



Mathematisch-Naturwissenschaftliche Fakultät



Department of Mathematics

Module Handbook Mathematical Physics Master of Science

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1 Program description

1.1 Study Concept

The Master of Science Mathematical Physics is an international research-oriented two year master's program offered jointly by the departments of Mathematics and Physics within the Faculty of Science of the University of Tübingen starting every year in the winter semester. It is geared towards students with a solid background in Mathematics as well as in Physics, and it requires a bachelor's degree in physics or mathematics or an equivalent degree. The scientific discipline "Mathematical Physics" is concerned with the mathematically rigorous formulation and analysis of physical theories and models. In this master's program students will thus deepen and broaden their knowledge of Mathematics and Physics in interdisciplinary courses in Mathematical Physics as well as in disciplinary courses in Mathematics and Theoretical Physics. At the end of the program they are particularly well prepared for jobs where the typical competences of mathematicians are needed in combination with applications of physics. The program is international and cannot be pursued without a solid knowledge of the English language. Language skill on the level of B2 according to the European Framework of Reference for Languages are therefore required. All mandatory modules and a large number of facultative modules are offered only in English. Some facultative modules may sometimes be offered only in German.

1.2 Qualification Goals

Students deepen and broaden their theoretical knowledge of different areas of mathematical physics, mathematics and theoretical physics. They become proficient in general and specific methods and principles in these areas. They can connect problems and questions from physics with their counterparts in mathematical models and are able to judge and critically question the relevance and adequacy of mathematical models and the derived consequences. They are able to report on and scrutinize the current state of research in the area of their specialisation. Graduates can apply their expanded knowledge in order to develop and successfully handle their own research projects. They are able to present, discuss, and defend the results of their research in writing and orally in front of a scientific audience. In the course of the Mathematical Physics Colloquium students practice scientific collaboration and discourse in interdisciplinary and internationally mixed groups.

Their education enables graduates in mathematical physics to successfully and professionally tackle complex mathematical modelling problems in physics and, after an appropriate familiarization with the subject, also in other areas of technology, finance or economics. They are moreover well prepared for interdisciplinary and international collaborations in mixed teams of different specialists from different cultural backgrounds, as are common nowadays in all areas of research and development.

1.3 Program Structure

The Master's Program is a two year (four terms) consecutive study program with a modular structure. Based on the foundational modules "Geometry in Physics", "Functional Analysis in Geometry", "Mathematical Quantum Theory", and "Mathematical Relativity", to be attended during the first year, students can specialise rather freely according to their personal preferences in one or more areas of Mathematical Physics, Mathematics and/or Theoretical Physics. The few restrictions are that every student must take at least one module from the Mathematics master's program and one module from the Theoretical Physics master's program, as well as a seminar. As a consequence, all graduates of the Master's Program have proven their ability to successfully conduct mathematical studies and theoretical physics studies at the master's level. A Scientific Project in the third term typically serves as a preparation for the Master Thesis (M.Sc. Thesis, 30 ECTS-points) written during the final term. During the second year students are also required to attend the Mathematical Physics Colloquium. This is a weekly colloquium where specialists lecture about recent developments in Mathematical Physics, and students have the opportunity to meet and discuss with international guest scientists and local researchers about current topics. The prescribed period of study is two years corresponding to a total of 120 ECTS points.

1.4 Mentoring

At the start of the program every student will be assigned to a mentor from the group of professors involved in the master's program for the whole duration of his/her studies. Students meet their mentor at the beginning and later at least once per term in order to plan and discuss the progress of their studies. In particular, at these meetings the study and examination plan in compliance with the examination regulations is discussed. The module selection is documented and passed on to the head of the examinations board for approval. During the first meeting possible gaps in the knowledge should be discussed in order to fill them by taking appropriate courses within the area of elective specialisation. The study and examination plan is then updated every semester during the meetings with the mentor. The mandatory mentoring program assures that students specialise in a purposeful way and select accordingly goal-oriented combinations of modules from mathematics and physics.

During the meetings with the mentor also possible time slots for a study period at a university abroad can be discussed. In principle, every semester is suitable, depending on the study progress of the student and the courses avaliable at the other institution. It is also possible to write the master's thesis during a stay abroad under the cosupervision of a scientist there.

1.5 Information for students with a bachelor's degree in Physics at the University of Tübingen

Graduates of the 4-year degree program Bachelor of Science in Physics at the University of Tübingen can already gain up to 60 credit points for the degree program Master of Science in Mathematical Physics during their bachelor studies.

In particular,

• the module BMTPKFT Klassische Feldtheorie from the bachelor's program can be credited with

9 credit points for the module MAT-40-32 Advanced Topics in Theoretical Physics in the master's program, and

• up to 21 credit points in the section Vertiefungsfach in the bachelor's program can be credited in the section Elective Studies, provided the choice is suitable.

Moreover,

- up to 27 credit points in the section Ergänzungsmodule in the bachelor's program can be gained via the modules MAT-65-11 Geometry in Physics, MAT-65-12 Mathematical Quantum Theory, MAT-65-13 Mathematical Relativity or MAT-65-14 Mathematical Statistical Physics from the master's program, and
- the bachelor's thesis can be credited with 9 credit points in the module Scientific Project.

In order to finish the Master of Science in Mathematical Physics subsequently to the bachelor's degree in Physics at the University of Tübingen it is recommended to choose in the section Vertiefungsfach in the bachelor's program courses in theoretical physics, which can be credited in the section Elective Studies in the master's program in Mathematical Physics. Moreover, it is recommended to choose in the section Ergänzungsmodule in the bachelor's program at least two of the modules MAT-65-11, MAT-65-12, MAT-65-13 or MAT-65-14 from the master's program in Mathematical Physics. Good choices would be the combinations MAT-65-11 + MAT-65-13 and MAT-65-12 + MAT-65-14. Also the combination MAT-65-11 and MAT-65-12 would be suitable.

2 Study Plans

2.1 Overview by Modules

We provide here an overview of the study plan as a table showing the modules to be taken.

Suggested Term	Module Number	Module Title	Type of Course	Type of Module	Course- work	Type of Exam	ECTS- Points
Section 1: I	oundations						
1	MAT-65-11	Geometry in Physics	L+E	PM	EC	wr. o. or.	9
1	MAT-65-12	Mathematical Quantum Theory	L+E	PM	EC	wr. o. or.	9
2	MAT-65-13	Mathematical Relativity	L+E	PM	EC	wr. o. or.	9
Section 2: I	Knowledge Ex	xpansion					
1–3	MAT-40-31	Advanced Topics in Mathematics	L+E	PMW	EC	wr. o. or.	9
1–3	MAT-40-32	Advanced Topics in Theoretical Physics	L+E	PMW	EC	wr. o. or.	9
2–3	MAT-40-33	Seminar Knowledge Extension	S	PMW	s.M.	Pr	3
Section 3: I	Elective Spec	ialisation					
1-3		Modules from the master's programmes of the Department of Mathematics or Physics according to Section 3.		WPM			30
Section 4: 9	Scientific Wo	rk					
3	MAT-40-41	Scientific Project	Р	PM	s.M.	-	9
3–4 MAT-40-42 Mathematical Physics Colloquium			C+C	PM	-	-	3
4	MAT-40-43	Master Thesis M.Sc. Mathematical Physics	MT	PM	s.M.	MT	30

Abbreviations:

Course Work

Type of Module : PM=compulsory module, PMW=compulsory module with choice, WPM=elective module Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

: EC=exercise certificate

Other : h=hours, o.=or, s.M.=see module description

2.2 Overview by the Course of Studies

We first provide a general study plan showing the distribution of credit points over the different areas and the general time line. On the following pages example study plans for different types of specialisation are provided, where possible courses are assigned to the modules MAT-40-31 and MAT-40-32 as well as the modules from the area of Elective Specialisation.

Term	СР	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work
1.	27	27 CP			
2.	30	27 01	21 CP	30 CP	
3.	31			30 01	42 CP
4.	32				1 2. 01

Figure 2.1: General Study Plan

2.3 Example Study Plans

The example study plans shown below shall give an idea how the individual study in the different specialisations could look like. They are not meant as a recommendation, and it is neither guaranteed that the courses listed will be offered each year, nor that they all will be given in English.

Example Study Plan without Specialisation

Term	СР	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientif	ic Work	
		Geometry in Physics (9 CP)	Linear Partial				
1.	27	Mathematical Quantum Theory (9 CP)	Differential Equations (9 CP)				
		Mathematical		Advanced Topics in Mathematical Quantum Theory (9 CP)			
2.	30	Relativity (9 CP)	Seminar(3 CP)	Mathematical Statistical Physics (9 CP)			
			Quantum Field	Advanced Topics in Mathematical Relativity (6 CP)			
3.	31		Theory and Particle Physics (9CP)	Advanced Topics in Mathematical Statistical Physics (6 CP)	Mathe- matical Physics	Scientific Project (9 CP)	
4.	32				Colloquium (3 CP)	Master Thesis (30 CP)	

Figure 2.2: The program Mathematical Physics can be completed to a large extent also without choosing a particular specialisation. In this case we recommend taking all four foundational modules and also all advanced courses offered. The modules from the area Knowledge Expansion should then be chosen in accordance with the planned specialisation in the Scientific Project and the Master Thesis, cf. e.g. the following study plans.

