Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 2 — due 3.11. — T. Piecha

Exercise 1 (4 points)
Prove that

$$M[P/x][Q/x] \equiv_\alpha M[(P[Q/x])/x]$$

holds for all λ-terms M, P, Q.

Exercise 2 (7 points)
We consider the following λ-terms:

(1) $(\lambda x.y)x$

(2) $(\lambda x.x(xy))z$

(3) $(\lambda x.xxy)(\lambda y.xyy)$

(4) $(\lambda x.xyy)(\lambda x.xxy)$

(5) $(\lambda yx.xy)((\lambda z.z)y)(\lambda xz.x)$

(6) $(\lambda xyz.xz)((\lambda zy.yy)z)((zz)(zz))(\lambda x.xx)$

(a) Which terms have a β-normal form? (Give successive β-contractions.) (3 points)

(b) Which terms are strongly normalisable? (2 points)

(c) Which terms are β-equal? (2 points)

Exercise 3 (6 points)
Give β-reduction series for the following λ-terms, where $\mathbf{K} :\simeq \lambda xy.x$ and $\mathbf{\Omega} :\simeq (\lambda x.xx)(\lambda x.xx)$:

(a) $\mathbf{KK}(\mathbf{KK})$ (2 points)

(b) $\mathbf{K\Omega}(\mathbf{K\Omega})$ (2 points)

(c) $\mathbf{\Omega K}(\mathbf{\Omega K})$ (2 points)

Exercise 4 (3 points)
Which of the following statements hold for arbitrary λ-terms M and N? Give a short justification or present a counterexample.

(a) If $M[N/x]$ is in β-normal form, then M is in β-normal form. (1 point)

(b) If $M[N/x]$ has a β-normal form, then M has a β-normal form. (1 point)

(c) If M has a β-normal form, then $M[N/x]$ has a β-normal form. (1 point)