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Qualitative Reasoning with Story-Based Motion Representations: Inverse and Composition

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Abstract

Representations of motion that are story-based constitute a promising tool to categorise the motion of entities, because they can be generated using any qualitative spatial representation, and they consider explicitly the speed of the entities. Up to the present, mainly categorisation properties of the story-based representations have been presented. In this paper we show how story-based representations allow for the reasoning operations that the qualitative calculi possess, namely, inverse and composition—We provide a method to compute the inverse, and a method that notably simplifies the computation of the composition.

1 Introduction and Related Work

Qualitative representations provide a way to transform and represent quantitative into qualitative knowledge [Hernandez, 1994; Dylla *et al.*, 2013; Dylla *et al.*, 2016]. In addition, they provide instruments for "reasoning" (in the broad sense of the term): the *'conceptual neighbourhood diagrams'* [Freksa, 1992], which enable decision-making [Dylla *et al.*, 2007]); also, operations between qualitative relations, such as *'inverse'* (also called *'converse'*), and *'composition'* are the base for reasoning methods—mostly constraint based techniques—in qualitative reasoning [Renz and Nebel, 2007; Cohn and Renz, 2008; Ligozat, 2012, Intr.].

To show that a qualitative representation is suitable for qualitatively representing quantitative knowledge, and to present its conceptual neighbourhood diagrams, e.g., [Van de Weghe and Philippe De Maeyer, 2005], is a more straightforward task than to show its suitability for reasoning in constraint based techniques, e.g., [Van De Weghe *et al.*, 2005a]—some qualitative representations remain, so far, without such a reasoning apparatus, e.g., [Glez-Cabrera *et al.*, 2013; Wu *et al.*, 2014], even when steps in this direction were taken [González-Cabrera *et al.*, 2010]. A cause is that the *composition* can only be computed by using the semantics of the relations [Renz and Nebel, 2007], and this often requires a burdensome manual case analysis, e.g., [Van de Weghe *et al.*, 2005b; Mossakowski and Moratz, 2010].

In previous work a novel method, the *'story-based'* approach, was presented to generate qualitative representations of motion [Purcalla Arrufi and Kirsch, 2017; Purcalla Arrufi and Kirsch, 2018] from any qualitative spatial representation. As example, the method was used to generate two representations of motion: Motion-RCC and Motion-OPRA$_1$, which are suitable to represent qualitative knowledge and perform decision-making. In this paper, we examine the possibility that such story-based representations may be suitable for qualitative reasoning: we provide a method for computing the inverse of relations and a method that notably simplifies the manual computation of the composition.

The paper is structured as follows: firstly, we introduce the story-based representations of motion by means of two examples. Secondly, we show how to compute the inverse of the story-based motion relations. Thirdly, we introduce our approximation method—an upper bound—to the composition set of *motion* relations; we call it *'narrative composition'* of motion relations; this method is based on another definition of composition between *spatial* relations. Further, the narrative composition of motion relations is also practically performed using paths on a matrix that we call *'composition matrix'*. Finally, we apply the method to concrete composition examples to show its validity.

2 Story-Based Representations of Motion

In this Section, we aim at categorising *'motion scenarios'* (Figure 1): pairs of two entities, k and l, that are described by two instantaneous pairs position-velocity vectors—one pair for each entity: $(\vec{x}_k, \vec{v}_k; \vec{x}_l, \vec{v}_l)$.

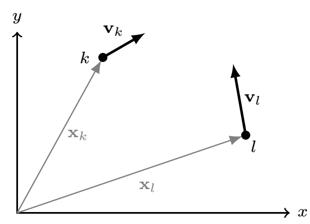


Figure 1: A *motion scenario*—two entities k and l, described by their instantaneous positions, $\vec{x}_k$ and $\vec{x}_l$, and velocities, $\vec{v}_k$ and $\vec{v}_l$.

Story-based representations of motion provide a categorisation for motion scenarios, moreover, these representations can be generated from any qualitative spatial representation $\mathcal{R}$.

In a story-based representation a relation is represented as $S_i(R_j)$; it relates the pair of entities (k, l). We call R_j the *'position component'*, and it is the qualitative spatial relation between (k, l) that is given by the generating spatial representation, $\mathcal{R}$. We call S_i the *'story component'* or just *'story'*: This is the full temporal (past; present; future) sequence of spatial relations, $(R_{i_1}, R_{i_2}, \dots, R_j, \dots, R_{i_m})$, that originates from a motion scenario by *assuming* uniform motion [Purcalla Arrufi and Kirsch, 2017] (See examples in Figures 2 and 4).

The total number of possible stories generated by a certain spatial representation is finite [Purcalla Arrufi and Kirsch, 2017, Math. App.], we call it the *'stories set'*. $\Sigma = \{S_1, S_2, \dots, S_i, \dots, S_n\}$.

