Theoretical Foundations of Logic Programming — SS 2018

Exercise sheet 1 — due 27.4. — T. Piecha

Exercise 1 (4 points)
Write the following formulas with all missing parentheses, and present their respective syntax trees.

(a) $p_0 \wedge (p_1 \wedge p_2 \wedge p_3) \wedge \neg p_4$ (2 points)

(b) $\neg(p_1 \wedge p_2) \to \neg p_1 \vee \neg p_2$ (2 points)

Exercise 2 (4 points)
Show that $(A \vee C) \wedge (B \vee \neg C) \mathrel{=\!\mid\!\models} (A \vee C) \wedge (B \vee \neg C) \wedge (A \vee B)$.

Exercise 3 (6 points)
Show that for every set of formulas Γ and any formulas A, B, C the following holds:

(a) If $\Gamma, A \models B$ and $\Gamma, A \models \neg B$, then $\Gamma \models \neg A$. (3 points)

(b) If $\Gamma, A \models C$ and $\Gamma, B \models C$, then $\Gamma, A \vee B \models C$. (3 points)

Exercise 4 (6 points)
We consider only $\to$-free formulas A, i.e. formulas in the fragment $\{\wedge, \vee, \neg\}$.
We define the *dual* A^* of a formula A recursively as follows:

$$\begin{aligned} A^* &:= A \quad \text{if } A \in \{p_0, p_1, p_2, \ldots\} \\ (A \wedge B)^* &:= (A^* \vee B^*) \\ (A \vee B)^* &:= (A^* \wedge B^*) \\ (\neg A)^* &:= \neg A^* \end{aligned}$$

(a) Prove by induction that for any two formulas A and B: $A \mathrel{=\!\mid\!\models} B \iff A^* \mathrel{=\!\mid\!\models} B^*$. (4 points)

(b) We now change the above definition of A^* by replacing the first clause by

$$A^* := \neg A \quad \text{if } A \in \{p_0, p_1, p_2, \ldots\}$$

Prove that $A^* \mathrel{=\!\mid\!\models} \neg A$ for every formula A. (2 points)