Example Study Plan Quantum Theory

Term	СР	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientif	ic Work
		Geometry in Physics (9 CP)				
1.	27	Mathematical Quantum Theory (9 CP)	Operator Theory (9 CP)			
2.	30	Mathematical Relativity (9 CP)	Quantum Field Theory and Particle Physics (9 CP)	Functional Analysis (9 CP)		
			Seminar(3 CP)			
				Advanced Topics in Mathematical Quantum Theory (9 CP)		
3.	31			Computational Methods in Physics / Astrophysics (6 CP)	Mathe-	Scientific Project
				Theoretical Condensed Matter Physics (6 CP)	matical Physics Colloquium (3 CP)	(9 ČP)
4.	32					Master Thesis (30 CP)

Figure 2.3: The mathematical foundations of quantum theory are predominantly allocated to areas of analysis. Thus we recommend that those specialising in one of the areas Mathematical Quantum Theory, Quantum Field Theory, Condensed Matter, Many-Body Quantum Systems, or Quantum Information attend mathematical courses from analysis, e.g. Operator Theory, Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Relativity

Term	СР	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work		
		Geometry in Physics (9 CP)	Astronomy and				
1.	27	Mathematical Quantum Theory (9 CP)	Astrophysics (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Introduction to Partial Differential Equations (9 CP)	Riemannian Geometry (9 CP)			
	00		Seminar(3 CP)				
				Advanced Topics in Mathematical Relativity (9 CP)			
3.	31			Theoretical Astrophysics (6 CP)	Mathe-	Scientific Project	
				Computational methods in Physics / Astrophysics (6 CP)	matical Physics Colloquium (3 CP)	(9 ČP)	
4.	32					Master Thesis (30 CP)	

Figure 2.4: The mathematical foundations of relativity are predominantly allocated to areas of geometry and analysis. Thus we recommend that those specialising in one of the areas Mathematical Relativity, Astronomy, Cosmology, or Astro Physics attend mathematical courses from geometry, e.g. Riemannian Geometry and Lorentz Geometry, and from analysis, e.g. Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Statistical Physics

Term	СР	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientif	ic Work	
		Geometry in Physics (9 CP)					
1.	27	Mathematical Quantum Theory (9 CP)	Probability Theory (9 CP)				
		Mathematical	Advanced Statistical	Mathematical Statistical Physics (9 CP)			
2.	30	Relativity (9 CP)	Physics (9 CP)	Density Functional Theory (6 CP)			
				Advanced Topics in Mathematical Statistical Physics (6 CP)		Scientific	
3.	31		Seminar (3CP)	Mathematical Statistics (9 CP)	Mathe- matical Physics	Project (9 CP)	
4.	32				Colloquium (3 CP)	Master Thesis (30 CP)	

Figure 2.5: The mathematical foundations of statistical physics are predominantly allocated to areas of probability. Thus we recommend that those specialising in one of the areas Mathematical Statistical Physics, Soft Matter, or Density Functional Theory attend mathematical courses from probability, e.g. Probability Theory and Mathematical Statistics.

2.4 Overview by Study Progress and Credit Requirements

Ove	rview by Study Progress an	d Credi	it Requirer	nents	;								
			Exam				Teaching						
		Exam	(min)		Weight in the final grade	Type of Course			Points (CP)	is a red Composite are ma allocate to countion on	points to commenulsory al arked as ion of Er rses is fo ily. Cred	of examo semes idation of locations such. TCTS point or informatics are o complet	ters Inly. S he nts a- nly
		Type of Exam	Duration (min)	Grading	Weight	Type of	Status	SWS	ECTS	1. CP			4. CP
Fou	ndations of Mathematical Pl	nysics:							27				
MAT	-65-11 Geometry in Physics							6	9				
1.	Lecture	Wr. or	90–120 or	g	9	L	О	4		6			
2.	Exercises	Or.	20–30	9		E	О	2		3			
MAT	-65-12 Mathematical Quantun	n Theor	у					6	9				
1.	Lecture	Wr.	90–120	g	9	L	o	4		6			
2.	Exercises	or Or.	or 20–30	9		Е	0	2		3			
MAT	-65-13 Mathematical Relativity	/						6	9				
1.	Lecture	Wr.	90–120	_	9	L	0	4			6		
2.	Exercises	or Or.	or 20–30	g		E	0	2			3		
Kno	wledge Expansion:								21				
MAT	-40-31 Advanced Topics in Ma	athemat	ics					6	9				
1.	Lecture	Wr.	90–120	_	9	L	0	4		6			
2.	Exercises	or Or.	or 20–30	g	9	E	0	2		3			
MAT	-40-32 Advanced Topics in Ph							6	9				
1.	Lecture	Wr.	90–120	~	9	L	0	4			6		
2.	Exercises	or Or.	or 20–30	g	9	Е	0	2			3		
MAT	-40-33 Seminar							2	3				
1.	Seminar	Pres.	45–90	g	3	S	0	2				3	
Elec	ctive Specialisation:								30				
	Here the modules MAT-65-15 and Master's Programs in Mathem be discussed and agreed upor board.	d Astr	o and	Partic	cle Physics	s, car	be cl	nosen. 1	he choi	ces nee	d to		
MAT	-65-14 Mathematical Statistica	al Physi	cs					6	9				
1.	Lecture	Wr.	90–120			L	f	4			6		
2.	Exercises	or Or.	or 20–30	g	9	E	f	2			3		
	-65-21 Advanced Topics in Ma			um Th	eory			6	9				
1.	Lecture	Wr.	90–120			L	f	4			6		
	I.	or Or	or 20_30	^l g	9		<u> </u>			<u> </u>	<u> </u>	I	1

Or.

20-30

			Exam				Teaching				Те	rm	
		Type of Exam	Duration (min)	D	Weight in the final grade	f Course			Points (CP)	is a recomposite are material allocate to countries on the countries of th	points to commenulsory all arked as ion of Ed rses is fo ily. Cred	of examo semes dation of location of such. TCTS point or informatics are occupled	ters only. s he nts a- nly
		be o	ıratic	Grading	eigh.	Type of	Status	SMS	ECTS	1.	2.	3.	4.
		Ţ	ă	ত্র	>	1≥	S.	S	Ш	CP	CP	CP	CF
2.	Exercises					E	f	2			3		
MAT-65-22 Advanced Topics in Mathematical Quantum Th				eory	(short	version)	4	6			Г		
1.	Lecture	Wr. or	90–120 or	g	6	L	f	2			3		
2.	Exercises	Or.	20–30			E	f	2			3		
MAT-65-23 Advanced Topics in Mathematical Relativity				ity	I	ı		6	9				1
1.	Lecture	Wr. or	90–120 or	g	9	L	f	2				3	
2.	Exercises	Or.	20–30			E	f	2				3	
MAT	-65-24 Advanced Topics in M	athemat	ical Relativ	ity (sh	ort ve	ersion)	4	6				
1.	Lecture	Wr. or	90–120 or	g	6	L	f	2				3	
2.	Exercises	Or.	20–30			Е	f	2				3	
Scie	entific Work								42				
MAT	-40-41 Scientific Project								9				
1.	Project	Proj.		ng	9		0					9	
MAT	-40-42 Mathematical Physics	Colloqu	ium				1		3				
1.	1. Colloquium ng		ng			0					1	2	
MAT-40-43 Master Thesis									30				
1.	Thesis	g	30		0						30		

: o=obligatory, f=fakultative : o.=or, SWS=hours in class per week, CP=credit points=ECTS points Status Other

3 Module Descriptions

Section 1: Foundations

In the case that some of the mandatory modules in this section or modules, which are essentially identical as far as the contents and competences are concerned, have been part of the Bachelor studies, which are the prerequisite for this Master's Degree Program, according to the examination regulations these modules cannot be taken in the Master's Degree Program any more. They have to be replaced by other suitable modules in the framework of the studies and examination plan.

Module Number: MAT-65-11	Module Title: Geometry in Physics		Type of Module: Compulsory Module						
ECTS-Points	9								
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h						
Duration	1 Semester								
Frequency	regularly in Winter Semeste	1							
Term	1								
Language of Instruction	English								
Forms of Teaching and Learning	Lectures 4 SWS + Exercise	Classes 2 SWS, Homework A	ssignements						
Content	relevance for physics. Partic	cular topics are manifolds, diff	ods of differential geometry and their ferential forms, Riemannian metrics ry of submanifolds, real vector bun- Physics are discussed.						
Objectives	tions of differential geometrential and integral calculus are naturally applied within pacquaintance with the use of ticular, a deeper understance examples how the mathematem dents are able to name and well as to explain the contexto Through homework assignment independent acquaintal lectures. They learn how to develop solution strategies	y. They develop, in particular and experience through examply sical theories. Students ob if the listed notions of differential and integratical notions are naturally approve the essential statement developed in the lecture and nents and exercise classes stude with the notions, statement transfer these methods to ne	tance with the use of the listed nor, a deeper understanding of differples how the mathematical notions tain knowledge, understanding, and tial geometry. They develop, in parall calculus and experience through plied within physical theories. Stutes and concepts from the lecture as to put it into a larger framework. In Judents develop a confident, precise, ents, and methods explained in the ew problems, to analyse them and roup. They are able to present their cessary.						

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometry in Physics	L	0	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		Е	0	2	3		OI.	0. 20-30		
	In this module students need the exam. The type of examin							in order to	be adn	nitted to
Literature	Possible References:	man iffere	ifolds ntial	: An geor	intro metr	oduction y for ph	n. Springer ysicists. Wo	orld Scientifi		
Transfer	ativity. Successful completion	Participation in the module is a prerequisite for participation in the module Mathematical Relativity. Successful completion of the module may be a prerequisite for participation in the module Seminar Knowledge Extension and is so for the participation in the module Scientific Project.								
Prerequisites	There are no further prerequis	ites.								
Responsible Persons	Christoph Bohle, Carla Ceder	oaum	, Ste	fan	Teufe	el				

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-12	Module Title: Mathematical Quantum Theo	ory						of Module: ulsory Modu	le		
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		lass	:		Self-St 180 h	tudy:			
Duration	1 Semester regularly in Winter Semester										
Frequency	regularly in Winter Semester										
Term	1										
Language of Instruction	English										
Forms of Teaching and Learning	Lectures 4 SWS + Exercise (Classe	es 2 S	sws	, Hor	mework	Assignem	ents			
Content	The module provides an intro stationary and time-depende Rayleigh-Schrödinger pertur space formalism, and elementheory and semiclassical app	nt Sch bation nts of s	rödir thec scatt	nger ory a ering	equa nd H I theo	ation, fu lartree- ory. Op	ndamental resp. Har tionally, oth	approximati tree-Fock th	on met eory, t	hods as he Fock	
Objectives	Students obtain knowledge at them to analyse known and physical problems in atom, sepectral and interference the the mathematical model and amples how the mathematical they enhance their knowledge derstanding of the listed notion problems from quantum theostate and particle pysics and retical methods and to quest of the results derived from it notions are naturally applied on methods and subjects. Stoncepts from the lecture as it into a larger framework. Through homework assignmand independent acquaintar lectures. They learn how to to develop solution strategies solutions and to stand for the	new polid storetical of the of the on roons a cory. The distribution the well a certain and the certain storetics on the core with the certain storetics on the core of the certain storetics.	roble ate a calculation of the c	ems f frame	rom particles and deriv atura atura and s ds and deriv atura and s ds and atica ace are erien theo to n ain th se cl tions meth and w	quantucle pysid to quared from ally appeared can be interred. I mode acceptate through a see content assessing, statements to within a	m theory. Tos and their estion the rinit. Studer lied within particular within particular within particular within particular physical via specific pugh example prove the example prove the example e	They are ablar mathematic elevance and the experient obtain known of analyse known the emathematic eles how the essential seed in the lecture entry entry and analyse known of analyse ana	e to interest to i	derrelate dels via quacy of augh ex- Thereby, and unand new m, solid be theodel and dematical by	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Mathematical Quantum Theory	L	0	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
		In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	Successful completion of mo ticipation in the module Adva pletion of one of the modules prerequisite for the participat	nced Math	Topic ema	cs in tical	Math Quar	nematic ntum Tl	al Quantun neory and N	n Theory. Su	ccess	ful com-	

