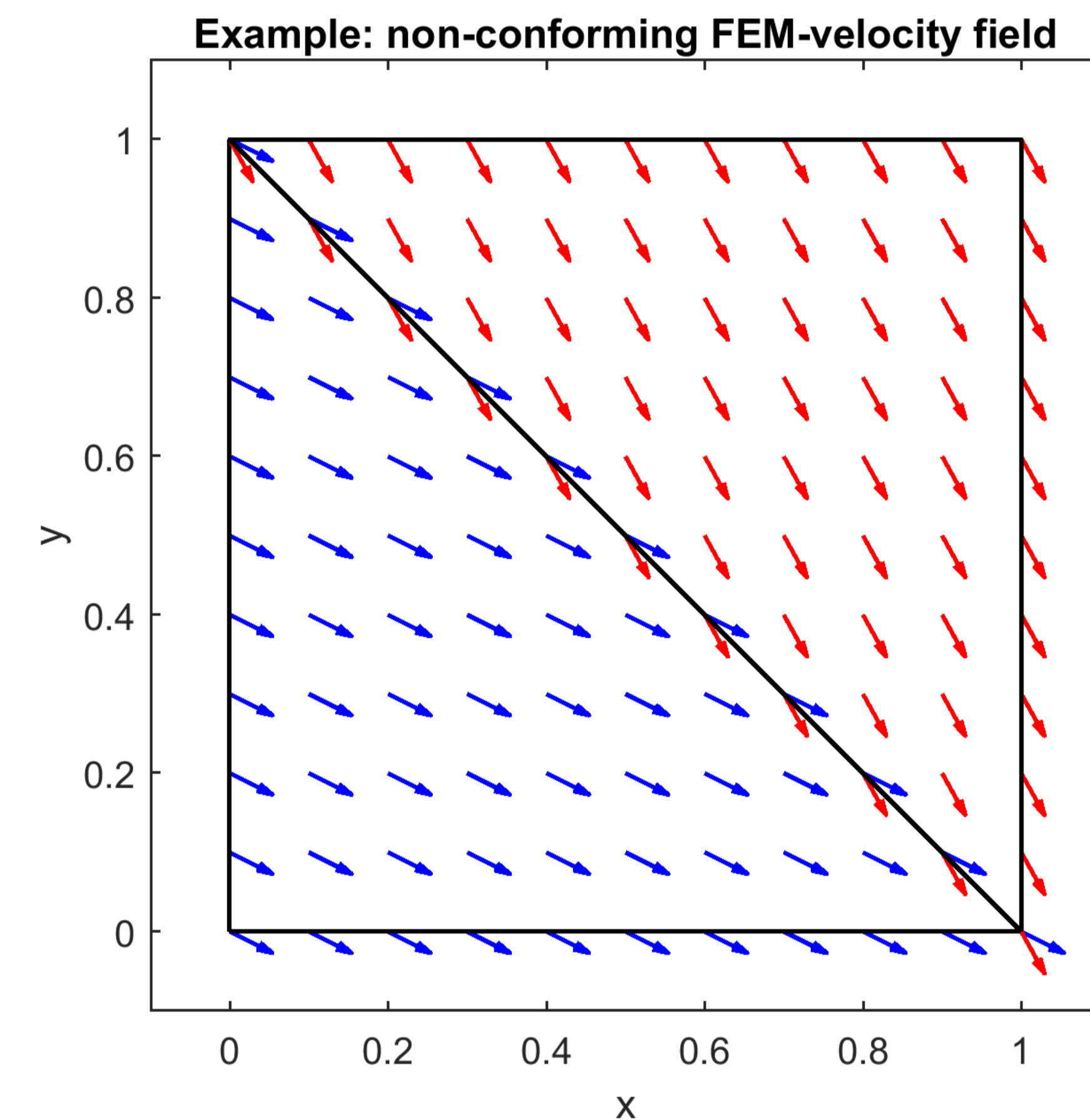


Figure 1: Standard FEM yields continuous heads but non-conforming velocity fields.

Approach

- Project the velocity field of an FEM solution onto a conforming one:

$$\pi^{RTN_0} : L^2(\Omega, \mathbb{R}^d) \rightarrow RTN_0$$
- Find analytical element-wise solutions for particle trajectories.

