Technical note 2: Collaborative Research Center 1070 – Geoscientific and archaeological research

Analyzing prehistoric weights and weight systems using Kendall‘s formula in the open source (geo-)statistical language R

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Introduction

Weight systems represent a fundamental precondition for the functioning of today‘s economic systems. The use of standardized weights and measures is important in order to assess the value of individual goods and to facilitate interregional trade and exchange. However, from an archaeological point of view the use of weights and weight systems is known as an important resource in prehistoric societies; but in many cases hard to identify and/or analyze. On the contrary, it can be traced back at least to the Bronze Age (Flinders Petrie 1926; Petruso 1981; Eiwanger 1989; Peroni 1998; Alberti et al. 2006; Michailidou 2008; Rahmstorf 2003, 2006a/b, 2010). In recent decades, it has been found that the archaeological records from the Mediterranean and the western Levant is well suited for studies on early weight systems: there are representations of weights, written records as well as archaeological finds, which can be identified as weights because of their shape and the basis of various markers as well. A large variety of studies were able to show that the introduction of weight systems and units in the Mediterranean and the Levant seems to be closely related to the development of city-like settlements and the development of hierarchical societies in which complex economies standardized weight units were used for administrative purposes and trade (Knapp 1988; Bass 1991; Wiener 1991; Petruso 1992; Hänsel 1995; Hesse 1995; Panagl 1995; Michailidou 2001; Ratnager 2003; Berg 2004; Bobokhyan 2009, 2010; Rahmstorf 2011, 2014). Probably one of the most famous examples for the importance of weight systems in Bronze Age trade is the shipwreck from Uluburun (Pulak 1996, 1998, 2000; Yalçin et. al 2005; Yalçin 2006).

However, when it comes to the Bronze and Iron Age cultures north of the Alps, the archaeological situation is quite different. Here, the detection of standardized weight units with archaeological methods is challenging, because these cultures left no written sources or pictorial representations of weights and scales behind, that could be used as a point of reference in the identification of archaeological finds as standardized weights. In recent years sophisticated statistical and mathematical approaches made it possible to identify individual artifacts as possible weights. In addition, there are finds, which can be interpreted as scales (Medovič 1995; Pare 1999; for Viking Age weights and scales see Steuer 1984, 1987). However, the question remains, on which units these weight systems are based. In this case archaeological research has to make use of quantitative methods in order to demonstrate the existence of standardized weight units in these non-literate societies. Hoards represent a useful resource for studies on weight systems, because they often contain several dozen artifacts made of bronze (Schwarz 1971; Menke 1978/79; Moosleitner 1988; Winghart 1990; Sommerfeld 1994; Lenerz-de Wilde 1995; Innerhofer 1997; Lenerz-de Wilde 2002; Hoßfeld 2006; Lenerz-de Wilde 2011).

Archaeological and mathematical hurdles

Although the existence of prehistoric units and weight systems has been intensively studied in archaeological research, there are only few studies in which they were analyzed using quantitative methods (Kyhlberg 1980; Sperber 1986; Holm 1987; Sperber 1988, 1989, 1991; Malmer 1992; Petruso 1992; Sperber 1993a/b, 1996; Pare 1999; Sperber 1999; Petruso 2003; Hoßfeld 2006). There are several reasons for this observation. When it comes to the statistical analysis of weight units and weight systems from prehistoric cultures, different source-critical factors must be considered. A basic prerequisite for the recognition of weights is that normalized weights existed and they were made of a material that has been conserved until today. It must also be noted that the use and distribution of weight systems is linked to cultural entities and presupposes politically stable communication spheres where their use is mandatory or generally accepted for other reasons. Consequently, for the study of prehistoric units there is the problem that standardized weight units and weight systems were subject to significant fluctuations not only on a spatial level but also on a temporal one. A statistical analysis is further complicated by the fact that the accuracy of prehistoric scales is unknown – therefore the methodological fault tolerance of the presumed weights themselves is also unknown. Another factor is the state of preservation of the archaeological finds: taxonomic processes such as post depositional fragmentation or corrosion can alter the mass of the weights – in particular when they are made of iron.

In an optimal case the quantitative analysis is based on artefacts with an unaltered mass, which presumably belong to a spatial and temporal coherent weight system of a studied society. Only in a few cases these prerequisite are met. Hence, we need a capable algorithm which can handle and/or communicate deviations within the artefact weights and the weight system.