Prerequisites	-
Responsible Persons	Christian Hainzl, Stefan Teufel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-13	Module Title: Mathematical Relativity							of Module: ulsory Modu	le		
ECTS-Points	9						·				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		lass	:		Self-St 180 h	tudy:			
Duration	1 Semester										
Frequency	regularly in Summer Semest	er									
Term	2										
Language of Instruction	English	inglish									
Forms of Teaching and Learning	Lectures 4 SWS + Exercise (ectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements									
Content	are Newton's theory of gravit tions, the Schwarzschild spa matter models, black holes,	the module provides an introduction to the mathematical theory of relativity. Particular topics are Newton's theory of gravity, special theory of relativity, relativistic effects, Einstein's equators, the Schwarzschild spacetime. Optionally, other topics such as cosmological models, latter models, black holes, Cauchy problem and ADM decomposition, singularity theorems of gravitational waves can be discussed.									
Objectives	use them to analyse known interrelate physical problems through methods from differe mathematical model and the on methods and subjects ga 65-11. Students are able to lecture as well as to explain framework. Through homework assignm and independent acquaintar lectures. They learn how to	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from the theory of relativity. They are able to interrelate physical problems in cosmology and astrophysics and their mathematical models through methods from differential geometry and to question the relevance and adequacy of the mathematical model and the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular in module MAT-65-11. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Mathematical Relativity	ш г Туре of Course	o o Status	SMS 4 2	6 3	Coursework	Type of Exam or. o.	90-180 o. 20-30	Grading	Weight for Grade	
	In this module students need the exam. The type of exami							s in order to	be adı	nitted to	
Transfer	in the module Advanced Top the modules Mathematical F	Successful completion of module Mathematical Relativity is a prerequisite for the participation in the module Advanced Topics in Mathematical Relativity. Successful completion of one of the modules Mathematical Relativity or Mathematical Quantum Theory is a prerequisite for the participation in the module Scientific Project.									
Prerequisites	Participation in the module Geometry in Physics is a prerequisite.										
Responsible Persons	Carla Cederbaum, Gerhard I	Huiske	en, Fr	ank	Loos	e					

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Section 2: Knowledge Expansion

Module Number:	Module Title:							f Module:		
MAT-40-31	Advanced Topics in Mathema	tics					Compu	ılsory Modu	le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	Every semester									
Term	1–3	3								
Language of Instruction	English or German	English or German								
Forms of Teaching and Learning	Lectures 4 SWS + Exercise C	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements								
Content	the correspondent SWS-cove mended subjects are for insta- tions, Harmonic analysis, Lie tic processes, Calculus of va- geometry. Further details car	is required to attend one or more lectures as well as the respective exercise classes with the correspondent SWS-coverage from the Master's degree program in Mathematics. Recombeneded subjects are for instance Partial differential equations, Numerics of differential equations, Harmonic analysis, Lie groups, Nonlinear functional analysis, Operator theory, Stochastic processes, Calculus of variations, Symplectic geometry, Algebraic topology or Algebraic eometry. Further details can be found in the catalogue of mathematical modules starting at age 40 and in the module handbook of the degree program M.Sc. Mathematics.								
Objectives	The students aquire deepend knowledge in one selected area of mathematics independently of physical applications. They broaden the basis of their mathematical knowledge and extend the methods at hand to tackle mathematical problems. The further qualification goals, in particular the concrete content related qualifaction goals, will follow from the module description of the chosen course in the module handbook for the M.Sc. Mathematics.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Advanced Topics in Mathematics	L E	0	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module students need the exam. The type of examin							in order to	be adr	nitted to
Transfer	The module may be a prereq	uisite	for th	ie m	aster	thesis				
Prerequisites	See prerequisites in the Mode	ıle Ha	ındb	ook N	И.Sc	. Mathe	ematics.			
Responsible Persons	The dean of studies at the de	partm	ent o	of ma	athen	natics				
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-40-32	Module Title: Advanced Topics in Theoreti	cal Ph	veice					of Module: ulsory Modu	le with	Choice	
ECTS-Points	9		yoloc				Compt	discry wiedd	ic with	Onoice	
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St	tudy:			
Duration	1 Semester										
Frequency	Every semester	very semester									
Term	1–3	-3									
Language of Instruction	English or German	nglish or German									
Forms of Teaching and Learning	Lectures 4 SWS + Exercise	ectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements									
Content	physics as well as the resp from the Master's degree pro- ticls Physics. Recommended physics, Theoretical astrophysics, Advanced statistical physics, tum optics, Quantum information trophysics, Current topics in	is required to attend one or more advanced-level lectures from the field of theoretical physics as well as the respective exercise classes with the correspondent SWS-coverage from the Master's degree program in Physics or the Master's degree program Astro and Parcicle Physics. Recommended subjects are for instance Quantum field theory and Particle Physics, Theoretical astrophysics, Relativistic astrophysics, Many-particle quantum systems, advanced statistical physics, Yang-Mills theory, Condensed matter physics, Theoretical quantum optics, Quantum information theory, Cosmology, Numerical methods in physics and astrophysics, Current topics in theoretical physics. Further details can be found in the module andbook of the corresponding degree programs.									
Objectives	The students aquire deeper pendently of rigorous mathe theoretical physics and exter qualification goals, in particu the module description of the the M.Sc. Astro and Particle	matica d the lar the chos	I fori meth cond en c	malis lods crete	m. 7 at ha	They brand to ta tent rela	oaden the l ackle proble ated qualifa	pasis of their ems in physic tion goals,	r know cs. The will foll	ledge in e further low from	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Advanced Topics in Theoretical Physics	L E	0	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module students need the exam. The type of exami	l to su nation	cces is se	sfully et by	con	nplete a	assignments or.	s in order to	be adr	nitted to	
Transfer	The module may be a prerec	uisite	for th	ne m	aste	r thesis					
Prerequisites	See prerequisites in the Module Handbook M.Sc. Physics or M.Sc. Astro and Particle Physics.										
Responsible Persons	Die Studiendekanin oder der Studiendekan des Fachbereichs Physik										
Abbroviations											

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-40-33	Module Title: Seminar Knowledge Extension							Type of Module: Compulsory Module with Choice			
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	:		Self-S 60 h	tudy:			
Duration	1 Semester						'				
Frequency	Every semester										
Term	2–3	-3									
Language of Instruction	English or German	nglish or German									
Forms of Teaching and Learning	Seminar: Presentation, Disci	eminar: Presentation, Discussion, Teamwork, Handout									
Content	Various topics from various areas of Mathematical Physics, Mathematics or Theoretical Physics.										
Objectives	The students have learnt to develop independently or in team an acquaintance with an advanced topic in Mathematics or Physics by applying scientific methods and to present it in form of an oral presentation. They have improved their skills in the presentation of mathematical or physical results and are able to argue for these results in critical discussions.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Seminar	S	0	2	3	yes	Pr	45–90	g	100	
Transfer	The module may be a prerec	uisite	for th	ne m	aste	thesis		-1	1		
Prerequisites	Successful completion of one Physics".	e of th	e mo	dule	s fro	m the S	Section "Fo	oundations o	f Math	ematical	
Responsible Persons	Stefan Teufel										
	graded, ng=not graded	am, wr	.=wr	tten	exar	n, Pr=p	resentatio	n, H=essay, l	P=port	folio	

: o=obligatory, f=facultative Status

Section 3: Elective Specialisation

Within the study area Elective Specialisation students can choose modules from the Master Programs Mathematical Physics, Mathematics, Physics, and Astro and Particle Physics according to their individual interests. In particular, courses listed in the module descriptions MAT-40-31 and MAT-40-32 but not chosen there, the module MAT-65-13 respectively MAT-65-14 not yet chosen in the area Foundations, the modules MAT-65-15 and MAT-65-21 to MAT-65-24, as well as other appropriate advanced modules from the programs Mathematical Physics, Mathematics (see ??), Physics, and Astro and Particle Physics are available. Note that not all modules can be offered every year, but there is always a broad choice. Also note that some modules from other programs might be offered only in German, but also here a choice of English courses is ensured. The selection of modules within the area Elective Specialisation must be discussed and decided together with the mentor. Each module can be selected only once. In agreement with the mentor and upon request at the examinations board, 9 ECTS points within the area of Elective Specialisation can be allocated for modules that serve to close knowledge gaps either in mathematics or physics.

Within the area of Elective Specialisation students obtain relevant skills. They learn to independently judge which additional qualifications and competences are relevant to their studies and to select courses accordingly. They are able to acquire specific knowledge also beyond the mandatory parts of the study program. Within the area of their specialisation they can report on and scrutinize the current state of research. In the exercise classes students learn to work confidently, precisely and independently with the notions, statements and methods presented during the lectures. They also learn how to apply methods to new problems and to analyse and solve them alone or in groups.

•	Advanced Topics in Mathematical Quantum Theory (MAT-65-21, 9 CP)	27
• ,	Advanced Topics in Mathematical Quantum Theory (short version) (MAT-65-22, 6 CP)	29
• ,	Advanced Topics in Mathematical Relativity (MAT-65-23, 9 CP)	31
• ,	Advanced Topics in Mathematical Relativity (short version) (MAT-65-24, 6 CP)	33
•	Foundations of Quantum Mechanics (MAT-65-15, 9 CP)	25
•	Mathematical Statistical Physics (MAT-65-14, 9 CP)	23

Module Number: MAT-65-14	Module Title: Mathematical Statistical Phy	rsics	Type of Module: Elective Module								
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h								
Duration	1 Semester										
Frequency	not regularly, in Summer Se	ot regularly, in Summer Semester									
Term	2-3	3									
Language of Instruction	English										
Forms of Teaching and Learning	Lectures 4 SWS + Exercise	Classes 2 SWS, Homework A	ssignements								
Content	concepts of probability theor bles, thermal equilibrium, E cesses, Wiener process), la phase transitions), statistica tion to thermal equilibrium, I	y, classical statistical mechani Boltzmann equation, entropy), attice models (Ising model, Gil al quantum mechanics (quant Bose-Einstein condensate). O t phenomena, renormalization	istical physics. Particular topics are cs of gases (equivalence of ensem-Brownian motion (stochastic probbs measure, thermodynamic limit, um mechanical ensembles, transiptionally, other topics such as open a group theory and the fluctuation-								