Numerical Results

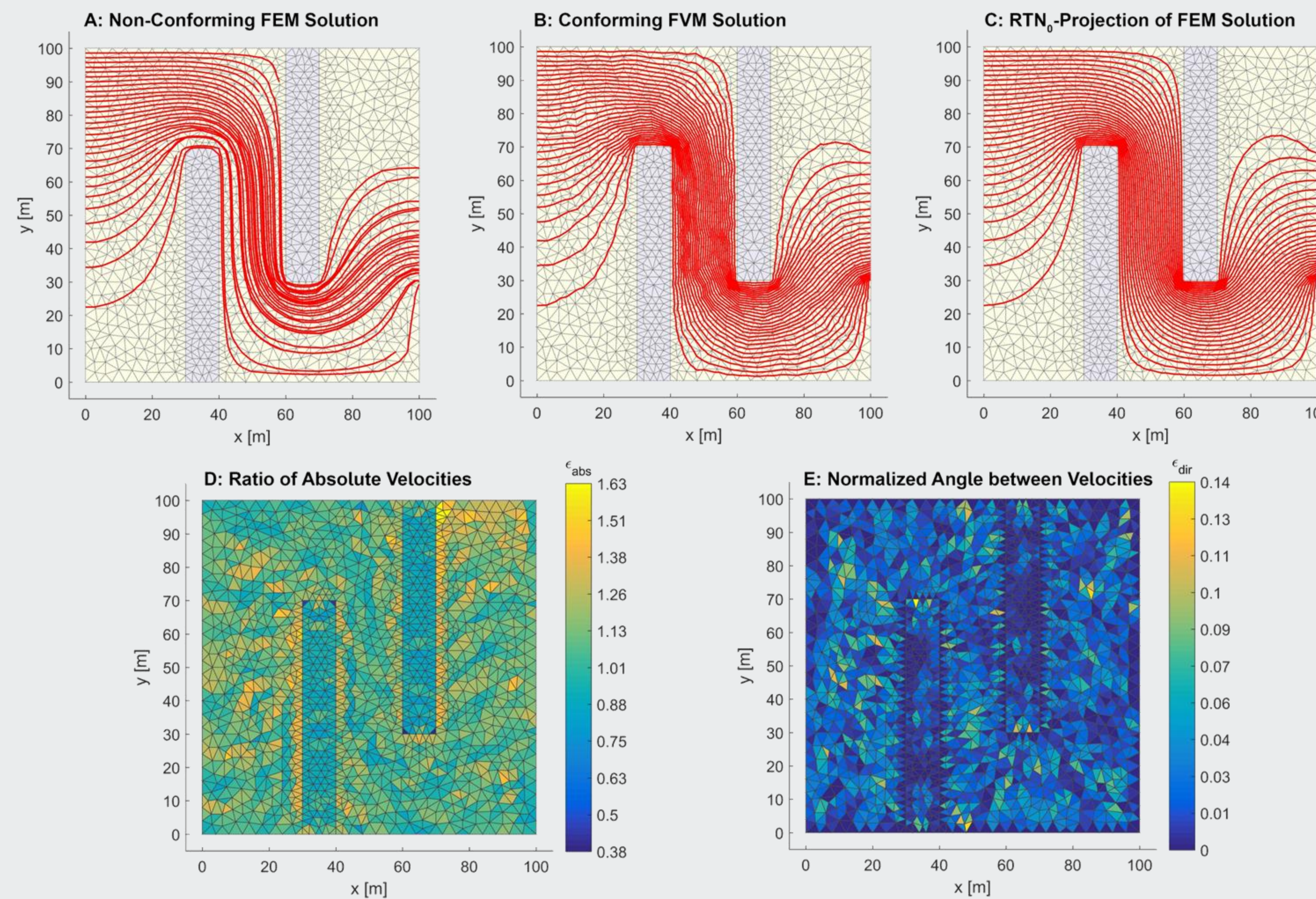


Figure 2: Particle trajectories and difference measures of the two conforming velocity fields for saturated and non-divergent flow in a 2-D domain with two nearly impermeable walls, and non-conforming particle tracking based on a standard FEM solution. (Selzer and Cirpka, 2020)

- Particle tracking based on a non-conforming FEM velocity field is inconsistent; the tracks stagnate; the overall pattern is erroneous.
- The RTN_0 –projection yields a smooth and conforming velocity field.
- Consistency and convergence of the RTN_0 –projection can empirically be shown.
- A cell-centered FVM reconstruction of the head problem also yields good velocity fields, with some grid effects.

Conclusions

- The RTN_0 –projection is smoother and more accurate.

