EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

MATHEMATISCH-
NATURWISSENSCHAFTLICHE
FAKULTÄT

Vorstellungsvortrag Habilitationsvorhaben:

Strong Interactions and the Origin of Mass

Davide Campagnari

Physikalisches Kolloquium
18. Januar 2017

Outline

1. Towards the Standard Model
2. Features of Quantum Chromodynamics
   - Asymptotic Freedom
   - Confinement
   - Chiral Symmetry (Breaking)
3. Variational principle and DSEs
   - A (very) short introduction to Quantum Chromodynamics
   - What we want to do…
   - … and how we plan to do it…
   - … and what we got out of it

Elementary (?) Particles

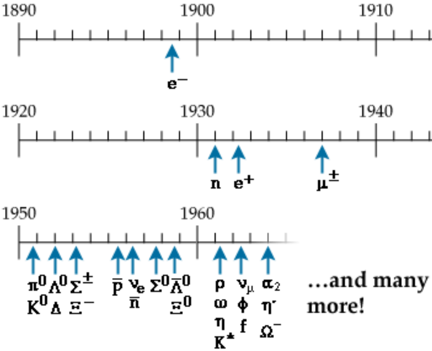
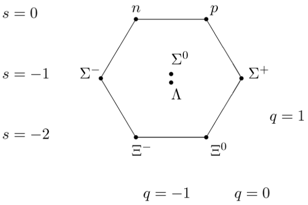


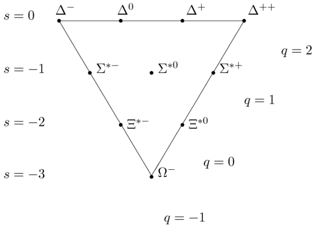
Bild: `particleadventure.org`

Finding some order in the Particle Zoo

Eightfold Way (1961)

Hadrons classified according to the representations of SU(3).





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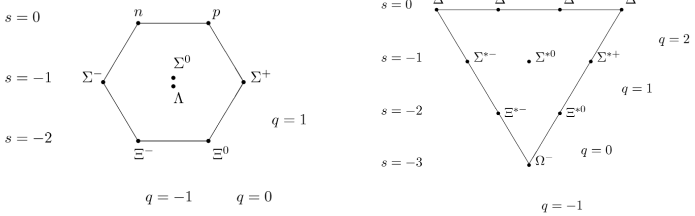

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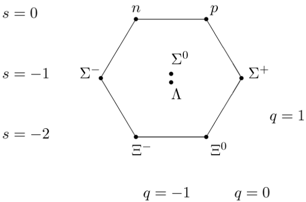

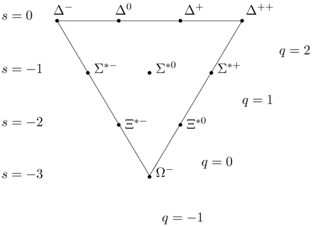

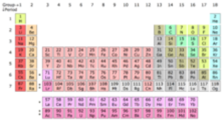

- predicts existence and mass of Ω^-!
- not quite a theory; rather a periodic system

Bilder: `wikipedia`

Let’s talk about quarks

Quarks proposed as actual particles (1964)

- Three quarks: up (charge $\frac{2}{3}e$), down ($-\frac{1}{3}e$), strange ($-\frac{1}{3}e$).
- Baryons made of three quarks: proton uud, neutron udd, …
- Mesons made of quark–anti-quark: pion $u\bar{d}$, kaon $u\bar{s}$, …

$\Rightarrow$ Naturally explains the observed Eightfold Way.

Quarks alone make no theory…

Open questions

OK, quarks *might* exist. But…

- how do they interact?
- what about the statistics of states like Δ^{++} (uuu)?
- why only qqq or $q\bar{q}$ states?
- why can't we observe them directly?

…they need an interaction

Some answers

1965 **colour** introduced to explain statistics

1968 point-like objects observed in SLAC experiments
maybe quarks do *really* exist…

1969 **colour** explains the $\pi^0 \to \gamma\gamma$ decay rate
(historically; nowadays we wouldn't say that any more)

1971 **colour** is used as starting point for an interacting theory:
Quantum Chromodynamics

In the meantime…

At the same time the unified theory of weak and electromagnetic interactions was being developed too. By the early ’70s the Standard Model existed (at least on paper).