Objectives Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from statistical physics. They are able to interrelate fundamental physical concepts, such as equilibrium, irreversibility and entropy, and their mathematical models via probabilistic methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular on probability theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary. Exam (min) Weight for Grade Type of Course Requirements Type of Exam Coursework for Obtaining Credit, Grading, Grading Dur. of I Status Weight if ECTS SWS applicable Title Mathematical Statistical L 4 6 90-180 wr. o. 100 yes g **Physics** o. 20-30 or. 2 Ε 3 0 In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor. **Transfer** Successful completion of module is a prerequisite for the participation in the module Advanced Topics in Mathematical Statistical Physics. **Prerequisites** Responsible Marcello Porta, Roderich Tumulka **Persons**

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-15	Module Title: Foundations of Quantum Me	chanic	cs					of Module: e Module			
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-Si 180 h	tudy:			
Duration	1 Semester						·				
Frequency	regularly every two years										
Term	2-3	-3									
Language of Instruction	English	English									
Forms of Teaching and Learning	Lectures 4 SWS + Exercise (Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements									
Content	ing its mathematical and phili Bohmian mechanics, many v sented and analysed mathen berg's uncertainty principle,	The module provides an introduction to fundamental questions of quantum mechanics, including its mathematical and philosophical aspects. Various interpretations such as Copenhagen, Bohmian mechanics, many worlds and spontaneous collapse of the wave function are presented and analysed mathematically and physically. Other topics include Born's rule, Heisenberg's uncertainty principle, the quantum measurement problem, Bell's non-locality theorem, dentical particles and no-hidden-variable theorems.									
Objectives	understand several important matical knowledge relevant to mathematical treatment with surprising phenomena and prontroversial about the orthodebate on fundamental issue results of the lecture as well in the exercise classes they the terms, statements and m	Students know and can apply the rules of quantum mechanics in different environments and understand several important theories of how the quantum world works. They acquire mathematical knowledge relevant to the application of these rules and theories and can connect the mathematical treatment with the physical meaning. They will familiarise themselves with the surprising phenomena and paradoxes of quantum mechanics. They will appreciate what is controversial about the orthodox interpretation and why, and will be able to follow the current debate on fundamental issues. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Foundations of Quantum Mechanics	L E	f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	The type of examination is se	et by tl	ne in	struc	tor.						
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.										
Prerequisites	The basic modules on Analys	sis an	d Lin	ear A	Algeb	ora are	required.				
Responsible Persons	Roderich Tumulka										

 $\label{eq:Grading System} \textbf{ : g=graded, ng=not graded} \\$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-21	Module Title: Advanced Topics in Mathematics	atical (Quar	ıtum	The	ory		of Module: re Module		
ECTS-Points	9						l l			
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-Si 180 h	tudy:		
Duration	1 Semester						·			
Frequency	not regularly									
Term	2-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements									
Content	like Hartree and Hartree-Fo mathematical models in qua tems. It will present both th	The module provides an introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent ferminon systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.								
Objectives	able to apply them in the anal ematical Quantum Theory. Soncepts from the lecture as it into a larger framework. The of research in the specific and Through homework assignment independent acquaintar lectures. They learn how to	Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Advanced Topics in Mathematical Quantum	L	0	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module students need the exam. The type of exami						ssignment:		be adr	nitted to
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	Knowledge from the module	Mathe	emati	cal C	Quan	tum Th	eory is ass	umed.		
Responsible Persons	Stefan Teufel									

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-22	Module Title: Advanced Topics in Mathema version)	itical C	Quan	tum ⁻	Γheo	ry (sho		of Module: re Module			
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass			Self-St 120 h	tudy:			
Duration	1 Semester										
Frequency	not regularly, in Summer Semester										
Term	2										
Language of Instruction	English	English									
Forms of Teaching and Learning	Lectures 2 SWS + Exercise Classes 2 SWS, Homework Assignements										
Content	theory, like Hartree and Har group, mathematical models systems. It will present both	The module provides a short introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent ferminon systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.									
Objectives	Students obtain knowledge a able to apply them in the ana ematical Quantum Theory. S concepts from the lecture as it into a larger framework. The current state of research in the Through homework assignment and independent acquaintar lectures. They learn how to develop solution strategies solutions and to stand for the	lysis of tudent well and are speents and transfer on the speents and the speents and the speents on the speents of the speents	of knows are able cific and extending the the cific and the cific are	own a able explained to a control of the control of	and retorn the torn the torn the	new proname and control aribe ar asses and control asses and control asses and control asses and control are assessment as a control are a control are as a control are a contro	oblems from nd prove the ext developed in parts a students de ments, and new proble group. The	n the specific e essential s ed in the lec also critically evelop a cont methods ex ems, to ana	tarea of tarea of tare are are are are are are are are are	of Math- ents and nd to put enge the precise, d in the em and	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Advanced Topics in Mathematical Quantum Theory	L E	0	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module students need the exam. The type of exami	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	The module may be a prerequisite for the master thesis.										
Prerequisites	Knowledge from the module	Mathe	mati	cal C	Quan	tum Th	eory is assi	umed.			
Responsible Persons	Stefan Teufel										

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-23	Module Title: Advanced Topics in Mathema	atical I	Relat	ivity				of Module:		
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-Si 180 h	tudy:		
Duration	1 Semester						'			
Frequency	not regularly, in Winter Seme	ster								
Term	3									
Language of Instruction	English	English								
Forms of Teaching and Learning	Lectures 4 SWS + Exercise (Classe	s 2 S	SWS	, Hoi	mework	Assignem	ents		
Content	The module provides an introduction to an advanced topic of mathematical theory of relativity. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.									
Objectives	learn analytic and geometric equations and to examine th mathematical solutions. Stuconcepts from the lecture as it into a larger framework. The of research in the specific are Through homework assignment independent acquaintar lectures. They learn how to	Students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Advanced Topics in Mathematical Relativity	L	0	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module students need the exam. The type of exami	to su	cces	sfully	con			s in order to	be adr	nitted to
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	Knowledge from the module Mathematical Relativity is assumed.									
Responsible Persons	Carla Cederbaum, Gerhard Huisken, Frank Loose									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-24	Module Title: Advanced Topics in Mathension)	natical	Rel	ativit	y (sł	nort ve		of Module: e Module				
ECTS-Points	6											
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h		lass	:		Self-St 120 h	udy:				
Duration	1 Semester						·					
Frequency	not regularly, in Winter Seme	ster										
Term	3	3										
Language of Instruction	English											
Forms of Teaching and Learning	Lectures 2 SWS + Exercise (ectures 2 SWS + Exercise Classes 2 SWS, Homework Assignements										
Content	relativity. It will present both t	The module provides a short introduction to an advanced topic of mathematical theory of elativity. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.										
Objectives	Students obtain deepend kn learn analytic and geometric equations and to examine the mathematical solutions. Students from the lecture as it into a larger framework. The current state of research in the Through homework assignment and independent acquaintant lectures. They learn how to to develop solution strategies solutions and to stand for the	techrese. No dents well a ney are specially a ce with transfer on the second se	nique Morec are a s to e able cific nd e th the fer the	es in over, able explained to area exercing the second to	orde they to na ain th desc se cl tions meth and v	er to produce do under control ar ar asses on the control assess on the control	ove existend the derstand the derstand the derstand the ext developed in parts a students dements, and new proble group. The	ce of solutice physical resential sed in the lecular critically velop a contimethods exems, to ana	ons of elevance tatementure ar challed fident, coplaine lyse th	Einstein the of the ents and ad to put enge the precise, d in the em and		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Advanced Topics in Mathematical Relativity	п Type of Course	o Status	SWS 2	s ECTS	Coursework	Type of Exam o. o.	Dur. of Exam (min) 0. 20-30	ص Grading	Weight for Grade		
	the exam. The type of exami	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor. — In exceptional cases the module can be offered by the lecturer without exercises, in this case, only 3 credit points are awarded										
Transfer	The module may be a prerequisite for the master thesis.											
Prerequisites	Knowledge from the module Mathematical Relativity is assumed.											
Responsible Persons	Carla Cederbaum, Gerhard I	luiske	ın, Fı	ank	Loos	е						

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Section 4: Scientific Work

Module Number: MAT-40-41	Module Title: Scientific Project							Type of Module: Compulsory Module			
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 15 h						Self-Study: 255 h			
Duration	1 Semester										
Frequency	Every semester										
Term	3										
Language of Instruction	English										
Forms of Teaching and Learning	Individual supervision by a mentor, study of scientific works.										
Content	 Definition of an advanced scientific project in coordination with the mentor. Independent search and study of the relevant scientific literature. Formulation of specific problems and methodical approach to their solution. Written presentation of the project in conext of current state of research on 5-10 pages. This module serves generally as a preparation for the Master Thesis 										
Objectives	 Students develop skills to systematically familiarize themselves with a new subject, learn to work critically and to form a substantiated, professional and interdisciplinary judgement, acquire qualifications in such areas as literature research, identification of relevant problems and appropriate methods, as well as in the written presentation of a research proposal. 										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Scientific Project	т Type of Course	o Status	SWS 1	© ECTS	Coursework	Type of Exam	Dur. of Exam (min)	g Grading	Weight for Grade	
Transfer	Successful completion of this module is a prerequisite for participation in module Master Thesis.										
Prerequisites	Successful completion of module Geometry in Physics and of one of the modules Mathematical Quantum Theory or Mathematical Relativity.										
Responsible Persons	Stefan Teufel, Werner Vogelsang.										