2.1 Motion-RCC

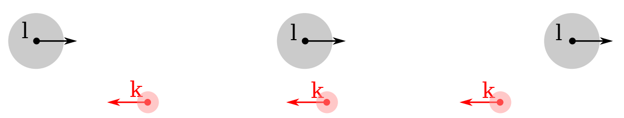
Motion-RCC [Purcalla Arrufi and Kirsch, 2018] is a story-based representation that is generated by the 8 spatial relations in the RCC representation [Randell *et al.*, 1992]: DC, EC, PO, TPP, NTPP, EQ, TPPI, NTPPI; they are concerned with the overlapping of entities.

Motion-RCC has 16 stories, which, for the sake of simplicity, we reduce to five in this paper, by requiring that both entities, k and l, move with different velocities, and that the first entity, k, is smaller than the second one, l: {(DC), (DC, EC, DC), (DC, EC, PO, EC, DC), (DC, EC, PO, TPP, PO, EC, DC), (DC, EC, PO, TPP, NTPP, TPP, PO, EC, DC)}—we name this stories $\{S_{11}, S_{12}, S_{13}, S_{14}, S_{15}\}$.

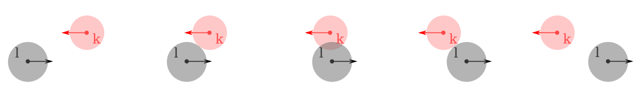
A Motion-RCC relation, e.g., $S_{13}(\text{PO})$, is consequently formed by a story, S_{13}, and one of its spatial relations, PO, the position component; as we see in the third motion scenario in Figure 2b. There is a caveat: if the qualitative relation repeats in the story—as DC in the story S_{13}—we use the sub-index '$-$' or '$+$' to denote the temporal precedence: the relation $S_{13}(\text{DC}_-)$ occurs previous to $S_{13}(\text{DC}_+)$ (See first and last motion scenario in Figure 2b).

2.2 Motion-OPRA$_1$

The Motion-OPRA$_1$[Purcalla Arrufi and Kirsch, 2018] is generated by the OPRA$_1$ relations, i.e., $\{\angle_a^b \mid a, b \in \{0, 1, 2, 3\}\}$, which are concerned with orientations of entities (See Figure 3) [Moratz, 2006]. In Table 1 we detail the Motion-OPRA$_1$ stories that are used in the examples throughout the paper, for example, the story S_{C21} (Figure 4), used in the examples of Section 5˙.



(a) Example of story $S_{11} = (\text{DC})$. Each motion scenario have motion relation $S_{11}(\text{DC})$



$S_{13}(\text{DC}_-)$ $S_{13}(\text{EC}_-)$ $S_{13}(\text{PO})$ $S_{13}(\text{EC}_+)$ $S_{13}(\text{DC}_+)$

(b) Example of story $S_{13} = (\text{DC}, \text{EC}, \text{PO}, \text{EC}, \text{DC})$ where each pictured motion scenario has a particular motion relation.

Figure 2: Stories generated by the RCC spatial representation. All the spatial relations forming the story are represented by the corresponding motion scenario.

3 Computing Inverse Relations

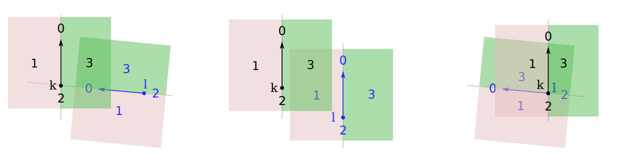
As we have explained in Section 2, each story-based relation is a binary relation between two entities, k and l, that has the form $S_i(R_j)$. S_i, is the story to which the motion scenario belongs, and R_j, the spatial relation of the motion scenario.

As $S_i(R_j)$ relates the pair (k, l), we solve a pertinent question: which is the story-based relation for the permuted pair, (l, k). This relation, $S_i(R_j)^{-1}$, is named *'inverse'* relation (or *'converse'*).

By definition, the terms S_i and R_j stand independently, as a Cartesian product, in the relations notation. Thus, $S_i(R_j)^{-1} = S_i^{-1}(R_j^{-1})$, and we can compute the inverse of each term just using the inverse of the generating spatial relation:

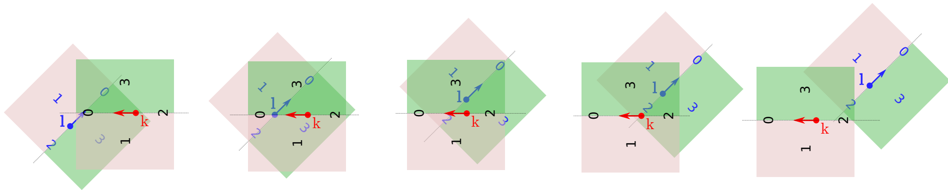
- R_j^{-1} is provided by the generating spatial representation. For example, $\text{DC}^{-1} = \text{DC}$, $\text{TPP}^{-1} = \text{TPPI}$, $(\angle_1^3)^{-1} = \angle_3^1$, or $(\angle 2)^{-1} = \angle 2$
- S_i^{-1} is computed by expressing the story as the list of spatial relations, then applying the inverse to any spatial relation on the list, and, finally, expressing the list of spatial relations as the corresponding story.
  For example, in Motion-RCC, the story S_{13}, is defined by the temporal sequence (DC, EC, PO, EC, DC), thus $S_{13}^{-1} = (\text{DC}^{-1}, \text{EC}^{-1}, \text{PO}^{-1}, \text{EC}^{-1}, \text{DC}^{-1}) = (\text{DC}, \text{EC}, \text{PO}, \text{EC}, \text{DC})$, which is the same story S_{13}. Consequently, $S_{13}^{-1} = S_{13}$
  In Motion-OPRA$_1$ the story S_{C10} (See Table 1), corresponds to the temporal sequence $(\angle_1^3, \angle 3, \angle_3^1)$, thus the inverse is computed by the same procedure above: $S_{\text{C10}}^{-1} = ((\angle_1^3)^{-1}, (\angle 3)^{-1}, (\angle_3^1))^{-1}) = (\angle_3^1, \angle 1, \angle_1^3) = S_{\text{C20}}$

