25 easy pieces in MATHSTAT

"I fear not the man who has practiced 10,000 kicks once, but I fear the man who has practiced one kick 10,000 times."

Bruce Lee

1: Write the expectation of a random variable (r.v.) Z, $\mathbb{E}[Z]$, extensively

a) for a discrete random variable,

b) for a continuous random variable.

2: $\text{Var}(Z)$ can be written as $\mathbb{E}[Y]$. What is Y?

3: Write $\text{Var}(Z)$ extensively

a) for a discrete random variable,

b) for a continuous random variable.

4: What does the cumulative density function or cumulative distribution function (c.d.f.) tell you?

5: X is a continuous r.v. How are the c.d.f. $F_X(x)$ and the density function (d.f.) $f_X(x)$ related?

6: $\text{Cov}(X, Y)$ can be written as $\mathbb{E}[Z]$. What is Z?

7: Write $\text{Cov}(X, Y)$ extensively for X and Y

a) as discrete random variables,

b) as continuous random variables.

8: Express $\mathbb{E}_{XY}[XY]$ as a function of $\text{Cov}(X, Y)$.

9: Write $\mathbb{E}_{XY}[XY]$ extensively for X and Y

a) as discrete random variables,

b) as continuous random variables.

10: $g(X)$ denotes a measurable function of the r.v. X (like e.g. X^2, $\ln(X)$). Write $\mathbb{E}[g(X)]$ extensively for a continuous r.v. X.

11: X and Y are continuous random variables. $Z = g(X,Y)$ is a measurable function. Write $\mathbb{E}[g(X,Y)]$ extensively.

12: X and Y are continuous random variables. What does the joint c.d.f. $F_{XY}(x,y)$ tell you? Write $F_{XY}(x,y)$ extensively. What does the joint p.d.f. $f_{XY}(x,y)$ (discrete case) tell you?

13: X and Y are continuous random variables. How are $F_{XY}(x,y)$ and $f_{XY}(x,y)$ related?

14: If X and Y are independent:

a) $F_{XY}(x,y) =$,

b) $f_{XY}(x,y) =$.

15: If X and Y are independent:

a) $\mathbb{E}_{XY}(XY) =$,

b) $\mathrm{Cov}(X,Y) =$.

16: If X and Y are independent:
$\mathbb{E}_{XY}[h(X)g(Y)] =$.

17: $\mathbb{E}_{XY}[X+Y] =$,
$\mathbb{E}_{XYZ}[X+Y+Z] =$,
$\mathrm{Var}(X+Y) =$.

18: Write extensively for X and Y

a) as discrete random variables,

b) as continuous random variables:

$f_{X|Y}(X|Y=y)$
$\mathbb{E}_{X|Y}[X|Y=y]$
$\mathbb{E}_{X|Y}[X^2|Y=y]$

19: $\mathbb{E}[aX] =$,
$\mathrm{Var}(aX) =$,
(a is non-random scalar).

20: For $\underline{X} = (X_1, X_2, \dots, X_n)'$

$\mathbb{E}[\underline{X}] = \mu$, $\mu =$?

$\text{Var}(\underline{X}) = \Sigma$, $\Sigma =$?

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

(A is a non-random matrix).

$\underline{Z} = A \cdot \underline{X}$,

$\mathbb{E}[\underline{Z}] =$

$\text{Var}(\underline{Z}) =$.

21: $Y = a + b \cdot X$

$\mathbb{E}[Y] =$

$\mathbb{E}[Y|X = x] =$.

22: Given the joint density $f_{XY}(x, y)$: how do you get $f_X(x)$ and $f_Y(y)$?

a) for discrete random variable,

b) for continuous random variable.

23: Under which conditions can $f_{XY}(x, y)$ be obtained from $f_X(x)$ and $f_Y(y)$?

24: X and Y are jointly normally distributed

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim BVN\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho_{XY}\right).$$

What is the relation of parameters and moments? $X \sim$

$Y \sim$

$X|(Y = y) \sim$

$Y|(X = x) \sim$

$\mathbb{E}[X|Y = y] =$

$\text{Var}(X|Y = y) =$

25: X, Y and Z are normally distributed.

$W = a \cdot X + b \cdot Y + c \cdot Z$

How is W distributed?