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-40-42	Module Title: Mathematical Physics Colloquium							of Module: ulsory Modu	le	
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h									
Duration	2 Semester									
Frequency	Every semester									
Term	3–4									
Language of Instruction	English									
Forms of Teaching and Learning	Presentations, discussions. Specific form of study: during	Presentations, discussions. Specific form of study: during the final semester students present their Master thesis.								
Content	During each semester on 15 appointed dates (2 h each) there will take place presentations and discussions on current topics in mathematical physics. Speakers are the researchers of the involved departments, guest scientists and master's students, who present the results of their Master Thesis.									
Objectives	Students gain an insight into the current development of mathematical physics beyond the area of their own specialization. They develop the ability to follow scientific presentations and to discuss and challenge them within a larger group of scholars. They therefore also obtain interdisciplinary and intercultural competencies through regular cooperation and discussion in mixed groups.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
аррисаые	Colloquium Winter Semester	С	0	2	1	no	-	-	ng	-
	Colloquium Summer Semester	С	0	2	2	no	-	-	ng	-
Transfer	-	1				1	1	1	1	1
Prerequisites	-									
Responsible Persons	Carla Cederbaum, Stefan Te	ufel								

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-40-43	Module Title:Type of Module:Master Thesis M.Sc. Mathematical PhysicsCompulsory Module											
ECTS-Points	30						·					
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 900 h											
Duration	1 Semester	1 Semester										
Frequency	Every semester											
Term	4											
Language of Instruction	English or German	English or German										
Forms of Teaching and Learning	Master thesis											
Content	Students are assigned to workgroups and participate in seminars of the group. Under the supervision of the mentor students have to handle a concrete problem from mathematical physics by applying scientific methods and present it in written form in English or German. In particular this includes: • Definition of an advanced scientific task in coordination with the mentor; • Independent search and study of the relevant scientific literature; • Formulation of appropriate questions and methodical approach to their answers; • Independent execution and written presention of the project and the results in the context of the current state of research; • Presentation of the results in English in Mathematical Physics Colloquium.											
Objectives	Students are able to develop acquaintance with a new problem within a given period of time and treat it with increasing independence by applying scientific methods; develop acquaintance with scientific literature on a new topic; critically interpret scientific results and integrate them into their state of knowledge; present their results in written form based on principles of Good Scientific Practice; present their work in an international scientific environment.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Master Thesis	M Type of Course	o Status	SWS	% ECTS	Coursework	Type of Exam	Dur. of Exam (min)	ص Grading	Weight for Grade		
Transfer	-											

Prerequisites	
	27 CP from the compulsory elective Section Foundations of Mathematical Physics,
	a total of 18 CP from the Sections Knowledge Expansion and Elective Specialisation,
	Successful completion of module Scientific Project.
Responsible Persons	Stefan Teufel, Werner Vogelsang.

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Catalogue of Mathematics Modules

This section lists modules from the M.Sc. Mathematics programme that can be included in the area Elective Specialisation and which are not yet listed in Section 3.

•	Abstract Dynamical Systems (MAT-55-33, 9 CP)	194
•	Algebraic Curves (MAT-45-14, 9 CP)	56
•	Algebraic Curves and Riemann Surfaces (MAT-50-29, 9 CP)	140
•	Algebraic Geometry (MAT-45-11, 9 CP)	50
•	Algebraic Geometry and Toric Varieties (MAT-45-12, 9 CP)	. 52
•	Algebraic Groups (MAT-45-16, 9 CP)	. 60
•	Algebraic Number Theory (MAT-45-21, 9 CP)	. 68
•	Algebraic Topology 1 (MAT-50-21, 9 CP)	124
•	Algebraic Topology 2 (MAT-50-22, 9 CP)	126
•	Algebraic Topology 3 (MAT-50-23, 3 CP)	128
•	Algebraic Transformation Groups (MAT-45-13, 9 CP)	54
•	Algorithms of Numerical Mathematics (MAT-70-01, 9 CP)	281
•	Applied Topology 2 (MAT-50-26, 3 CP)	134
•	Applied topology 1 (MAT-50-25, 3 CP)	132
•	Area Minimising Flows (MAT-55-43, 5 CP)	202
•	Automorphic Forms (MAT-55-53, 5 CP)	220
•	Bayesian Networks and Causality (MAT-75-21, 5 CP)	345
•	Calculus of Variations (MAT-55-49, 5 CP)	214
•	Cohomology and Sheaves (MAT-55-61, 9 CP)	224
•	Combinatorics (MAT-75-02, 9 CP)	321
•	Commutative Algebra (MAT-45-02, 9 CP)	. 46
•	Computer Algebra (MAT-45-03, 9 CP)	. 48
•	Consistency Proofs (MAT-55-62, 6 CP)	226
•	Control Theory (MAT-55-06, 9 CP)	158
•	Convex Geometry (MAT-50-02, 9 CP)	92
•	Cox Rings (MAT-45-18, 9 CP)	. 62
•	Elastic Curves (MAT-55-46, 3 CP)	208
•	Elementary Number Theory (MAT-45-25, 6 CP)	. 74
•	Elliptic Curves and Cryptography (MAT-45-27, 9 CP)	. 78
•	Elliptic Curves and Taniyama-Shimura (MAT-45-28, 9 CP)	80
•	Elliptic Functions and Elliptic Curves (MAT-45-24, 9 CP)	. 72
•	Ergodic Theory (MAT-55-05, 9 CP)	156
•	Explicit Mathematics (MAT-55-65, 6 CP)	231
•	Financial Mathematics and Numerics (MAT-70-51, 6 CP)	317
•	Foundations of Discrete Mathematics (MAT-75-12, 9 CP)	341

•	Fully Non-Linear Elliptic Equations (MAT-55-27, 5 CP)	188
•	Fully Non-Linear Elliptic and Parabolic Partial Differential Equations (MAT-60-36, 3 CP)	263
•	Functional Analysis (MAT-55-01, 9 CP)	148
•	Game Theory (MAT-70-40, 3 CP)	316
•	Geometric Evolution Equations (MAT-60-01, 3 CP)	237
•	Geometric Group Theory (MAT-50-30, 9 CP)	142
•	Geometric Measure Theory (MAT-55-42, 9 CP)	200
•	Geometric Measure Theory – Flows (MAT-55-48, 5 CP)	212
•	Geometric Measure Theory – Varifolds (MAT-55-47, 5 CP)	210
•	Geometric Variation Problems (MAT-60-02, 3 CP)	239
•	Geometry of Manifolds 1 (MAT-50-10, 9 CP)	102
•	Geometry of Manifolds 2 (MAT-50-11, 9 CP)	104
•	Graph Theory (MAT-75-10, 9 CP)	337
•	Gravitational Collapse and Singularities in General Relativity (MAT-60-30, 3 CP)	259
•	Gromov-Witten Theory (MAT-50-40, 6 CP)	144
•	Groups and Representations (MAT-65-05, 9 CP)	265
•	Hamiltonian Systems (MAT-65-38, 9 CP)	277
•	Harmonic Analysis in Euclidean Space (MAT-55-12, 9 CP)	170
•	Harmonic Analysis on Abelian Groups (MAT-55-13, 9 CP)	172
•	Harmonic Analysis on General Groups (MAT-55-14, 9 CP)	174
•	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic (MAT-50-50, 9 CP)	146
•	Information Geometry (MAT-50-12, 3 CP)	106
•	Information Geometry and Neural Data Processing 2 (MAT-50-13, 3 CP)	108
•	Information Theory (MAT-75-07, 9 CP)	331
•	Integrable Systems (and Infinite Dimensional Lie Algebras) (MAT-50-18, 9 CP)	118
•	Introduction to Analytic Number Theory (MAT-45-26, 3 CP)	. 76
•	Introduction to Berkovich Geometry (MAT-45-20, 3 CP)	. 66
•	Introduction to Combinatorial Birational Geometry (MAT-45-40, 9 CP)	. 88
•	Introduction to Combinatorial Mirror Symmetry (MAT-45-41, 6 CP)	. 90
•	Introduction to Commutative Algebra and Algebraic Geometry (MAT-45-01, 9 CP)	. 44
•	Introduction to Dynamical Systems (MAT-55-34, 3 CP)	196
•	Introduction to Geometric Measure Theory (MAT-55-41, 9 CP)	198
•	Introduction to Geometric Measure Theory – Measure Theoretic Methods (MAT-55-44, 5 CP)	204
•	Introduction to Geometric Measure Theory – Varifolds (MAT-55-45, 5 CP)	206
•	Introduction to Harmonic Analysis (MAT-55-11, 9 CP)	168
•	Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory) (MAT-50-CP)	
•	Introduction to K-theory (MAT-50-24, 3 CP)	130

•	Introduction to Mathematical Logic (MAT-55-60, 3 CP)	222
•	Introduction to Modular Forms (MAT-45-29, 3 CP)	. 82
•	Introduction to Optimisation (MAT-70-20, 6 CP)	300
•	Introduction to Partial Differential Equations (MAT-55-21, 9 CP)	178
•	Introduction to Partial Differential Equations – Part 1 (MAT-55-25, 5 CP)	184
•	Introduction to Riemann Surfaces (MAT-50-15, 5 CP)	112
•	Introduction to Stochastic Differential Equations - Part 1 (MAT-70-12, 5 CP)	295
•	Introduction to Tropical Enumerative Geometry (MAT-50-05, 5 CP)	. 98
•	Introduction to set theory (MAT-55-63, 3 CP)	228
•	Lie Groups (MAT-55-51, 9 CP)	216
•	Limits of Spaces (MAT-60-05, 6 CP)	245
•	Linear Control Theory (MAT-55-07, 6 CP)	160
•	Markov Chains and Applications (MAT-75-11, 9 CP)	339
•	Mathematical Aspects of Neuronal Information Processing 1 (MAT-50-14, 3 CP)	110
•	Mathematical Aspects of Neuronal Information Processing 2 (MAT-50-19, 3 CP)	120
•	Mathematical Aspects of the Quantum Hall Effect (MAT-65-32, 6 CP)	269
•	Mathematical Methods for Condensed Matter Physics (MAT-65-31, 6 CP)	267
•	Mathematical Population Genetics (MAT-75-08, 6 CP)	333
•	Mathematical Statistics (MAT-75-03, 9 CP)	323
•	Matrix Analysis and Applications (MAT-65-37, 6 CP)	275
•	Morse Theory (MAT-55-28, 3 CP)	190
•	Non-Commutative Ergodic Theory (MAT-55-09, 9 CP)	164
•	Non-Linear Elliptic and Parabolic Partial Differential Equations (MAT-60-35, 6 CP)	261
•	Non-Linear Functional Analysis (MAT-55-02, 9 CP)	150
•	Non-Linear Optimisation (MAT-70-21, 9 CP)	302
•	Nonlinear Elliptic Partial Differential Equations in Minimal Surface Theory (MAT-55-24, 9 CP)	182
•	Null Geometry in General Relativity (MAT-60-08, 5 CP)	251
•	Number Theory and Cryptography (MAT-45-22, 9 CP)	. 70
•	Numerical Optimisation (MAT-70-25, 5 CP)	306
•	Numerics of Differential Equations of Surfaces (MAT-70-06, 6 CP)	291
•	Numerics of Instationary Differential Equations (MAT-70-03, 9 CP)	285
•	Numerics of Stationary Differential Equations (MAT-70-02, 9 CP)	283
•	Numerics of Stochastic Differential Equations (MAT-70-15, 3 CP)	297
•	Operator Algebras (MAT-55-04, 9 CP)	154
•	Operator Algebras and their Applications to Statistical Mechanics (MAT-55-71, 6 CP)	235
•	Operator Theory (MAT-55-03, 9 CP)	152
•	Optimal Control Theory with Ordinary Differential Equations (MAT-70-05, 5 CP)	289
•	Optimisation with Differential Equations (MAT-70-22, 9 CP)	304