Thus, full examples of inverse relations in Motion-RCCare $S_{12}(\text{EC}_+)^{-1} = S_{12}(\text{EC}_+)$, $S_{13}(\text{PO})^{-1} = S_{13}(\text{PO})$. Examples in Motion-OPRA$_1$are $S_{\text{C10}}(\angle_1^3)^{-1} = S_{\text{C20}}(\angle_3^1)$, or, $S_{\text{C10}}(\angle 3)^{-1} = S_{\text{C20}}(\angle 1)$.



(a) $\angle_3^0$ (b) $\angle_3^1$ (c) $\angle 1$

Figure 3: Examples of OPRA$_1$ spatial relations.
$\angle_a^b$, between two entities k and l that are at different points. The syntax is $\angle_{l \text{ with respect to } k}^{k \text{ with respect to } l}$
$\angle a$, between two entities k and l that are at the same point. The syntax is $\angle\text{region of } k \text{ to which } l \text{ points}$



$S_{C21}(\angle_3^1)$ $S_{C21}(\angle_0^1)$ $S_{C21}(\angle_1^1)$ $S_{C21}(\angle_1^2)$ $S_{C21}(\angle_1^3)$

Figure 4: Illustration of the story $S_{C21} = (\angle_3^1, \angle_0^1, \angle_1^1, \angle_1^2, \angle_1^3)$, with motion scenarios representing the full sequence of relations that constitute it.

4 Computing Composition of Story-Based Relations

The composition of qualitative relations is related to three entities k, l, and m. If we know the relation between the pair (k, l), say, R_A, and the relation between the pair (l, m), say, R_B, what would be then the possible relation for the pair (k, m)? Such relation or relations $\widetilde{R_C} = \{R_{C_1}, \ldots, R_{C_N}\}$ is called the *'composition'* of R_A and R_B—we express it symbolically as, $\widetilde{R_C} = R_A \circ R_B$. In the case of story-based relations we write $\widetilde{S_C(R_C)} = S_A(R_A) \circ S_B(R_B)$, i.e., $\widetilde{S_C(R_C)} = \{S_{C_1}(R_{C_1}), \ldots, S_{C_N}(R_{C_N})\}$

Usually, the result of the composition is not a single relation, but a set of relations. For example, in RCC, if the entities (k, l) fulfil the relation PO, and (l, m) fulfil TPP, then (k, m) may fulfil one of these relations: PO, TPP, or NTPP. We express that as, $\{\mathrm{PO}, \mathrm{TPP}, \mathrm{NTPP}\} = \mathrm{PO} \circ \mathrm{TPP}$. The full RCC composition values are displayed in [?], and an algorithm to find the composition in OPRA_1 is presented in [Mossakowski and Moratz, 2012].

Finding the composition is an arduous task that must be tailored to every representation. Though we do not exactly solve the composition for story-based relations, we take a step towards the computation of the composition by limiting its possible results. Indeed, we build an operation with story-based relations, the *'narrative composition'*, $S_A(R_A) \triangledown S_B(R_B)$, that yields a small superset of the standard composition, $S_A(R_A) \circ S_B(R_B) \subset S_A(R_A) \triangledown S_B(R_B)$, and usually does not coincide with it.

4.1 The Narrative Composition

In this Section, we describe how we build the narrative composition of story-based relations, $S_A(R_A) \triangledown S_B(R_B)$. We do it by requiring *necessary* conditions that the standard composition, i.e., $S_A(R_A) \circ S_B(R_B)$, must fulfil. Unfortunately, these conditions are *not sufficient*, i.e., the narrative compositions often adds extra relations to the standard composition (as shown in the example of Section 5.2). Nevertheless, the narrative composition gives a good first approach to the standard composition, that can be refined.

As a first step, we try to compute the story component of the composition, that is, we want to compute the stories S_C in the composition $\widetilde{S_C(R_C)} = S_A(R_A) \circ S_B(R_B)$. Stories are expressed as a finite sequence of spatial relations: $S_A = (R_{A_1}, R_{A_2}, \ldots, R_{A_i}, \ldots, R_{A_n})$ and $S_B = (R_{B_1}, R_{B_2}, \ldots, R_{B_j}, \ldots, R_{B_m})$. We notice that, by definition, the position component must be part of the corresponding story component, i.e., $R_A \in S_A$, and $R_B \in S_B$.

