EBERHARD KARLS UNIVERSITÄT TÜBINGEN

WIRTSCHAFTS- UND SOZIALWISSENSCHAFTLICHE FAKULTÄT

Chair of Statistics, Econometrics and Empirical Economics

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S414
Advanced Mathematical Methods

Exercises

WS 2019/20

Linear Algebra

Exercise 1 Eigenvalues

Devise the characteristic equations for the matrices from exercise a)-c) and determine the eigenvalues.

a) $\mathbf{B} = \begin{pmatrix} 4 & 1 \\ -2 & -0,5 \end{pmatrix}$

b) $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ c) $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -3 \\ 0 & -1 & 3 \end{pmatrix}$

Exercise 2 Eigenvalues and Eigenvectors

Given the matrix:

$$\mathbf{A} = \begin{bmatrix} -3 & 2 \\ -2 & 2 \end{bmatrix}$$

a) Calculate the eigenvalues and the respective eigenvectors of $\mathbf{A}$.

b) Use the eigenvalues to calculate the determinant of $\mathbf{A}$.

Exercise 3 Eigenvalues

A 3×3 matrix $\mathbf{A}$ has the eigenvalues $\lambda_1 = 1$, $\lambda_2 = 3$ and $\lambda_3 = 4$. Compute the determinant of $\mathbf{A}$, $\text{rg}(\mathbf{A})$, the determinant of $\mathbf{A}^{-1}$ and the eigenvalues of $\mathbf{A}^{-1}$. What can be said about the quadratic form $\mathbf{x}'\mathbf{A}\mathbf{x}$ of the matrix $\mathbf{A}$ for any vectors of $\mathbf{x}$?

Exercise 4 Eigenvalues

Find the characteristic vectors of the matrix $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$:

Solution Exercise 1:

a) $\lambda_1 = 3.5$ and $\lambda_2 = 0$

b) $\lambda_1 = 2$ and $\lambda_2 = -5$

c) $\lambda_1 = 4.30278$; $\lambda_2 = 0.69722$; $\lambda_3 = 1$

Solution Exercise 2:

a) Eigenvector for $\lambda_1 = 1$:

$$\Rightarrow \begin{pmatrix} a \\ 2a \end{pmatrix} \qquad \text{for } a \in \mathbb{R} \setminus \{0\}$$

Eigenvector for $\lambda_2 = -2$:

$$\Rightarrow \begin{pmatrix} b \\ \frac{1}{2}b \end{pmatrix} \qquad \text{for } b \in \mathbb{R} \setminus \{0\}$$

b) $\det(\boldsymbol{A}) = -2$

Solution Exercise 4:

$$v_1 = \begin{pmatrix} -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}, \qquad v_2 = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$