	Ordinary Differential Equations - Analysis and Numerics (MAT-70-04, 9 CP)	287
	Partial Differential Equations (MAT-55-22, 9 CP)	180
	• Partial Differential Equations in Conformal Geometry: the Yamabe Problem (MAT-55-26, 3 CP)	186
	Percolation Theory (MAT-75-05, 3 CP)	327
	Point Processes (MAT-75-09, 6 CP)	335
	• Primes of the form \mathbf{x}^2+ny^2 and $ClassFieldTheory(MAT-45-30,3CP)$ 84 $ProbabilityDistances for DataSci_{75-20,6CP}$	
•	Probability Theory (MAT-75-01, 9 CP)	319
•	Propagation of Chaos (MAT-65-39, 9 CP)	279
•	Pseudo Differential Operators (MAT-55-10, 3 CP)	166
•	Quantum Information Theory (MAT-65-36, 9 CP)	273
•	Real Algebraic Geometry (MAT-45-19, 6 CP)	. 64
•	Representation Theory of Finite Groups (MAT-45-31, 6 CP)	. 86
•	Riemannian Geometry (MAT-50-16, 6 CP)	114
•	SL2(R) (MAT-55-52, 3 CP)	218
•	Selected Chapters from Dynamical Systems Theory (MAT-55-32, 3 CP)	192
•	Selected Chapters from Functional Analysis (MAT-55-70, 6 CP)	233
•	Selected Chapters from Operator Theory (MAT-55-15, 9 CP)	176
•	Space-Like Hypersurfaces in Lorentzian Manifolds (MAT-60-04, 6 CP)	243
•	Special Relativity (MAT-60-07, 3 CP)	249
•	Special Topics in Evolution Equations for Submanifolds (with Exercise Class) (MAT-60-10, 6 CP)	255
•	Special Topics in Evolution Equations for Submanifolds (without Exercise Classes) (MAT-60-11, 3 CP)	257
•	Spectral Theory of Positive Operators (MAT-55-08, 6 CP)	162
•	Statistical Learning Theory for Nonparametric Regression 1 (MAT-70-31, 9 CP)	310
•	Statistical Learning Theory for Nonparametric Regression 2 (MAT-70-32, 9 CP)	312
•	Stochastic Analysis (MAT-75-06, 9 CP)	329
•	Stochastic Differential Equations (MAT-70-11, 9 CP)	293
•	Stochastic Optimal Control in Infinite Dimensions (MAT-70-16, 3 CP)	299
•	Stochastic Processes (MAT-75-04, 9 CP)	325
•	The Einstein Constraint Equations (MAT-60-09, 6 CP)	253
•	The Ricci Flow of Riemannian Metrics (MAT-60-06, 6 CP)	247
•	Theoretical Aspects of Machine Learning (MAT-70-30, 6 CP)	308
•	Theory and Numerics for Constrained Optimisation Problems (MAT-70-33, 9 CP)	314
•	Theory of Mathematical Proofs (MAT-55-64, 6 CP)	229
•	Topics in Mathematical Relativity (MAT-60-03, 3 CP)	241
•	Topological Vector Spaces and Distributions (MAT-50-27, 6 CP)	136
•	Topology (MAT-50-20, 6 CP)	122
	Toric Varieties and Mori Dream Spaces (MAT-45-15, 9 CP)	58

Tropical Enumerative Geometry (MAT-50-04, 9 CP)	96
Tropical Enumerative Geometry - Part 2 (MAT-50-06, 5 CP)	100
Tropical Geometry (MAT-50-03, 9 CP)	94
Uniformisation of Riemann Surfaces (MAT-50-28, 5 CP)	138
Wave Equations of Relativistic Quantum Mechanics (MAT-65-33, 6 CP)	271

Module Number: MAT-45-01	Module Title: Introduction to Commutative ometry	e Algebra and Algebraic Ge-	Type of Module: Compulsory Module with Choice				
ECTS-Points	9						
Workload - Time in Class - Self-Study	Workload: 270 h						
Duration	1 Semester						
Frequency	regularly in Winter Semeste	r					
Term	1-3						
Language of Instruction	German						
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS						
Content	 Rings and ideals. Gröbner bases. Localization. Noetherian rings and modules. Integral ring extensions. Krull's principal ideal theorem and dimension theory. Hilbert's Nullstellensatz and Noether normalisation. Affine varieties, Zariski topology, morphisms. 						
Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.						

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Commutative Algebra and Algebraic Geometry	L E	f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									
Literature	Possible References :									
	 Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997. 						Springer Springer rie. Vie-			
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Commutative Algebra' due to the large overlap in content.									
Prerequisites	There are no further prerequis	sites.								
Responsible Persons	Jürgen Hausen									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio										

 $\label{eq:continuous} \begin{tabular}{ll} Teaching Format : L=lecture, & LE=lecture & with integrated exercises, & SL=seminar & or lecture, & E=exercise & class, & T=tutorial, & P=project, & S=seminar, & IC=inverted & classroom & or lecture, & E=exercise & class, & SL=seminar & or lecture, & E=exercise & class, & SL=seminar & or lecture, & E=exercise & class, & SL=seminar & or lecture, & E=exercise & class, & SL=seminar & or lecture, & E=exercise & class, & SL=seminar & or lecture, & SL=seminar & or$

Status : o=obligatory, f=facultative

Module Number: MAT-45-02	Module Title: Commutative Algebra							Type of Module: Compulsory Module with Choic			
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in Cl	lass:			Self-Si 180 h	tudy:			
Duration	1 Semester						<u>'</u>				
Frequency	regularly in Winter Semester										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS									
Content	 Rings and Ideals. 										
	 Localisation and local 	rings.									
	 Noetherian and Artinia 	an ring:	s and	d mo	dule	s.					
	 Integral ring extension 	s and	Cohe	en-S	eide	nberg t	heorems.				
	 Krull's principal ideal t 	heoren	n and	d din	nens	ion the	ory.				
	 Primary decomposition 	n.									
	 Normality, regularity a 	nd disc	crete	valu	atior	n rings.					
	Hilbert's Nullstellensa	tz and	Noet	ther	norm	nalisati	on.				
Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Commutative Algebra	L E	f	4	6	yes	wr. o.	90-180 o. 20-30	g	100	

Literature	Possible References :
	 Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969.
	 David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
	 David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995.
	 Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980.
	Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Commutative Algebra and Algebraic Geometry' due to the large overlap in content.
Prerequisites	There are no further prerequisites.
Responsible Persons	Victor Batyrev, Thomas Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-03	Module Title: Computer Algebra							of Module:	e with	Choice
ECTS-Points	9	9								
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 180 h									
Duration	1 Semester									
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS								
Content	Computation of impor Syzygies, free resolut	 Canonical forms and standard bases for ideals and modules. Computation of important operations for ideals and modules. Syzygies, free resolutions and the proof of the Buchberger criterion. Calculation of the primary decomposition of ideals. Hilbert functions. 								
Objectives	algebraic geometry as well a familiar with the theory of sta with important software pack algorithms in these. The stu the lecture as well as assess In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	Students are familiar with important problems in the interplay of commutative algebra and algebraic geometry as well as algorithmic approaches to solving them. In particular, they are familiar with the theory of standard bases and their diverse applications. They are also familiar with important software packages in the field of symbolic computing and have implemented algorithms in these. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse. They also learnt about important software packages in the field of symbolic computing and								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Computer Algebra	L E	f f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	ave	bee	n acc	uired.	Whether th	e examination		

Literature	Possible References :
	 Gert-Martin Greuel, Gerhard Pfister: A SINGULAR Introduction to Commutative Algebra. Springer 2008.
	 Wolfram Decker, Christoph Lossen: Computing in algebraic geometry. A quick start using SINGULAR. Springer 2006.
	 Wolfram Decker, Gerhard Pfister: A first Course in computational algebraic geometry. Cambridge University Press 2013.
	David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Commutative Algebra and Algebraic Geometry are helpful however not absolutely necessary for participation in the module Computer Algebra
Responsible Persons	Hannah Markwig, Thomas Markwig
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-11	Module Title: Type of Module Algebraic Geometry Compulsory Mo								le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 180 h									
Duration	1 Semester	1 Semester								
Frequency	regularly in Summer Semes	regularly in Summer Semester								
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS								
Content	Prevarieties and varieties.	Prevarieties and varieties.								
	Projektive varieties as	nd hom	oge	neou	s spe	ectrum				
	Finite and proper more	Finite and proper morphisms.								
	Blow-up and Grassm	Blow-up and Grassmannians.								
	Rational maps.									
	Divisors and line bund	dles, cl	ass (grou	o and	d Picar	d group.			
Objectives	they develop a deeper under The students are capable of assessing and explaining the In the exercise classes they the terms, statements and m on new problems, to analyse	The students learn central terms, results and methods of modern Algebraic Geometry and they develop a deeper understanding of the interconnections between Geometry and Algebra. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	examination the coursework	n this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.								

Literature	Possible References :
	Robin Hartshorne: Algebraic geometry. Springer 2006.
	Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012.
	Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997.
	David Mumford: The red book of varieties and schemes. Springer 1999.
	Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988.
	Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry and Toric Varieties' due to the large overlap in content.
Prerequisites	Essential knowledge from the module Commutative Algebra is assumed.
Responsible Persons	Victor Batyrev, Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-12	Module Title: Algebraic Geometry and Toric Varieties Type of Module: Compulsory Module with Cho							Choice			
ECTS-Points	9							,	<u> </u>		
Workload - Time in Class - Self-Study	Workload: 270 h	,									
Duration	1 Semester	1 Semester									
Frequency	regularly in Summer Semes	regularly in Summer Semester									
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	6							
Content	Projective space.	Projective space.									
	Prevarieties, morphis	ms, tan	ngent	spa	ce a	nd sing	ularities.				
	Products and separate	ion.									
	Projective varieties as	Projective varieties and Grassmannians.									
	Divisors and line bund	Divisors and line bundles, class group and Picard group.									
	Toric varieties.										
Objectives	Students learn the central country develop an advanced upon the commerce of th	ndersta lass of nvestig Algebra ing the nnection have a nethods them a	Indination ation ation a and e essens. Indicate the according at the according according to the according according to the according according to the according according according to the according to the according according to the according according to the	g of to various of a discontinuity of a discontinuity of the less	the retrietie in im omet al reservation corrections the reservation in the retrievation in the retrievatio	elations s, they portan ry by a sults of nfident, e. They solution	ships between also learn t example confurther confurther lecture precise and have learn on strategies	en Geometr how methodelass of Algelenponent. The as well as and independent ed to transfess on their owner in critical and independent ed to transfess on their owner in critical ed.	y and / ods of braic v e stude assess ent har er the r	Algebra. Convex arieties, ents are ing and adling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Algebraic Geometry and Toric Varieties	L E	f f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	examination the coursework	n this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature	Possible References :
	 David A. Cox, John B. Little, Henry K. Schenck: Toric varieties. American Mathematical Society 2011:
	Robin Hartshorne: Algebraic geometry. Springer 2006.
	Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012.
	Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997.
	David Mumford: The red book of varieties and schemes. Springer 1999.
	Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988.
	Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry' due to the large overlap in content.
Prerequisites	Essential knowledge from the module Introduction to Commutative Algebra and Algebraic Geometry is assumed.
Responsible Persons	Jürgen Hausen

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-13	Module Title: Algebraic Transformation Groups Type of Module: Compulsory Module with Choice									Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS								
Content	Elements of the struct Elements of the representation Quotients in algebraich Classical invariant the Geometrical invariant tients. Additionally certain as — Toric varieties; — Spheric varieties	Additionally certain aspects of topics from the following list are covered:								
Objectives	tures. At the same time, the example from group and ringing and proving the essention presented connections. In the exercise classes they the terms, statements and mon new problems, to analyse	The students learn basic methods for mathematical work with symmetries on geometric structures. At the same time, they experience the interaction of different algebraic concepts, for example from group and ring theory, in algebraic geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Transformation Groups	L E	f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.								