Any story $S_C \in \widetilde{S_C}$ must be necessarily related to some composition of the elements of S_A and S_B that keeps the temporal order. However, we show that such composition of elements is not directly given by the *standard* composition, i.e., $R_{A_i} \circ R_{B_j}$, but by a new type of composition of spatial relations, the *'narrative composition'*, which we express as $R_A \triangledown R_B$.

For example, if, in Motion-RCC, we want to compute the composition $\widetilde{S_C(R_C)} = S_A(R_A) \circ S_B(R_B)$, where $S_A = (\mathrm{DC}, \mathrm{EC}, \mathrm{DC})$ and $S_B = (\mathrm{DC})$, then, the pairwise standard composition of spatial relations produces this result: $(\mathrm{DC} \circ \mathrm{DC}, \mathrm{EC} \circ \mathrm{DC}, \mathrm{DC} \circ \mathrm{DC})$, which yields only *three-element* sequences: $(\mathrm{DC}, \mathrm{NTPPI}, \mathrm{PO})$, $(\mathrm{EC}, \mathrm{DC}, \mathrm{DC})$, and so on. This is unsatisfactory, because $S_C = (\mathrm{DC}, \mathrm{EC}, \mathrm{PO}, \mathrm{EC}, \mathrm{DC})$ is also a possible composition story that can never be obtained this way.

Summarising, we need to define new two operations: first, between spatial relations, $R_A \triangledown R_B$, second, between story-based motion relations, $S_A(R_A) \triangledown S_B(R_B)$. We name them both *'narrative composition'*—the operands, either motion or spatial relations, determine the way they are computed.

Narrative Composition of Spatial Relations: $\mathbf{R_A \triangledown R_B}$ The key to obtain all possible results, when composing stories, is that we have to use an *extended composition* between the elements, i.e., the spatial relations of the sequence. Indeed, in uniform motion, while (k, l) fulfil DC and (l, m) fulfil DC, it may happen that (k, m) goes through a sequence of relations, e.g., $(\mathrm{DC}, \mathrm{EC}, \mathrm{PO})$, where each element of the sequence is a possible result of the standard composition $\mathrm{DC} \circ \mathrm{DC}$.
Consequently, we define the *'narrative composition'*, $R_A \triangledown R_B$ of two spatial relations, R_A and R_B, as all possible subsequences of stories formed by relations that belong to the standard composition $R_A \circ R_B$.

Example 1: The standard composition of DC and EC yields five possible relations: $\mathrm{DC} \circ \mathrm{EC} = \{\mathrm{DC}, \mathrm{EC}, \mathrm{PO}, \mathrm{TPPI}, \mathrm{NTPPI}\}$. Consequently, $\mathrm{DC} \triangledown \mathrm{EC}$ is formed by all the combinations of relations in $\{\mathrm{DC}, \mathrm{EC}, \mathrm{PO}, \mathrm{TPPI}, \mathrm{NTPPI}\}$ that form story subsequences, i.e.,

Non-Parallel velocities $\vec{v}_k \nparallel \vec{v}_l$				
Σ_C	$\vec{v}_k \neq 0, \vec{v}_l \neq 0$			
	$\angle_1^3\ \angle_1^0\ \angle_1^1\ \angle_2^1\ \angle_3^1$	$S_{C1\text{-}1}$	$\angle_3^1\ \angle_3^0\ \angle_3^3\ \angle_2^3\ \angle_1^3$	$S_{C2\text{-}1}$
	$\angle_1^3\ \angle 3\ \angle_3^1$	S_{C10}	$\angle_3^1\ \angle 1\ \angle_1^3$	S_{C20}
	$\angle_1^3\ \angle_0^3\ \angle_3^3\ \angle_3^2\ \angle_3^1$	S_{C11}	$\angle_3^1\ \angle_0^1\ \angle_1^1\ \angle_1^2\ \angle_1^3$	S_{C21}
Σ_B	$\vec{v}_k \neq 0, \vec{v}_l = 0$			
	$\angle_1^3\ \angle_1^0\ \angle_1^1$	$S_{B1\text{-}1}$	$\angle_3^1\ \angle_3^0\ \angle_3^3$	$S_{B2\text{-}1}$
	$\angle_0^3\ \angle 3\ \angle_2^1$	S_{B10}	$\angle_0^1\ \angle 1\ \angle_2^3$	S_{B20}
	$\angle_3^3\ \angle_3^2\ \angle_3^1$	S_{B11}	$\angle_1^1\ \angle_1^2\ \angle_1^3$	S_{B21}
	$\vec{v}_k = 0, \vec{v}_l \neq 0$			
	$\angle_1^1\ \angle_2^1\ \angle_3^1$	$S_{B3\text{-}1}$	$\angle_3^3\ \angle_2^3\ \angle_1^3$	$S_{B4\text{-}1}$
	$\angle_1^0\ \angle 3\ \angle_3^2$	S_{B30}	$\angle_3^0\ \angle 1\ \angle_1^2$	S_{B40}
	$\angle_1^3\ \angle_0^3\ \angle_3^3$	S_{B31}	$\angle_3^1\ \angle_0^1\ \angle_1^1$	S_{B41}
Parallel velocities $\vec{v}_k \parallel \vec{v}_l$				
Σ_T	The entities' trajectories are superposed			
	$\angle_0^2\ \angle 0\ \angle_2^0$	$S_{T\text{-}1}$		
	$\angle_0^0\ \angle 2\ \angle_2^2$	S_{T0}		
	$\angle_2^0\ \angle 0\ \angle_0^2$	S_{T1}		
Σ_P	The entities' trajectories are not superposed			
	singleton stories			
	$\angle_1^3$	S_{P11}	$\angle_3^3$	S_{P21}
	$\angle_3^1$	S_{P12}	$\angle_1^1$	S_{P22}

