

- WAGNER DE CAMPOS SANZ, THOMAS PIECHA, *A criticism of the BHK interpretation.*

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We criticize the BHK interpretation by considering basic and non-basic extensions of natural deduction for intuitionistic logic NJ. For any basic extension we present a construction (which depends on the disjunction property) that converts each proof of the premiss  $\neg A \rightarrow (B \vee C)$  of Harrop's rule into a proof of its conclusion  $(\neg A \rightarrow B) \vee (\neg A \rightarrow C)$ . According to the BHK interpretation for implication the corresponding Kreisel–Putnam formula  $(\neg A \rightarrow (B \vee C)) \rightarrow ((\neg A \rightarrow B) \vee (\neg A \rightarrow C))$  is then assertable. However, the Kreisel–Putnam formula is not derivable in any basic extension of NJ. Therefore the BHK interpretation and NJ mismatch for basic extensions.

Next we consider non-basic extensions. In intuitionistic (Heyting) arithmetic HA the open formula  $Odd(x) \vee Even(x)$  has a proof in strictly normal form whose last step is an application of the induction rule. The disjunction property does not hold for this formula since neither  $Odd(x)$  nor  $Even(x)$  are provable in HA. Thus the construction used in the case of basic extensions fails for non-basic extensions like HA. That is, if the BHK interpretation is intended to be adequate for all intuitionistically acceptable extensions, including non-basic ones, then the aforementioned mismatch disappears. However, the BHK interpretation cannot then be taken as the correct description of what constitutes an assertion of a complex formula anymore, since  $Odd(x) \vee Even(x)$  is provable in HA and thus HA assertable, but its assertion is not obtained in accordance with the BHK interpretation for disjunction. Therefore the BHK interpretation is at least incomplete with respect to HA. As a consequence, the thesis that the BHK interpretation gives the operational meaning of the logical constants would have to be given up if the induction rule is considered an acceptable extension.