Kendall's formula

Consider a data set of different measured weights (artefacts, fragments assumed to be part of a former weight system) $x_1, x_2, \ldots, x_n$ where each of the weights is an approximate integral multiple of one positive number $q_1, q_2, \ldots, q_k$ where typically k = 1 or another small integer. The numbers q_k are related to the proposed weight or currency system and consists of an artificial vector of numbers (same units like x), where each of these numbers should be tested as potential candidate.

David G. Kendall (1918–2007) proposed a 'cosine quantogram' in the following form:

$$\varphi(q)=\sqrt{\frac{2}{n}}\sum_{i=0}^{n}\cos 2\pi\epsilon(i)/q$$

where $0 \leq \epsilon(i) < q i$ is the remainder when x_i is divided by q. This formula works well if we are dealing with the k = 1 case, i.e. we assume one distinct multiple in a given dataset defining a weight system. To avoid misinterpretations, the k > 1 case should be regarded with caution. The quantogram responds with a positive peak at 1/q if a true quantum q > 0 is present. In the Null case the cosine function results in a fixed large value and declines later on, while for larger q values the curve is close to random oscillations comparable to a standardized stationary Gaussian time series ($\acute{x} = 0$; $\sigma^2=1$) (Kendal 1974, 1977, 1986). Our best estimator should be something in between, if present. Thus selecting a meaningful quantum hunting search range [$q_1, \ldots, q_k$] is essential (excluding zero and infinity) and should be carefully selected. Kendall recommend strongly to select various artificial search ranges and different scales of plotting to reveal the necessary density to identify all relevant peaks (Kendall 1974). Finally, the highest peak (H) in combination with the smallest base (potential indicator of deviation) should be selected and can be regarded as one multiplier within the analyzed weights. The significance can be tested according to Kendall (1974, 1977, 1986) proposed methodological workflow:

I. Arrange the data values in increasing order of size as $x_1, x_2, \ldots, x_n$

II. Put yj = x_j+1 − x_1 for j = 1,2, ..., N-l.

III. For consecutive values on the y-scale, starting at zero, reassign all the data values found in each cell to values randomly and uniformly distributed within that cell, thus getting $y_1^*, y_2^*, \ldots, y_{N-1}^*$

IV. Now put $x_1^* = x_1$ and $x_j^* = x_1 + \lambda y_{j+1}^*$ for j >= 2. Choose the coefficient λ so that $x_1^*, x_2^*, \ldots, x_N^*$ have the same sum of squares as $x_1, x_2, \ldots, x_N$

V. Draw the quantogram for the starred data, and record the height H* of the highest peak in the interval [$1/q_0, 1/q_1$], proceeding exactly as in the analysis of the real data.

VI. Repeat steps III to V a large number of times (at least 100), and then assess the significance of the observed peak H against the data-based simulation set of peaks H*.

Methodological workflow and script example

Different methodological steps are necessary to calculate Kendall’s formula with R. First of all it is needed to populate the given weights from your finds by using the read.table() function or if easier create your own vector of weights. Both steps are provided within the script and can easily be adjusted with your own dataset (cf. Line [15–17], Line [19]). For Line [20] it is necessary to select a potential test sequence of weights to analyze. Line [23–25] are general setting for the analysis of significance.

– R-SCRIPT Part 1 – Little helpers

```
[1] push <- function(vec, item) {
[2] vec=substitute(vec)
[3] eval.parent(parse(text = paste(vec, ‚ <- c(„ vec, „ „ item, ‚)‘, sep = ‚‘)), n = 1)
[4] }

[5] pop <- function(vec) {
[6] tmp <- vec[length(vec)]
[7] vec=substitute(vec)
[8] eval.parent(parse(text = paste(vec, ‚ <- „ vec, ‚[-length(„ vec, ‚)]‘, sep = ‚‘)), n = 1)
[9] tmp
[10] }
```

– R-SCRIPT Part 2 – User settings

```
[11] # Working directory
[12] setwd(„C:/Path/to/your/data“)
[13] # Read list of weights by predifined *.csv / *.txt
[14] table <- „Testdata_Mauthausen.txt“
[15] dsIn = read.table(table, header=TRUE, sep=“\t“, dec=“.“, strip.white=TRUE )
[16] names(dsIn)
[17] x = dsIn$Mauthausen_weights
[18] # Hardcoded list of weights
[19] x = c(119.50, 121.00, [...])
[20] # artificial weight list [!ADJUST]
[21] q = seq(2, 40, length.out = 800)
[22] # Test of significance based on t.test()
[23] Initial_Lambda      = 10                  # cf. KENDAL, 1973, S.439 - Workflow [IV]
[24] Initial_Adjustment = 0.001
```

The next part performs the cosine quantogram function on your dataset which you can visualize by the plot() function and different style settings (**Fig. 1**).