Table 1: Part of the OPRA$_1$ stories set, Σ, divided into meaningful subsets of stories: Σ_C, Σ_B, Σ_T, Σ_P

$\mathrm{DC} \triangledown \mathrm{EC} = \{(\mathrm{DC}), (\mathrm{DC}, \mathrm{EC}, \mathrm{DC}), \ldots, (\mathrm{EC}, \mathrm{PO}, \mathrm{EC}), \ldots, (\mathrm{EC}, \mathrm{PO}, \mathrm{TPPI}, \mathrm{PO}), \ldots\}$. Indeed (DC) is, for example, subsequence of story S_{12} (See section 2.1); (EC, PO, EC) is subsequence of S_{13}; and so on. However, a sequence such as (EC, TPPI, EC), though it is a combination of relations from the composition $\mathrm{DC} \circ \mathrm{EC}$, does not belong to the narrative composition $\mathrm{DC} \triangledown \mathrm{EC}$, because it is not the subsequence of any story.

Narrative composition of Motion Relations: $\mathbf{S_A(R_A) \triangledown S_B(R_B)}$ As a necessary condition, any story S_C belonging to the composition set $\widetilde{S_C(R_C)} = S_A(R_A) \circ S_B(R_B)$, must be the result of the narrative composition of the motion relations, i.e., $S_A(R_A) \triangledown S_B(R_B)$, which is obtained by concatenation of narrative composition of combinations of the spatial relations pairs constituting both stories S_A and S_B. Two further conditions complete the method: first, the composition of the position components, $R_A \triangledown R_B$, must always be present, because it corresponds to the position component of the composition, i.e., R_C; second, the concatenation of the narrative composition of spatial relations is only acceptable, if it is a story—not any sequence of concatenated narrative compositions is accepted.

The computation of the narrative composition is illustrated by means of Example 2, which is displayed in the composition matrix of Table 2.

Example 2: if we have two stories, $S_A = (R_{A_1}, R_{A_2}, R_{A_3})$, $S_B = (R_{B_1}, R_{B_2}, R_{B_3}, R_{B_4}, R_{B_5})$, the narrative story composition $S_A(R_{A_1}) \triangledown S_B(R_{B_3})$ is obtained by concatenating the narrative composition of story pairs of relations, keeping the temporal order, where the position components are also narratively composed, $R_{A_1} \triangledown R_{B_3}$ (grey boxes in Equations (1a) to (1c)). We present some possible results of narratively composed stories, which can be visualised as paths in a matrix (Table 2)

$$(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_1} \triangledown R_{B_4}, R_{A_1} \triangledown R_{B_5}, R_{A_2} \triangledown R_{B_5}, R_{A_3} \triangledown R_{B_5}) \tag{1a}$$

$$(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_5}) \tag{1b}$$

$$(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_5}) \tag{1c}$$

4.2 The Narrative Composition Matrix: Graphically computing the $\mathbf{S_A(R_A) \triangledown S_B(R_B)}$

We have explained above how to compute the *narrative composition of motion relations*. In this Section we present the

$S_A(R_{A_1}) \triangledown S_B(R_{B_3})$	R_{B_1}	R_{B_2}	R_{B_3}	R_{B_4}	R_{B_5}
R_{A_1}	$R_{A_1} \triangledown R_{B_1}$	$R_{A_1} \triangledown R_{B_2}$	$R_{A_1} \triangledown R_{B_3}$	$R_{A_1} \triangledown R_{B_4}$	$R_{A_1} \triangledown R_{B_5}$
R_{A_2}	$R_{A_2} \triangledown R_{B_1}$	$R_{A_2} \triangledown R_{B_2}$	$R_{A_2} \triangledown R_{B_3}$	$R_{A_2} \triangledown R_{B_4}$	$R_{A_2} \triangledown R_{B_5}$
R_{A_3}	$R_{A_3} \triangledown R_{B_1}$	$R_{A_3} \triangledown R_{B_2}$	$R_{A_3} \triangledown R_{B_3}$	$R_{A_3} \triangledown R_{B_4}$	$R_{A_3} \triangledown R_{B_5}$

