

[2] F. MUNINI and F. PARLEMENTO, *Admissibility of the structural rules in Kanger's sequent calculus for first order logic with equality*, *Logic Colloquium 2015*, this BULLETIN, vol. 22 (2016), no. 3, p. 414.

[3] V. P. OREVKOV, *On nonlengthening applications of equality rules*. *Zapiski Nauchnyh Seminarov LOMI*, vol. 16 (1969), pp. 152–156 (In Russian) English translation in: *Seminars in Mathematics: Steklov Math. Inst.* (A. O. Slisenko, editor), Studies in Constructive Logic, vol. 16, Consultants Bureau, NY–London, 1971, pp. 77–79.

[4] F. PARLEMENTO and F. PREVIALE, *The cut elimination and nonlengthening property for the sequent calculus with equality*, *Logic Colloquium*, 2016, arXiv 1705.00693.

- THOMAS PIECHA AND PETER SCHROEDER-HEISTER, *Intuitionistic logic is not complete for standard proof-theoretic semantics*.

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Prawitz conjectured that intuitionistic first-order logic is complete with respect to a notion of proof-theoretic validity [1,2,3]. We show that this conjecture is false. The notion of validity obeys the following standard conditions, where  $S$  refers to atomic bases (systems of production rules):

1.  $\models_S A \wedge B \iff \models_S A$  and  $\models_S B$ .
2.  $\models_S A \vee B \iff \models_S A$  or  $\models_S B$ .
3.  $\models_S A \rightarrow B \iff A \models_S B$ .
4.  $\Gamma \models A \iff$  For all  $S$ : ( $\models_S \Gamma \implies \models_S A$ ).
5. If  $\Gamma \models A$  and  $\Gamma, A \models_S B$ , then  $\Gamma \models_S B$ .

Any semantics obeying these conditions satisfies the generalized disjunction property:

For every  $S$ : if  $\Gamma \models_S A \vee B$ , where  $\vee$  does not occur positively in  $\Gamma$ , then either  $\Gamma \models_S A$  or  $\Gamma \models_S B$ .

This implies the validity ( $\models$ ) of Harrop's rule  $\neg A \rightarrow (B \vee C) / (\neg A \rightarrow B) \vee (\neg A \rightarrow C)$ , which is admissible but not derivable in intuitionistic logic.

[1] D. PRAWITZ, *Towards a foundation of a general proof theory*, *Logic, Methodology and Philosophy of Science IV* (P. Suppes et al., editors), North-Holland, 1973, pp. 225–250.

[2] ———, *An approach to general proof theory and a conjecture of a kind of completeness of intuitionistic logic revisited*, *Advances in Natural Deduction* (L. C. Pereira, E. H. Haeusler, and V. de Paiva, editors), Springer, Berlin, 2014, pp. 269–279.

[3] P. SCHROEDER-HEISTER, *Validity concepts in proof-theoretic semantics*. *Synthese*, vol. 148 (2006), pp. 525–571.

- EDOARDO RIVELLO, *On extending the general recursion theorem to non-wellfounded relations*.

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The principle of definition by recursion on a wellfounded relation [1], can be stated as follows: Let  $A$  be any set and let  $P$  be the set of all partial functions from  $A$  to some set  $B$ . Let  $G: A \times P \rightarrow B$  be any function and let  $R \subseteq A \times A$  be any binary relation.

Fact 1 (Montague): If  $R$  is wellfounded on  $A$  then there exists a unique function  $f: A \rightarrow B$  such that

$$\forall x \in A (f(x) = G(x, f \upharpoonright x^R)), \quad (1)$$

where  $x^R = \{y \in A \mid y R x\}$ .

If  $R$  is not wellfounded on the entire domain  $A$ , an obvious way of extending this method of definition is to identify a proper subset  $W$  of  $A$  on which  $R$  is wellfounded and to apply the principle to this set. The usual choice for  $W$  is the *wellfounded part* of  $R$ , defined as the set of all  $R$ -wellfounded points of  $A$ .

In my talk, after examining several different strategies to prove Fact 1, I will present a new approach to extend this method of definition to all kinds of binary relations. We look at subsets  $X$  of  $A$  on which  $R$  is not necessarily wellfounded, yet there exists a unique function