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>
> *The fact that the Standard Model had all the chances of being "right" was at the time only obvious to some field theory addicts. (De Rújula)*

Acceptance of the Standard Model

Both experimental and theoretical evidence in 1973:

- Measurement of strangeness-conserving neutral currents
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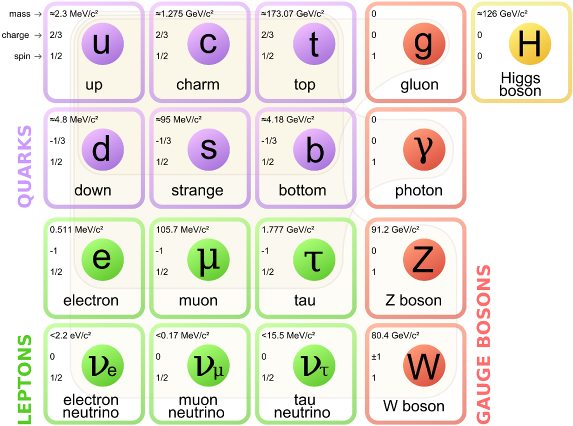
> *Quarks, somewhat reluctantly invented during the 1963/64 Christmas holidays, turned out to be for real. QCD and the rest of the Standard Model evolved from being considered a tropical disease affecting an overwhelmed minority of field theorists to being spoused by practically "everybody". (De Rújula)*

The Standard Model of Elementary Particles

a.k.a the usual picture you see in every talk about particle physics



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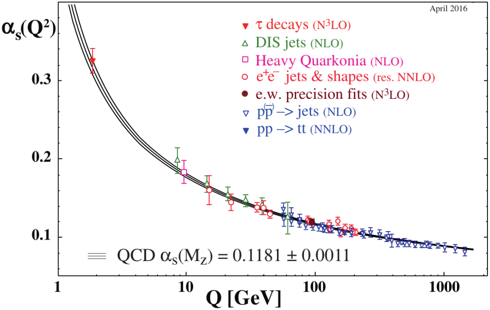
Asymptotic Freedom



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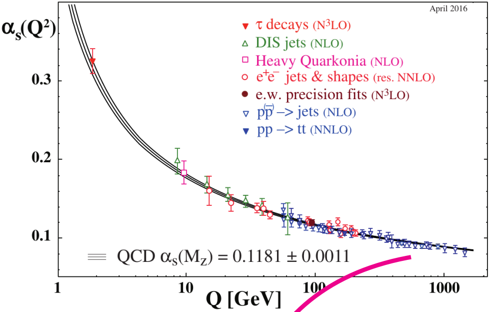

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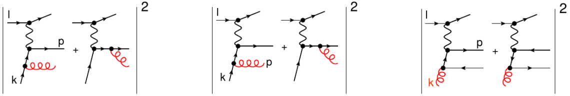
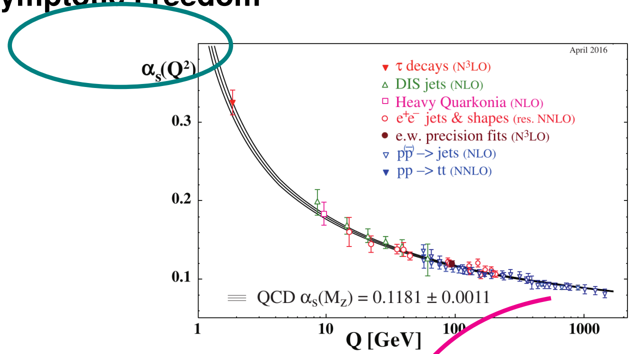


(a) (b) (c)

Schlegel, Vogelsang, Hinderer

Asymptotic Freedom



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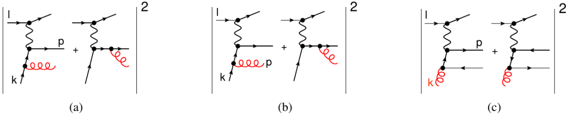

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Where are quarks?