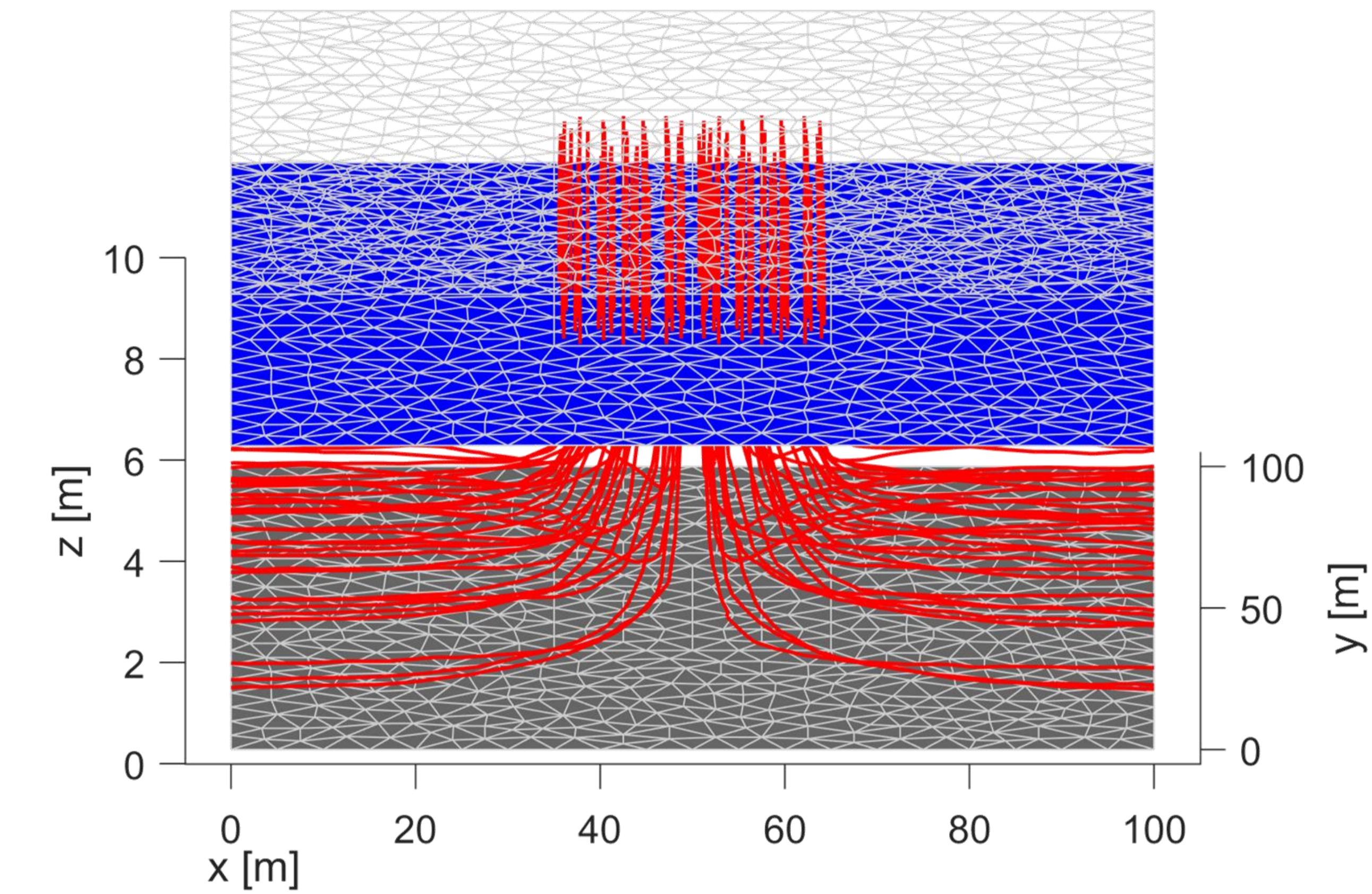


Figure 3: Particle tracking in variably saturated flow.

- The FVM reconstruction is numerically more robust, faster to compute, and accurate enough for catchment-scale simulations.

Outlook

- Parallelization of the particle tracking code for computation on GPUs
- Including all functionalities needed for subsurface modeling

Selzer, P., Cirpka, O.A.: Postprocessing of standard Finite-Element velocity fields for accurate particle tracking applied to groundwater flow, *Computational Geosciences*, 2020. (in press)