Literature	Possible References :
	Armand Borel: Linear algebraic groups. Springer 1991.
	Jean A. Dieudonne, James B. Carrell: Invariant theory. Academic Press 1971.
	David Mumford: Geometric invariant theory. Springer 1965.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Groups' due to the large overlap in content.
Prerequisites	Knowledge of the Commutative Algebra and Algebraic Geometry modules is helpful, but not a prerequisite for participation in the Algebraic Transformation Groups module.
Responsible Persons	Victor Batyrev, Jürgen Hausen

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom$

Status : o=obligatory, f=facultative

Module Number: MAT-45-14	Module Title: Algebraic Curves							of Module: ulsory Modu	le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h									
Duration	1 Semester	1 Semester								
Frequency	not regularly									
Term	1-3	1-3								
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise CI	ecture 4 SWS + Exercise Class 2 SWS								
Content	 Ramified coverings, The Linear systems, emberon Singularities of plane of the content of the	 Projective curves, divisors, Theorem of Riemann-Roch. Ramified coverings, Theorem of Hurwitz. Linear systems, embeddings, Castelnuovo inequality. Singularities of plane curves, Puiseux expansions. Classification and moduli spaces, Jacobi variety. 								
Objectives	selected sub-area of algebra oped an in-depth understand capable of naming and provi explaining the presented con In the exercise classes they I the terms, statements and me	Students have familiarised themselves with the central concepts, results and methods in a selected sub-area of algebraic geometry. familiarised themselves with it. They have developed an in-depth understanding of algebraic curves and their classification. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Curves	E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruct	must l	nave	beer	n acc	uired.	Whether th	e examination		
Literature	Gerd Fischer: Ebene a	Possible References: Robin Hartshorne: Algebraic geometry. Springer 2006. Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994. Rick Miranda: Algebraic Curves and Riemann Surfaces. AMS 1995.								

Transfer	The module belongs to the Study Specialisation Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Essential knowledge from the module Commutative Algebra as well as basic knowledge from Algebraic Geometry and Complex Analysis is required.
Responsible Persons	Victor Batyrev, Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-45-15	Module Title: Toric Varieties and Mori Dream Spaces Type of Module: Compulsory Module with Choice									Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	S						
Objectives	Geometry and combine Divisors on toric varie Quotient representation Sheaves of divisorial Cox sheaves and chate Quotients of H-factorial Shaded rings. Varieties with torus of the shaded rings	In the lecture Mori Dream Spaces are considered as generalisations of toric varieties: Geometry and combinatorial theory for toric varieties and Mori Dream Spaces. Divisors on toric varieties and Mori Dream Spaces. Quotient representation and Cox ring for toric varieties and Mori Dream Spaces. Sheaves of divisorial algebras. Cox sheaves and characteristic space. Quotients of H-factorial affine varieties. Shaded rings. Varieties with torus operations. Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra. With the class of Mori dream spaces, they have become familiar with a generalisation of toric varieties and their investigation using methods of convex geometry. In doing so, they have added another important methodological component to the interplay between algebra and geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.								
	on new problems, to analyse They are able to present the	them a	and t	o wo	rk on	solutio	on strategie:	s on their ow nem in critica	n or in	a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Toric Varieties and Mori Dream Spaces	L E	f	4 2	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	bee	n acc	uired.	Whether th	e examination		

Literature	Possible References :
	 Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface. Cox rings. Cambridge University Press 2014.
	Yi Hu, Sean Keel. Mori dream spaces and GIT. Michigan Math. J. 48: 331-348, 2000.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Essential knowledge from the modules Introduction to Commutative Algebra and Algebraic Geometry as well as Algebraic Geometry and Toric Varieties is assumed.
Responsible Persons	Jürgen Hausen

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-16	Module Title: Algebraic Groups		Type of Module: Compulsory Module with Choice							
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h							
Duration	1 Semester	1 Semester								
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS								
Content	Definition and examp	Definition and examples of algebraic groups.								
	Hopf algebras.									
	Operations of algebra	nic groups on varieties.								
	Linearisation of algeb	oraic groups.								
	Group closure.									
	Resolvable and nilport	tent groups.								
	The Lie algebra of an	algebraic group.								
	Examples of Lie alge	bras.								
	Convolutions and cor	nmutators.								
	The adjoint represent	ation and its differential.								
	The Jordan decompo	sition in affine algebraic group	S.							
	Characters of an alge	ebraic group.								
	Semi-invariants of a r									
		uction of quotients with applica	ations.							
	Diagonalisable group									
	Rigidity of diagonalisa									
	Theorem of Lie-Kolch									
	Structure of affine res									
		imple elements of algebraic gr	oups.							
	Borel subgroups and	•								
	Structure and classifi	cation of semisimple algebraic	groups.							

Objectives	Students have learnt about a large class of important groups and algebraic varieties that play an essential role in many mathematical fields. They have learnt how methods of group theory and algebraic geometry complement each other and can lead to a deeper understanding. They have learnt about the approach to classifying mathematical objects using an important example class and have acquired knowledge of methods that also play a key role in classification in completely different mathematical areas. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Groups	L	f	4	6	yes	wr. o.	90-180		100
	/ ligobialo Groups	Е	f	2	3	yes	or.	o. 20-30	g	100
	In this module an exercise cerexamination the coursework noral is decided by the instructed	nust ł	nave	beer	n acc	uired.	Whether th	e examination		
Literature	Possible References: James E. Humphreys: 1981. Armand Borel: Linear a			_		·	, -	1975. 21, S	pringe	r-Verlag
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Transformation Groups' due to the large overlap in content.									
Prerequisites	Knowledge of the modules Co a prerequisite for participation							ometry are I	nelpful	, but not
Responsible Persons	Victor Batyrev, Jürgen Hauser	Victor Batyrev, Jürgen Hausen								
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio										

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

: o=obligatory, f=facultative Status

Module Number: MAT-45-18	Module Title: Cox Rings		Type of Module: Compulsory Module with Choice							
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h							
Duration	1 Semester	1 Semester								
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	English	English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS								
Content	Divisorial algebras.									
	Cox rings.									
	Charakteristic spaces	i.								
	Good quotients.									
	Geometric invariant the Gold display.	neory.								
	Gale-duality.Connections to toric g	noomatry								
		peometry. eties with finitely generated Co	ny ring							
	Singularities.	siles with limitery generated oc	ing.							
	Picard group.									
	Basis locus.									
	Ampleness.									
	Kanonical class.									
	Intrinsic quadrics.									
	• k*-surfaces.									
	Varieties with torus ac	ction.								
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra and combinatorics. They have familiarised themselves with the Cox ring as an algebraic object for investigating special classes of geometric spaces. In doing so, they expand the interplay between algebra and geometry with another important methodological component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									

Requirements for Obtaining Credit, Grading, Weight if applicable	Title Cox Rings In this module an exercise cerexamination the coursework roral is decided by the instruction.	nust ł	nave	beer	n acc	uired.	Whether the	e examinatio		
Literature	2014.	Possible References: • Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, Antonio Laface: Cox Rings. CUP								
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge of commutative algebra and algebraic geometry to the extent of the module Introduction to commutative algebra and algebraic geometry is assumed.									
Responsible Persons	Jürgen Hausen									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-19	Module Title: Real Algebraic Geometry							of Module: ulsory Modul	le with	Choice	
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h					Self-St 120 h	Self-Study: 120 h			
Duration	1 Semester	1 Semester									
Frequency	not regularly	not regularly									
Term	1-3	1-3									
Language of Instruction	English										
Forms of Teaching and Learning	Lecture 3 SWS	Lecture 3 SWS									
Content	varieties. This involves quest of topological types for real a	This course aims to dive into different aspects of the study of the topology of real algebraic arieties. This involves questions related to the 16th Hilbert problem: we look at obstructions of topological types for real algebraic varieties and at the realisation of topological types via different construction techniques, with special emphasis to low dimensional cases.									
Objectives	Students learn about some fundamental differences in algebraic geometry over the complex and real numbers. They are familiar with the application of topological and algebraic methods to the study of real algebraic varieties. They have learnt how modern methods can be used to investigate and answer unsolved scientific questions of the late 19th century. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Real Algebraic Geometry	L	f	3	4,5 1,5	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acq	juired.	Whether th	e examination			
Literature	Possible References :										
	Frederice Mangolte: F	Real A	gebr	aic '	Variet	ies. Sp	oringer 2020).			
	Robert Silhol: Real Al	gebrai	c Su	rfac	es. Sp	oringer	1989.				
	Riccardo Benedetti, Editions Herrmann 19		acqu	es	Risler	: Rea	l Algebraic	and Semi-a	ılgebra	ic Sets.	
	Alex Degtyarev, Viatc eties: du côté de chez							rties of real	algebr	aic vari-	

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.	
Prerequisites	Basic knowledge in algebraic geometry or algebraic topology is helpful, but not mandatory.	
Responsible Persons	Hannah Markwig	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-20	Module Title: Introduction to Berkovich Geometry							Type of Module: Compulsory Module with Choice			
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass			Self-St 120 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	English	English									
Forms of Teaching and Learning	Lecture 3 SWS	Lecture 3 SWS									
Content	 Non-Archimedean fields, valuations, and absolute value functions. Ultrametric triangle inequality and induced topology. Affinoid domains. Berkovich affine and projective line. Analytification of algebraic varieties. 										
Objectives	fields and their induced top challenges in developing a Berkovich's approach to add line in Berkovich's frameworthey have encountered a typ encountered in their studies familiar with the connections students are capable of nail assessing and explaining the lin the exercise classes they the terms, statements and more to new problems, to analys	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Berkovich Geometry	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or (oral	s de	cided I	by the instru	uctor with a	pprova	I by the	
Literature	Possible References :										
	Annette Werner: Nich	ıtarchiı	medi	sche	Geo	metrie	. Vorlesung	sskript.			