Quark masses from high-energy physics

$$m_u \simeq 2.3\,\mathrm{MeV}, \quad m_d \simeq 4.8\,\mathrm{MeV}, \quad m_s \simeq 95\,\mathrm{MeV}.$$

Open questions — reprise

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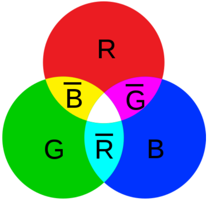
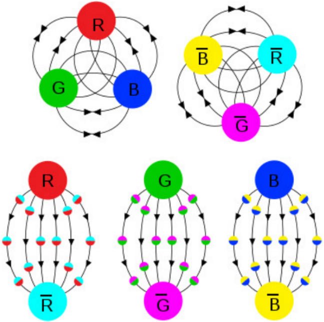
Coloured quarks but only white hadrons

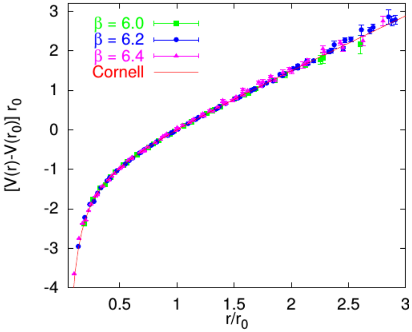
Quarks exist in three colours but hadrons consist only of *colourless* combinations.



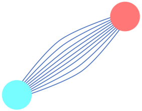


String Tension

Phenomenologically we can describe mesons as built out of (almost massless) quarks joined by a strong *flux tube*.



G. Bali, Phys. Rept. 343,1



$$V(r) = -\frac{\alpha_s}{r} + \sigma r$$

$$\sigma = (440\,\mathrm{MeV})^2$$

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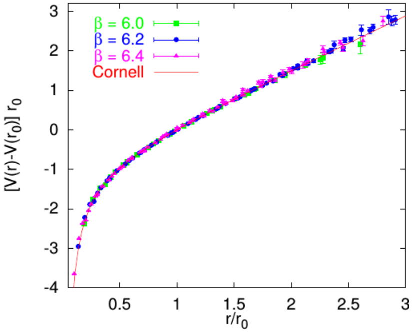

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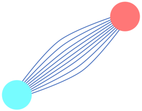


$$V(r) = -\frac{\alpha_s}{r} + \sigma r$$

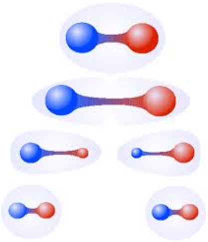
$$\sigma = (440\,\text{MeV})^2$$

$$= 156\,960\,\text{N}$$

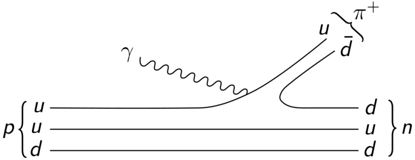
(i.e. 16 tons!)

String breaking

Try and take quarks apart: at some point it becomes energetically favourable to create a quark–anti-quark pair.



$$p + \gamma \to \pi^+ + n$$



Quite analogously $p + \gamma \to \pi^- + \Delta^{++}$

Constituent quarks

Quark “masses”

$$m_u \simeq 2.3\,\text{MeV}, \quad m_d \simeq 4.8\,\text{MeV}, \quad m_s \simeq 95\,\text{MeV}.$$

Particle	Quark content	Mass in MeV
p	uud	938
n	udd	939
ρ^+	$u\bar{d}$	775
ω	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	782
ϕ	$s\bar{s}$	1020

Constituent quarks

Kind of looks like $m_u \simeq m_d \simeq 300\,\text{MeV}$, $m_s \simeq 500\,\text{MeV}$.

The mass puzzle

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Particle	Quark content	Mass in MeV
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K^+	$u\bar{s}$	490
η	$.51(u\bar{u}+d\bar{d})-.68s\bar{s}$	548
η'	$.48(u\bar{u}+d\bar{d})+.73s\bar{s}$	958

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Two questions

If quarks get only a couple of MeV from the Higgs mechanism

- why is the pion so light?
- why is the proton so heavy?