Table 2: *Narrative composition matrix* of the story-based motion relations $S_A(R_{A_1})$ and $S_B(R_{B_3})$. The corresponding stories are $S_A = (R_{A_1}, R_{A_2}, R_{A_3})$ and $S_B = (R_{B_1}, R_{B_2}, R_{B_3}, R_{B_4}, R_{B_5})$. The possible results of the narrative composition are obtained through paths in the matrix fulfilling the following properties: The yellow cells are start and end steps, the orange cell is a necessary middle step, because it is the narrative composition of the position components in $S_A(R_{A_1})$ and $S_B(R_{B_3})$. The three examples of valid paths (Equation (1)) are given, the *green* path corresponds to $(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_1} \triangledown R_{B_4}, R_{A_1} \triangledown R_{B_5}, R_{A_2} \triangledown R_{B_5}, R_{A_3} \triangledown R_{B_5})$ (Equation (1a)); the *blue* path to $(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_5})$ (Equation (1b)); and the *black* corresponds to $(R_{A_1} \triangledown R_{B_1}, R_{A_1} \triangledown R_{B_2}, R_{A_1} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_3}, R_{A_2} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_4}, R_{A_3} \triangledown R_{B_5})$ (Equation (1c))

$S_{12}(\mathrm{DC}_-) \triangledown S_{13}(\mathrm{PO})$	DC_-	EC_-	PO	EC_+	DC_+
DC_-	$\mathrm{DC}_- \triangledown \mathrm{DC}_-$	$\mathrm{DC}_- \triangledown \mathrm{EC}_-$	$\mathrm{DC}_- \triangledown \mathrm{PO}$	$\mathrm{DC}_- \triangledown \mathrm{EC}_+$	$\mathrm{DC}_- \triangledown \mathrm{DC}_+$
EC	$\mathrm{EC} \triangledown \mathrm{DC}_-$	$\mathrm{EC} \triangledown \mathrm{EC}_-$	$\mathrm{EC} \triangledown \mathrm{PO}$	$\mathrm{EC} \triangledown \mathrm{EC}_+$	$\mathrm{EC} \triangledown \mathrm{DC}_+$
DC_+	$\mathrm{DC}_+ \triangledown \mathrm{DC}_-$	$\mathrm{DC}_+ \triangledown \mathrm{EC}_-$	$\mathrm{DC}_+ \triangledown \mathrm{PO}$	$\mathrm{DC}_+ \triangledown \mathrm{EC}_+$	$\mathrm{DC}_+ \triangledown \mathrm{DC}_+$

Table 3: This table exemplifies a real Motion-RCC case of the general case in Table 2. We compose narratively the motion relations $S_{12}(\mathrm{DC}_-)$ and $S_{13}(\mathrm{PO})$; where the stories components are $S_{12} = (\mathrm{DC}_-, \mathrm{EC}, \mathrm{DC}_+)$, $S_{13} = (\mathrm{DC}_-, \mathrm{EC}_-, \mathrm{PO}, \mathrm{EC}_+, \mathrm{DC}_+)$. The difference are the *grey cells*—they correspond to *punctual relations*—therefore, we cannot directly step from grey cell into grey cell; this makes the black path an invalid path in this representation. However, the narrative compositions given by the blue and green are perfectly valid.

'narrative composition matrix', as a tool to perform the narrative composition of story-based relations of motion in a more visual way.

The narrative composition matrix of two motion relations, e.g., $S_A(R_{A_1}) \triangledown S_B(R_{B_3})$. is the table formed by the Cartesian product of all narrative compositions of the spatial relations that form the story components (Table 2).

Now we can see that the narrative composition of two motion relations are all possible stories defined by paths in the narrative composition matrix. The paths must fulfil the following conditions:

i) Every path begins in the upper left corner and ends in the lower right corner (yellow coloured cells in Tables 2 and 3)

ii) Every path can only be generated by moving from every cell either rightwards, downwards or diagonally rightwards downwards.

iii) Every path must pass through the cell containing the narrative composition of the position components, which is the spatial relation corresponding to the position component of the composition, i.e., R_C; (orange coloured cell in Tables 2 and 3). For example, in Table 2 is $R_{A_1} \triangledown R_{B_3}$.

4.3 Additional Constraints in the Composition Matrix

We can reduce the large number of possible paths, i.e., narrative compositions, if we have information about topological properties of the spatial relations.

In most spatial representations, amongst others RCC and OPRA, some relations can only occur at single time instant when they are part of a story. These *'punctual relations'* are

EC and TPP, in RCC, and every OPRA relation $\angle_x^y$ that contains 0 or 2 (e.g., $\angle_0^3$, or $\angle_2^2$).

Because every story is a continuous movement of entities in space (indeed, a uniform motion), we cannot transition directly between punctual relations, unless another relation is in between. As example, in Table 3, the punctual relations of Motion-RCC are marked in grey. Thus, the black path is no more a possible narrative composition of the stories, because it transitions from one grey cell into a another neighbouring grey one.

