Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 13 — due Wednesday 31.1. at 18h — T. Piecha

Exercise 1 (5 points)
Solve the type inhabitation problem for the following types; that is, for each type construct a term having this type.

(a) $(\alpha \to \beta) \to ((\beta \to \gamma) \to (\alpha \to \gamma))$ (2 points)

(b) $(\alpha \to (\beta \to \gamma)) \to (\beta \to (\alpha \to \gamma))$ (3 points)

Exercise 2 (5 points)

(a) Show that the type $((\alpha \to \beta) \to \alpha) \to \alpha$ is *not* inhabited. (3 points)

(b) Is the type $((((\alpha \to \beta) \to \alpha) \to \alpha) \to \beta) \to \beta$ inhabited? (2 points)

— Exercises 3 and 4 are not to be handed in —

Exercise 3
Show that the following sequents are derivable in $\lambda 2$.

(a) $\vdash \lambda x.xx : \forall \beta.(\forall \alpha.\alpha \to (\beta \to \beta))$

(b) $\vdash \lambda x.x : \forall \beta.(\forall \alpha.\alpha \to \beta)$

(c) $\vdash \lambda x.x : \forall \alpha.(\alpha \to \alpha)$

(d) $\vdash \lambda x.x : \forall \alpha.\alpha \to \forall \beta.\beta$

Considering (b)-(d), explain why a λ-term can have different types in $\lambda 2$.

Exercise 4
Show that in $\lambda 2$ every Church numeral $\underline{n}$ has type $\forall \alpha.((\alpha \to \alpha) \to (\alpha \to \alpha))$.