Chiral symmetry

Mass term in the Lagrangian

$$-\mathcal{L}_m = m_u\,\bar{u}\,u + m_d\,\bar{d}\,d + m_s\,\bar{s}\,s = \left(\bar{u}\;\bar{d}\;\bar{s}\right)\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

- does not change if the column vector $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$ is multiplied by a constant phase
- for equal quark masses it does not matter what I call u, d, or s
- for vanishing quark masses ($\equiv$ *chiral quarks*) I can rotate the left- and right-handed components independently

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Admittedly, quarks are not exactly massless, but their masses are smaller than the typical QCD scale, so chiral symmetry should be approximately good.

Symmetries

Noether theorem

For every continuous symmetry of the Lagrangian there exists a conserved quantity.

In the quantum world, life is more involved: a symmetry of the Lagrangian has three possible realizations:

Wigner–Weyl	the symmetry is manifest and corresponds to some conserved quantity
Nanbu–Goldstone	the symmetry is *spontaneously broken* for every broken generator there is a massless *Goldstone boson* (notable exception: Higgs mechanism)
anomalous breaking	the symmetry is explicitly broken by quantum effects

How is chiral symmetry realized?

Manifest symmetries

- global phase rotation ⇒ baryon number conservation
- exchanging u, d, s ⇒ Eightfold Way

Broken (?) symmetries

- axial SU(3) ⇒ 8 Goldstone bosons
  got them: π, K, and somehow η
- axial U(1) ⇒ 1 Goldstone boson
  uhm... η' is the lightest, but it's not so light

Chiral symmetry (breaking) and hadron masses

The chiral condensate $\mathcal{X}$ is an order parameter for chiral symmetry:

$\mathcal{X} = 0$ chiral symmetry manifest

$\mathcal{X} \neq 0$ chiral symmetry broken

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Gell-Mann–Oakes–Renner relation

$$m_\pi^2 = \frac{m_u + m_d}{f_\pi^2}\,\mathcal{X}$$

Ioffe formula

$$m_\mathrm{N}^3 = 4\pi^2\mathcal{X} + \mathcal{O}(m_q)$$

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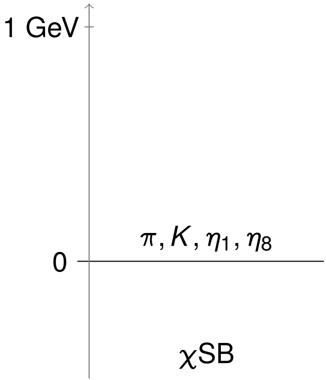
Turns out that the remaining axial U(1) symmetry is *anomalously* broken: this explains the outstandingly large mass of the η'.

Chiral symmetry breaking pattern



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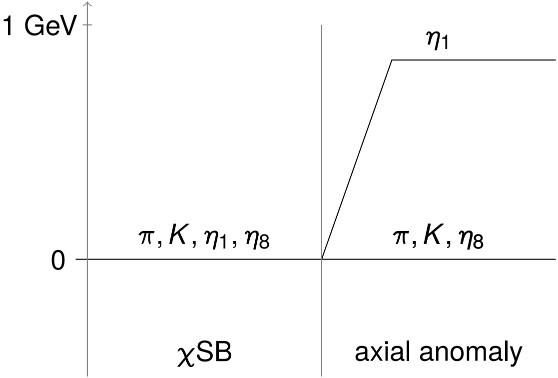

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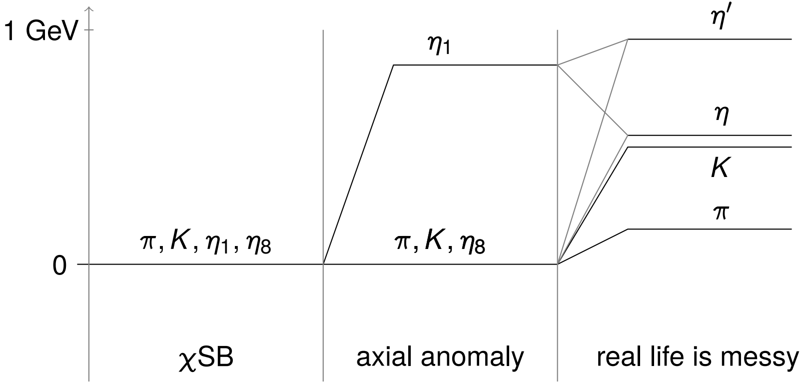

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How do we quantize classical field theory?

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More or less like we quantize classical mechanics.