5 Examples of Narrative Composition in Story-Based Representations

In this Section we offer two full examples of *narrative* composition in the story-based relations of motion Motion-OPRA$_1$. Furthermore, we refine the results to obtain the *standard* composition between such relations.

5.1 Narrative Composition $\mathbf{S_{C21}}(\angle_1^3) \triangledown \mathbf{S_{T\text{-}1}}(\angle_0^2)$

We want to compute the narrative composition $S_{C21}(\angle_1^3) \triangledown S_{T\text{-}1}(\angle_0^2)$ (See Figures 5 and 6)

Firstly, we express the stories as sequence of spatial relations: $S_{C21} = (\angle_3^1, \angle_0^1, \angle_1^1, \angle_1^2, \angle_1^3)$ and $S_{T\text{-}1} = (\angle_0^2, \angle 0, \angle_2^0)$. Secondly, we compute the narrative composition of stories by means of the narrative composition matrix (Table 4). The only possible path in the matrix that passes through the composed spatial components, i.e., $\angle_1^3 \triangledown \angle_0^2$ (blue cell), is the blue path.

The blue path in Matrix 4b generates many temporal sequences of relations, for example, $(\angle_0^1, \angle_1^1, \angle_2^1, \angle_1^2, \angle_0^3, \angle_1^3, \angle_3^3)$, and $(\angle_1^1, \angle_1^2, \angle_1^3)$. Only two of all generated sequences are Motion-OPRA$_1$ stories (see Table 1): S_{C21} and S_{B31}. The value of $\angle_1^3 \triangledown \angle_0^2$ compatible with the story yields the spatial component of the relation. Accordingly, $S_{C21}(\angle_1^3) \triangledown S_{T\text{-}1}(\angle_0^2) = \{S_{C21}(\angle_1^3), S_{B31}(\angle_1^3)\}$

However, looking at certain kinetic properties, we see that S_{B31} is not possible; because it implies that $\vec{v}_k = 0$, which contradicts $S_{C21}(\angle_1^3)$, where $\vec{v}_k \neq 0$. Therefore, the only possible result of the standard composition is the relation $S_{C21}(\angle_1^3)$ (Figure 7). Thus, finally, the standard composition yields $S_{C21}(\angle_1^3) \circ S_{T\text{-}1}(\angle_0^2) = S_{C21}(\angle_1^3) \subsetneq S_{C21}(\angle_1^3) \triangledown S_{T\text{-}1}(\angle_0^2) = \{S_{C21}(\angle_1^3), S_{B31}(\angle_1^3)\}$.

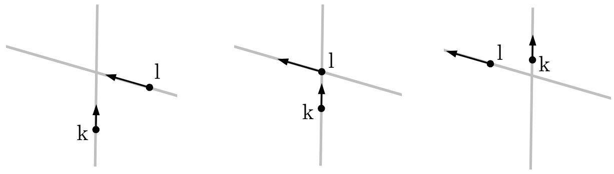
5.2 Narrative Composition $\mathbf{S_{C21}}(\angle_3^1) \triangledown \mathbf{S_{T\text{-}1}}(\angle_2^0)$

Proceeding as in Section 5.1, we compute the narrative composition of relations using the narrative composition matrix (Table 4). In this case, the only possible path in the matrix that passes through the composed spatial components, i.e., $\angle_3^1 \triangledown \angle_2^0$ (red cell), is the red path.

If we analyse all possible sequences of relations that the red path generates by narrative composition, we obtain only five stories, which correspond to five motion relations: $S_{C21}(\angle_3^1) \triangledown S_{T\text{-}1}(\angle_2^0) = \{S_{C2\text{-}1}(\angle_3^1), S_{C20}(\angle_3^1), S_{C21}(\angle_3^1), S_{B2\text{-}1}(\angle_3^1), S_{B3\text{-}1}(\angle_3^1)\}$.

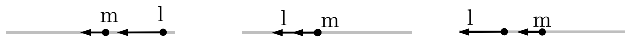
Analogous as in Section 5.1, we see that the relation $S_{B3\text{-}1}(\angle_3^1)$, is impossible—it implies that $\vec{v}_k = 0$, which is contradictory with one of the relations being composed, i.e., $S_{C21}(\angle_1^3)$, which implies that $\vec{v}_k \neq 0$. For the remaining compositions, we find possible realisations.

Therefore, the composition yields, $S_{C21}(\angle_3^1) \circ S_{T\text{-}1}(\angle_2^0) = \{S_{C2\text{-}1}(\angle_3^1), S_{C20}(\angle_3^1), S_{C21}(\angle_3^1), S_{B2\text{-}1}(\angle_3^1)\} \subsetneq S_{C21}(\angle_3^1) \triangledown S_{T\text{-}1}(\angle_2^0)$



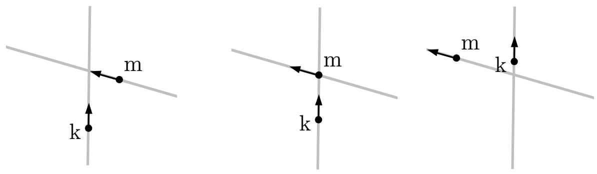
$S_{C21}(\angle_3^1)$ $S_{C21}(\angle_0^1)$ $S_{C21}(\angle_1^3)$

