

Exercise 1 (2 points)

Show: If $\Gamma \vdash M : \sigma$, then $\Gamma|_{\text{FV}(M)} \vdash M : \sigma$.

What has to be done to assign a type to a term M if $\text{FV}(M) \neq \emptyset$?

Exercise 2 (9 points)

For each of the sequents below formulate its associated system of equations.

Then use the unification algorithm to find an mgu, if it exists.

(a) $\vdash \lambda xy.x(yx) : \alpha \rightarrow \sigma$ (3 points)

(b) $x : \alpha \vdash \lambda y.xyx : \sigma$ (3 points)

(c) $x : \sigma \rightarrow \tau \vdash \lambda z.xzz : \tau$ (3 points)

Exercise 3 (9 points)

Which of the following terms are typable? Show this.

(a) $\lambda xy.xyx$ (2 points)

(b) $\lambda z.z(\lambda y.z)$ (2 points)

(c) $\lambda z.z(\lambda y.zy)$ (2 points)

(d) $(\lambda xz.x)(\lambda x.xxy)$ (3 points)