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Concealing some nasty details, the Hamiltonian of electrodynamics reads

$$H = \frac{1}{2}\int \left(\mathbf{E}_\perp^2 + \mathbf{B}^2\right) + \frac{1}{2}\int \frac{\rho(\mathbf{x})\,\rho(\mathbf{y})}{4\pi|\mathbf{x}-\mathbf{y}|}.$$

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The electric field **E** is the canonical momentum, so we can promote the classical quantities to operators

$$\left[E_i, A_j\right] = \mathrm{i}\delta_{ij}$$

and set up the Schrödinger equation.

The colour fields

Instead of one photon field A_μ

$$\begin{aligned} \mathbf{E} &= -\boldsymbol{\nabla} A_0 - \partial_t \mathbf{A} \\ \mathbf{B} &= \boldsymbol{\nabla} \times \mathbf{A} \end{aligned}$$

The colour fields

Instead of one photon field A_μ there are 8 different gluon fields

$$\mathbf{E}^a = -\boldsymbol{\nabla} A_0^a - \partial_t \mathbf{A}^a$$

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Instead of one photon field A_μ there are 8 different gluon fields

$$\mathbf{E}^a = -\boldsymbol{\nabla} A_0^a - \partial_t \mathbf{A}^a + g\, f^{abc} \mathbf{A}^b A_0^c$$

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The Maxwell equations are also more involved, e.g.

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \rho^a - f^{abc} \mathbf{A}^b \cdot \mathbf{E}^c$$

What we would like to do…

- solve the Schrödinger equation and
- find the ground state $|0\rangle$
- with the ground state evaluate all correlation functions

$$\langle 0|AA|0\rangle, \quad \langle 0|q\bar{q}|0\rangle, \quad \langle 0|q\bar{q}A|0\rangle, \quad \langle 0|q\bar{q}q\bar{q}|0\rangle, \ldots$$

$\Rightarrow$ access (in principle) to all measurable quantities (masses, cross sections, decay rates)

. . . and what we can do

- solve the Schrödinger equation order by order in perturbation theory
- find approximately the ground state $|0\rangle$
- with the ground state we can approximately evaluate the propagators

$$\langle 0|AA|0\rangle, \qquad \langle 0|q\bar{q}|0\rangle$$

Why the propagators?

Propagators are input for *n*-body problem (Bethe–Salpeter & Faddeev equations).

Vanishing propagator at zero momentum implies positivity violation.

From the quark propagator

$$\langle 0|q\bar{q}|0\rangle \propto \boldsymbol{\alpha}\cdot\mathbf{p}+\beta M(\mathbf{p})$$

we can calculate the chiral condensate

$$\mathcal{X} \propto \int M(\mathbf{p}).$$

Variational principle

The true ground state has minimal energy

$$E_0 \leq \langle \Psi | H | \Psi \rangle, \qquad \forall \quad |\Psi\rangle.$$

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Theoretical physicists attain their highest satisfactions from the harmonic oscillator, so we usually start with a Gaussian.

$$\Psi[A] \propto \exp\left\{ -\frac{1}{2} \int A(\mathbf{x})\, \omega(\mathbf{x}-\mathbf{y})\, A(\mathbf{y}) \right\}$$

- Wick’s theorem holds
- $\omega(\mathbf{x}-\mathbf{y})$ is a variational kernel
- $\omega(\mathbf{p})$ represents the quasi-particle energy

Qualitatively is everything there! But we’d like to do more.

Dyson–Schwinger equations

Try something more involved

$$\Psi \propto \exp\left\{-\frac{1}{2}\gamma_2 A^2 - \frac{1}{3!}\gamma_3 A^3 - \frac{1}{4!}\gamma_4 A^4 - q^\dagger(\bar{\gamma} + \bar{\Gamma}_0 A)q + \ldots\right\}.$$

Expectation values in the ground state $\psi_0 \propto e^{-S/2}$

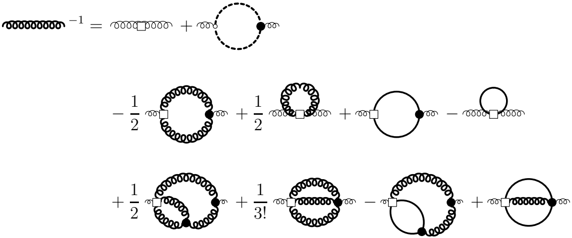
$$\langle f(x)\rangle = \int dx\, |\psi_0(x)|^2 f(x) = \int dx\, e^{-S(x)} f(x).$$