– R-SCRIPT Part 3 – Hunting quanta (Kendall 1974)

```
[25] nq = length(q)
[26] nx = length(x)

[27] Q = matrix(rep(q,nx), nrow=nx, ncol=nq, byrow= T)
```

```
[28] X = matrix(rep(x,nq), nrow=nx, ncol=nq, byrow= F)

[29] # Remainder
[30] Xr = X%%Q

[31] # Hunting Quanta function
[32] phi = rep(0,length(X[1,]))
[33] for (i in 1:length(X[1,])) phi[i] = sqrt(2 / nx) * sum(cos(2 * pi * Xr[,i] / Q[,i]))
```

– R-SCRIPT Part 4 – Visualization and report

```
[34] plot(q,phi, pch=19, col=“white“, main = „Cosine quantogram [Kendal, 1974]“,
     xlab=“Quantum (g)“, ylab=expression(paste(phi,“(„,tau,“)“)), cex.lab = 1.5)
[35] lines(q,phi, col=“red“, lwd=1, lty=3)
[36] abline(h=0, lty=“dashed“, col=“gray“)
[37] print(c(„Highest peak: „, max(phi)))
```

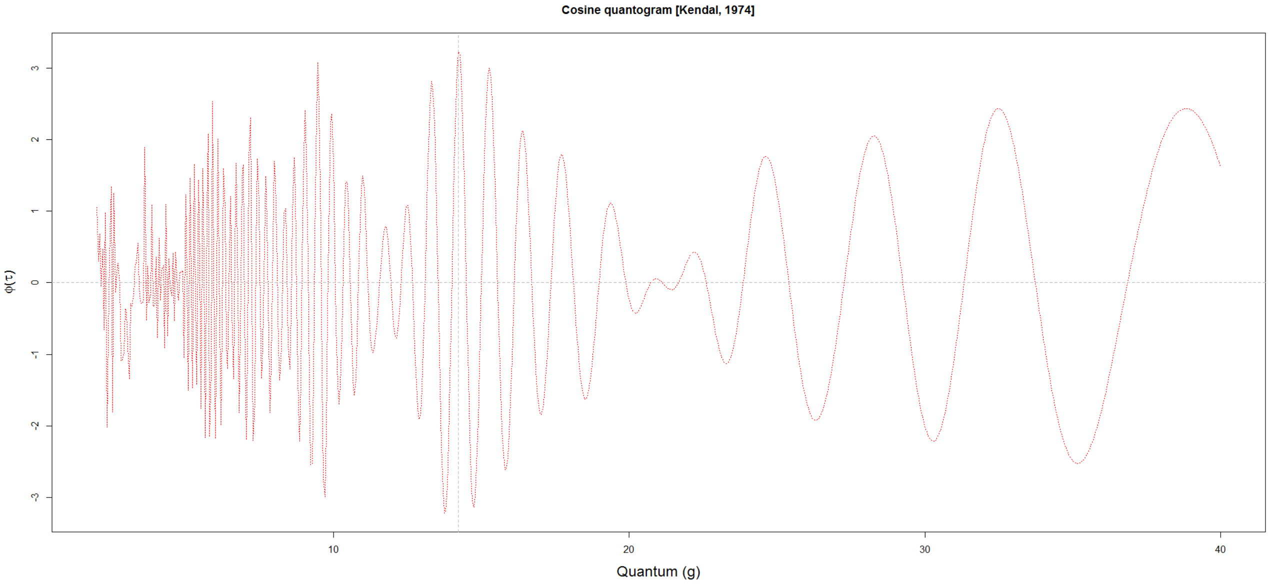

Fig. 1: Cosine quantogram highest peak with a small base by 14.2 g.

