

Exercise 1 (1 + 2 + 2 + 3 points)

Write the following abbreviated λ -terms with all missing parentheses and λ 's. Then write down all subterms, free variables and bound variables.

- (a) $\lambda x.(zy)$
- (b) $(\lambda x.xy)(\lambda y.yx)$
- (c) $(\lambda yx.xy)((\lambda z.z)y)(\lambda xz.x)$
- (d) $(\lambda xyz.xz)((\lambda zy.yy)z)((zz)(zz))$

Exercise 2 (2 points)

We consider the terms in Exercise 1. Rename, if necessary, all bound variables in such a way that no free variable has a bound occurrence.

Exercise 3 (2 points)

Evaluate the following substitutions:

- (a) $(\lambda y.x(\lambda w.vxwx))[(uv)/x]$
- (b) $((xy)(\lambda v.xv))[(\lambda y.vy)/x]$

Exercise 4 (6 + 2 points)

- (a) Prove that

$$M[P/x][Q/x] \equiv_{\alpha} M[(P[Q/x])/x]$$

holds for all λ -terms M, P, Q .

- (b) Why does $M[P/x][Q/x] \simeq M[(P[Q/x])/x]$ *not* hold in general?