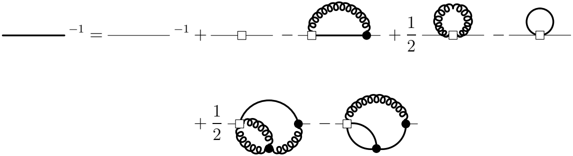
If the ground state is normalizable (which would be appealing)

$$\left\langle \frac{df(x)}{dx} \right\rangle = \left\langle \frac{dS(x)}{dx} f(x) \right\rangle.$$

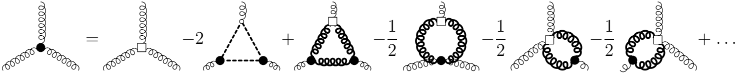
For a given wave function $S(x)$ this returns a series of equations relating expectation values.

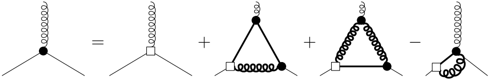
Propagator equations





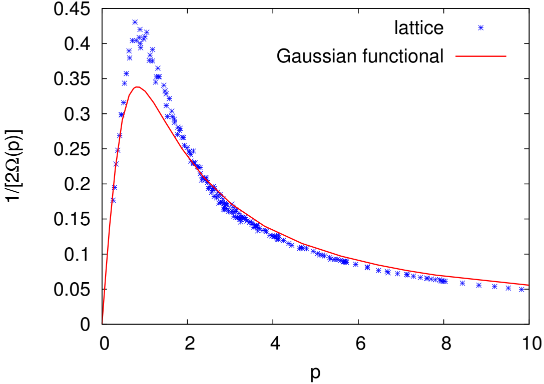
Vertex equations



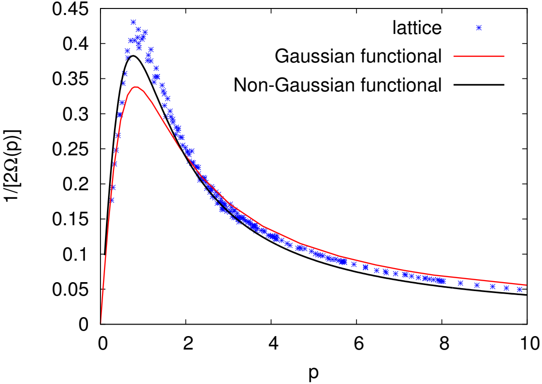


And so on. . .

