Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 1 — due 27.10. — T. Piecha

Exercise 1 (8 points)
Write the following abbreviated λ-terms with all missing parentheses and λ's. Then write down all subterms, free variables and bound variables.

(a) $\lambda x.(zy)$ (1 point)

(b) $(\lambda x.xy)(\lambda y.yx)$ (2 points)

(c) $(\lambda yx.xy)((\lambda z.z)y)(\lambda xz.x)$ (2 points)

(d) $(\lambda xyz.xz)((\lambda zy.yy)z)((zz)(zz))$ (3 points)

Exercise 2 (2 points)
We consider the terms in Exercise 1. Rename, if necessary, all bound variables in such a way that no free variable has a bound occurrence.

Exercise 3 (4 points)
Evaluate the following substitutions:

(a) $(\lambda y.x(\lambda w.vxwx))[(uv)/x]$ (2 points)

(b) $((xy)(\lambda v.xv))[(\lambda y.vy)/x]$ (2 points)

Exercise 4 (4 points)
Prove that for all λ-terms M: $\#\mathrm{FV}(M) \leq \mathit{length}(M)$.

(That is, show that the number of free variables of M is less than or equal to the length of M.)

Remark: The assertion is obviously trivial. The objective of this exercise is to present a clear proof by induction on λ-terms.

Exercise 5 (2 points)
Why does $M[P/x][Q/x] \simeq M[(P[Q/x])/x]$ *not* hold in general? (2 points)