– R-SCRIPT Part 4 – Test of significance [One sample t-test]

```
[38] H = max(phi)                              # reference peak
[39] sum_of_squares_x <- sum( (x-mean(x))^2)

[40] # Kendall‘s workflow [I]
[41] x <- sort(x)

[42] # Kendall‘s workflow [II]
[43] y<-rep(0,length(x)-1)
[44] for (i in 1:length(x)-1) y[i] <- x[i+1] - x[i]

[45] # Kendall‘s workflow [III] – [V]
[46] rH <- rep(0,100)
```

```
[47] for (reps in 1:100) {
[48]   xr <- x
[49]   yr <- runif(length(y),0, max(y))
[50]   found          = FALSE
[51]   Lambda         = Initial_Lambda
[52]   Sequence_Of_Lamba = Lambda
[53]  Adjust          = Initial_Adjustment
[54]   Index          = 0
```

The part below [55–68] calculates an adjusted random distribution, which meets the statistical requirements mention in Kendal 1974 – workflow [III] and [IV]. We adjust the necessary Lambda by using a greedy search over potential candidates, which needs to be initialized via user setting. As rule of thumbs start with a bigger initial adjustment, to reduce the number of iterations and have a look to the Lambda, if it is increasing stop the algorithm an set the initial Lambda accordingly higher and finally allow finer adjustments. This part needs a little bit of time to fit the function according to the initial weights.

```
[55]   while (!found) {
[56]     for (j in 2:length(x)) xr[j] <- xr[1] + Lambda*yr[j-1]
[57]     sum_of_squares_xr = sum( (xr-mean(xr))^2 )

[58]     if (sum_of_squares_xr < sum_of_squares_x) Lambda = Lambda + Adjust
[59]     if (sum_of_squares_xr > sum_of_squares_x) Lambda = Lambda - Adjust
[60]     if (round(sum_of_squares_xr) == round(sum_of_squares_x) ) found = TRUE

[61]     push(Sequence_Of_Lamba, Lambda)
[62]     if (duplicated(Sequence_Of_Lamba)[length(Sequence_Of_Lamba)] == TRUE)
[63]       {
[64]       Adjust = Adjust / 10
[65]       Index = Index + 1
[66]     }
[67]     if (Index == 10) found = TRUE
[68]   }

[69]   # Calculate cosine quantogram based on the random uniform distribution
[70]   nX = matrix(rep(xr,nq), nrow=nx, ncol=nq, byrow= F)
[71]   Xr = nX%%Q
[72]   phi_r = rep(0,length(nX[1,]))
[73]   for (i in 1:length(nX[1,])) phi_r[i] = sqrt(2 / nx) * sum(cos(2 * pi * Xr[,i] / Q[,i]))
[74]   rH[reps] = max(phi_r)
[75] }
```

For statistical testing we used a one sample t-test to examine the mean difference between the calculated H and the population mean of rH. Commonly in the one sample t-test, the population mean is known. We test the given H (as sample mean) against the population mean (mean of rH) to make a statistical decision as to whether or not H is different from rH.

```
[76] t.test(rH, mu=H)
```

In general, we formulate two hypotheses:

I. Null hypothesis: assumes that there are no significance differences between the rH mean and the H mean.
II. Alternative hypothesis: assumes that there is a significant difference between the rH mean and the H mean.

To avoid rejecting the null hypothesis by mistake an error probability of normally values (α) between 0.5 and 0.01 are chosen. This means if the t value of our H is close to 0 the null hypothesis is true. Respectively, we have to reject the null hypothesis if the t value is different from 0 in direction to our alternative hypothesis. In general, if we consider the p-value below 0.5 we reject our null hypothesis. Translating the null hypothesis in archeological terms means, we test the estimated weight to be part of a random distribution within the range of the given initial weights. If the null hypothesis is true, the estimated mean is in general part of a random distribution and cannot be considered as part of an underlying weight system. Respectively, the alternative hypothesis means we have significant differences between a random distribution mean and our calculated H, and thus we have most properly identified a weight system indicator.

This technical section is made to evolve! Please share your thoughts, requirements and suggestions regarding the described issues to Jan Ahlrichs (Jan(dot)Ahlrichs(at)uni-tuebingen(dot)de) and Karsten Schmidt (Karsten(dot)Schmidt(at)uni-tuebingen(dot)de.

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