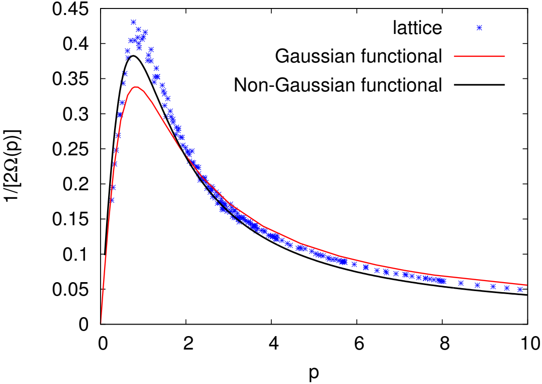
Gluon propagator



Gluon propagator

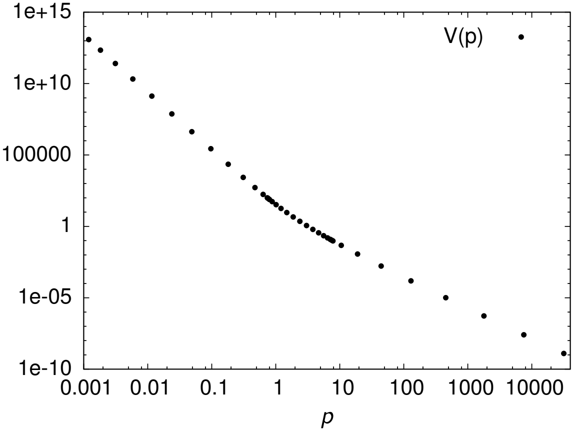


Gluon propagator

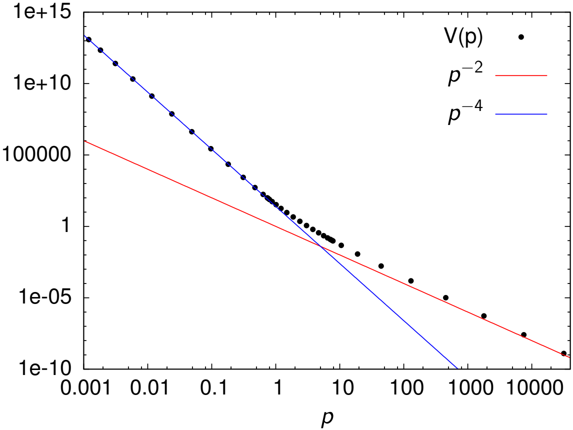


$$\Omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + \frac{m_A^4}{\mathbf{p}^2}}, \qquad m_A \simeq 880\,\mathrm{MeV}$$

Colour-Coulomb potential



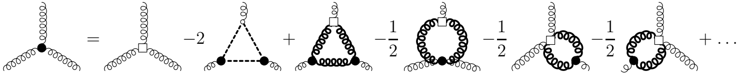
Colour-Coulomb potential

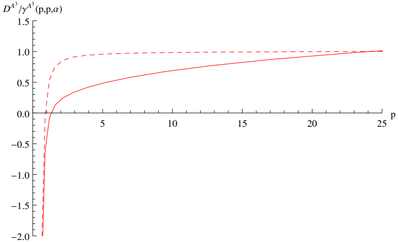


$$\frac{1}{p^4} \longleftrightarrow r$$

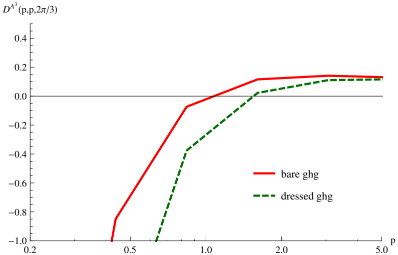
$$\frac{1}{p^2} \longleftrightarrow \frac{1}{r}$$

Three-gluon vertex





Dashed line: Ghost triangle only
Full line: Full DSE



Full red line: Bare ghost-gluon vertex
Dashed green line: Full ghost-gluon vertex

Coulomb gauge pairing model

Make the crudest approximations:

- QCD vacuum as BCS condensate of quark–anti-quark pairs
- keep only the confining Coulomb-like interaction $\sim \sigma r$

This alone is enough to give chiral symmetry breaking! (Albeit the numbers are too small.)

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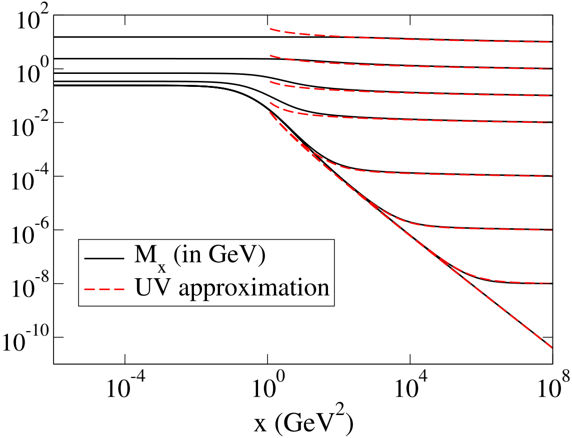
Adding the interaction to the transverse gluons with a bare vertex

$$M(0) \sim 135\,\text{MeV}, \qquad \mathcal{X} \sim (235\,\text{MeV})^3$$

- condensate OK, mass still small
- non-trivial quark-gluon vertex?
  almost certainly, but that's a tough one

Mass function



$M(0)$ independent of m_q for light quarks!

Conclusions

Mass generation is a strong interaction & strong coupling effect.

Confinement and DχSB are intimately linked in QCD.

The results presented here are the necessary input for calculations at finite temperature and density $\Rightarrow$ phase diagram of QCD.

DSEs can be consistently used to improve the vacuum wave functional.