Figure 5: The story S_{C21}, displayed with some of its motion relations



$S_{T\text{-}1}(\angle_0^2)$ $S_{T\text{-}1}(\angle 0)$ $S_{T\text{-}1}(\angle_2^0)$

Figure 6: The story $S_{T\text{-}1}$ displayed with all motion relations involved



$S_{C21}(\angle_3^1)$ $S_{C21}(\angle_0^1)$ $S_{C21}(\angle_1^3)$

Figure 7: The solution of the composition $S_{C21}(\angle_1^3) \circ S_{T\text{-}1}(\angle_0^2)$ is the Motion-OPRA$_1$ relation $S_{C21}(\angle_1^3)$, belonging to the story S_{C21}, shown here.

6 Conclusion

We showed that the standard operations used for reasoning—inverse and composition—can be computed in the story-based representations of motion. Accordingly, such representations constitute a qualitative calculus, a more powerful tool than a categorisation. We gave a method, the narrative composition of story-based motion relations, to approximately compute the composition of motion relations—with this method we obtain a superset of the possible solutions that can notably approach the exact result. Therefore, the most appealing task for future work is to refine the narrative composition, so that we obtain a general method, if possible, to exactly compute the standard composition in any story-based motion representation.

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$S_{\text{C21}} \triangledown S_{\text{T-1}}$	$\angle^{2}_{0}$	$\angle 0$	$\angle^{0}_{2}$
$\angle^{1}_{3}$	$\angle^{1}_{3} \triangledown \angle^{2}_{0}$	$\angle^{1}_{3} \triangledown \angle 0$	$\angle^{1}_{3} \triangledown \angle^{0}_{2}$
$\angle^{1}_{0}$	$\angle^{1}_{0} \triangledown \angle^{2}_{0}$	$\angle^{1}_{0} \triangledown \angle 0$	$\angle^{1}_{0} \triangledown \angle^{0}_{2}$
$\angle^{1}_{1}$	$\angle^{1}_{1} \triangledown \angle^{2}_{0}$	$\angle^{1}_{1} \triangledown \angle 0$	$\angle^{1}_{1} \triangledown \angle^{0}_{2}$
$\angle^{2}_{1}$	$\angle^{2}_{1} \triangledown \angle^{2}_{0}$	$\angle^{1}_{1} \triangledown \angle 0$	$\angle^{1}_{1} \triangledown \angle^{0}_{2}$
$\angle^{3}_{1}$	$\angle^{3}_{1} \triangledown \angle^{2}_{0}$	$\angle^{3}_{1} \triangledown \angle 0$	$\angle^{3}_{1} \triangledown \angle^{0}_{2}$

(a) *Narrative composition matrix* of the stories $S_{\text{C21}} = (\angle^{1}_{3}, \angle^{1}_{0}, \angle^{1}_{1}, \angle^{2}_{1}, \angle^{3}_{1})$ and $S_{\text{T-1}} = (\angle^{2}_{0}, \angle 0, \angle^{0}_{2})$.

$S_{\text{C21}} \triangledown S_{\text{T-1}}$	$\angle^{2}_{0}$	$\angle 0$	$\angle^{0}_{2}$
$\angle^{1}_{3}$	$\angle^{1}_{\{0,1,3\}}$	$\angle^{1}_{3}$	$\angle^{1}_{\{1,2,3\}}$
$\angle^{1}_{0}$	$\angle^{1}_{1}$	$\angle^{1}_{0}$	$\angle^{1}_{3}$
$\angle^{1}_{1}$	$\angle^{1}_{\{1,2,3\}}$	$\angle^{1}_{1}$	$\angle^{1}_{\{0,1,3\}}$
$\angle^{2}_{1}$	$\angle^{2}_{1}$	$\angle^{2}_{1}$	$\angle^{\{0,2\}}_{\{1,3\}}, \angle 1$
$\angle^{3}_{1}$	$\angle^{3}_{\{0,1,3\}}$	$\angle^{3}_{1}$	$\angle^{3}_{\{1,2,3\}}$

(b) We compute the composition in each cell of Matrix 4a (See the algorithm for OPRA_1 in [Mossakowski and Moratz, 2012]).The narrative composition on each cell is the combination of the relations in the cell

Table 4: We compute the two examples in Section 5: $S_{\text{C21}}(\angle^{3}_{1}) \circ S_{\text{T-1}}(\angle^{2}_{0})$, blue path passing through $\angle^{3}_{1} \triangledown \angle^{2}_{0}$; and $S_{\text{C21}}(\angle^{1}_{3}) \circ S_{\text{T-1}}(\angle^{0}_{2})$, red path passing through $\angle^{1}_{3} \triangledown \angle^{0}_{2}$.
We can use the same narrative composition matrix, because the motion components of the relations, i.e., S_{C21} and $S_{\text{T-1}}$, are the same. The start and end cell for both examples are the same, they are yellow coloured. The cell of the current spatial component is coloured according to the path.

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