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of topological concepts is assumed.
Responsible Persons	Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-21	Module Title: Algebraic Number Theory Type of Module: Compulsory Module with Choice									Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h									
Duration	1 Semester	Semester								
Frequency	not regularly									
Term	1-3									
Language of Instruction	German	German								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS								
Content		 Class numbers. Dirichlet's unit theorem. Extension of Dedekind rings. Valuation theory. Local fields. 								
Objectives	The students have learned the The students are capable of assessing and explaining the In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	namin prese have a ethoda thema	g and entectacques of t and t	d pro I con ired a he le o wo	oving necti a cor ecture rk or	the essions. Infident, They I solution	precise an have learn on strategie	Its of the lec d independe ed to transfe s on their ow	ture as ent har er the r n or in	well as adding of nethods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Number Theory	L E	f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	beer	n acc	uired.	Whether th	e examination		
Literature	Possible References: • Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007. • Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007. • Andre Weil: Basic number theory. Springer 1995.									

Transfer The module belongs to the Study Specialisation Algebra and Geometry. Taking the chosen personal Study Specialisation, it can be included in the Sections Advanced Knowledge in Mathematics or Elective Specialisation, in accordance strictive requirements of the respective section.						
	Prerequisites	There are no further prerequisites.				
	Responsible Persons	Victor Batyrev, Anton Deitmar, Jürgen Hausen				

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-22	Module Title: Number Theory and Cryptography Type of Module: Compulsory Module with Choice									Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS									
Content	RSA cryptosystem, primality tests, AKS algorithm.									
	Factorisation methods	s, numl	oer f	ield s	sieve					
	Quadratic reciprocity	in cryp	togra	aphy.						
	Evaluation of the disc	rete lo	garitl	hm.						
	Dynamical systems a	nd Poll	ard's	s rho	algo	rithm.				
	Elliptic curve cryptogr	aphy.								
	 Lattices and post-qua 	ntum c	rypt	ogra	phy.					
	Zero-knowledge proof	fs, digit	tal si	gnat	ures	and ha	sh function	S.		
Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	eCTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Number Theory and Cryptography	E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.								

Literature	Possible References :
	 Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008.
	Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011.
	Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992.
	 Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/).
	Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Elliptic Curves and Cryptography' due to the large overlap in content.
Prerequisites	The contents of the module Algebra from the study program Bachelor of Science are presumed.
Responsible Persons	Elena Klimenko, Thomas Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-24	Module Title: Elliptic Functions and Elliptic Curves							Type of Module: Compulsory Module with Choice			
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-St 180 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS										
Content	Elliptic functions, Weierstrass-P-function, Riemann surfaces, complex tori.										
	Plane projective curves, Theorem of Bezout, elliptic curves.										
	Curves over finite fields, rational points.Applications in cryptography.										
	Additionally a selection of the following: Additionally a selection of the following:										
	– Modular forms;– Classification of elliptic curves;										
	 Moduli spaces. 										
Objectives	The students have expanded their competence in mathematical-interdisciplinary studies. They have learnt about elliptic curves as a class of mathematical objects, which has comprehensive relevance in a wide spectrum of mathematical areas. The students have studied the notions, methods and results from the disciplines Complex Calculus, Algebraic Geometry, Number Theory, Topology and Cryptography, which are relevant in the given context, and they understand their mutual interrelations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Elliptic Functions and Elliptic	Type of Course	→ Status	SMS 4	e ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Curves	E	f	2	3	yes	or.	o. 20-30	g	100	
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										

Literature	Possible References :
	Wolfgang Fischer, Ingo Lieb: Funktionentheorie. Vieweg 2005.
	Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994.
	Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009.
	Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the lecutre Introduction to Complex Analysis is needed.
Responsible Persons	Jörg Zintl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-25	Module Title: Elementary Number Theory							of Module:	e with	Choice
ECTS-Points	6						<u> </u>			
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass:			Self-St 120 h	udy:		
Duration	1 Semester						'			
Frequency	not regularly									
Term	1-3									
Language of Instruction	German	German								
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 2 SWS								
Content	Divisibility in the integers.									
	Prime numbers.									
	Congruences.									
	 Quadratic residues. 									
	Arithmetic functions.									
	Multiplicative function:	3.								
	Classical Theorems.									
	Applications.									
Objectives	Students deepen their basic mathematical problems of varies essential results of the lectur Students will be able to reflect area. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	rious A e as we t and o have a ethods them a	kinds ell as critica acqui s of t and t	i. The assally a direct a line	e studes essir nalys a cor cture rk on	dents and and se the confident, e. They a solution	are capable explaining to current state precise and have learned attacks.	of naming a he presented of research d independed ed to transfes on their ow	nd prod conn in the ent har er the r	ving the ections. subject adling of nethods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Elementary Number Theory	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must ł	nave	beer	n acc	uired.	Whether the	e examinatio		

Literature	Possible References :
	Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001.
	Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, only basic knowledge of groups and rings from linear algebra is required.
Responsible Persons	Victor Batyrev, Thomas Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-26	Module Title: Introduction to Analytic Numb	er Th	eory	,				of Module: ulsory Modul	le with	Choice
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h					Self-St 120 h	Self-Study: 120 h		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3	1-3								
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS								
Content	Prime number theorer	 Arithmetic functions and Dirichlet series, Prime number theorem and Dirichlet's prime number theorem, Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula. 								
Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Analytic Number Theory	L	f	2	3	no	wr. o. or.	o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	orai	is at	eciaea	by the instr	uctor with a	pprova	ar by the
Literature	Possible References : • Komaravolu Chandras 1968.	sekha	ran:	Inti	oduc	tion to	Analytic N	umber The	ory.	Springer
Transfer	Differential Geometry. Taking be included in the Sections	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.								
Prerequisites	There are no further prerequi	sites								
Responsible Persons	Anton Deitmar									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-27	Module Title:Type of Module:Elliptic Curves and CryptographyCompulsory Module with Choice									Choice
ECTS-Points	9						·			
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	•		Self-St 180 h	udy:		
Duration	1 Semester						·			
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	3						
Content	Basic concepts of cry	ptograp	ohy.							
	Symmetric cryptosyst	ems, p	ublic	key	syst	ems, d	iscrete loga	rithm, RSA.		
	Factorisation into prin	nes, att	acks	on	crypt	osyste	ms.			
	Basic concepts of pla	ne proj	ectiv	e ge	ome	try.				
	Elliptic curves as Abe	_								
	Curves over finite field						•	· ·		
	Counting points, Hass				-			lgorithm.		
	Cryptosystems on elli	ptic cui	rves,	, aigo	oritnr	ns and	attacks.			
Objectives	Students are familiar with the cryptographically motivated advanced algebraic and geo lenges of algorithmic implem are capable of naming and and explaining the presented in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	questice metric to the metric	ons rateching the ection are ecti	relation nique essens. Ired a he le o wo	ng to es for e fan entia a cor cture rk on	elliption r answer niliar word resulting fident, e. They r solution	c curves an ering them. eith standard is of the lector precise an have learn on strategies.	d have an i They unders I algorithms. Sture as well d independe ed to transfe s on their ow	nsight stand to the stand to th	into the he chal- students esessing adling of methods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Elliptic Curves and	Type of Course	J Status	SMS 4	о ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Cryptography	E	f	2	3	yes	or.	o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	ave	beer	n acc	uired.	Whether th	e examination		

Literature	Possible References :
	 Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015.
	Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009.
	Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the modules 'Number Theory and Cryptography' due to the large overlap in content
Prerequisites	There are no further requirements.
Responsible Persons	Jörg Zintl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom \\$

Status : o=obligatory, f=facultative

Module Number: MAT-45-28	Module Title:Type of Module:Elliptic Curves and Taniyama-ShimuraCompulsory Module with Choice								Choice	
ECTS-Points	9	9								
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 180 h									
Duration	1 Semester						·			
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS								
Objectives	 Group-law, arithmetic Modular curves and for Riemann surfaces, at Geometric version of plained. 	Connection to Fermat's last theorem.								
Objectives	and geometry to answer prof of Taniyama-Shimura and its capable of naming and prov explaining the presented cor current state of research in t In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	ound resident application of the subsection of the subsection of the mount of the	mathe catio e essens. So ject acqu s of t and t	ematen to sential stude area ired he le o wo	ical of the pal resents v ents v a corecture rk or	question of or	ns using the Fermat's to the lecture able to refle precise are have learn on strategie	e example of heorem. The as well as ct and critical ad independent and transfers on their ow	the co e stude assess ally and ent har er the r	njecture ents are sing and alyse the adling of methods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Elliptic Curves and Taniyama-Shimura	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must	have	bee	n acc	quired.	Whether th	ie examinati		
Literature	Possible References : • Joseph H. Silverman:	The A	Arithn	netic	of E	lliptic C	Surves. Spr	inger 2009.		

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Introduction to Riemann surfaces and Algebraic number theory are assumed.
Responsible Persons	Ivo Radloff

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-45-29	Module Title: Introduction to Modular Forms							of Module: ulsory Modul	le with	Choice
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	_	lass	:		Self-St 120 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS								
Content	Gauss, Eisenstein and Ram have many surprising applic four square theorem and the course aims to give an introd • Modular forms for the • Examples: Eisenstein • Arithmetic application	The study of modular forms traces back its roots to the late 19th and early 20th century with Gauss, Eisenstein and Ramanujan, and is a fascinating blend of analysis and algebra. They have many surprising applications to number theory, including a beautiful proof of Lagrange's four square theorem and the ground-breaking proof of Fermat's last theorem in 1995. This course aims to give an introductory understanding of this broad topic. • Modular forms for the Modular group and congruence subgroups. • Examples: Eisenstein series, the Ramanujan Delta function, Theta series. • Arithmetic applications and conjectures. • Hecke operators and eigenforms.								
Objectives	Students have learnt the bas lar forms. They are familiar v The students are capable o as assessing and explaining critically analyse the current	ith an nami the p	alytic ng a rese	al, a nd p nted	lgebr rovin conr	raic and g the e nection	d geometric essential res s. Students	aspects of r sults of the l	nodula lecture	r forms. as well
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Modular Forms	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instru	uctor with a	pprova	l by the

Literature	Possible References :
	 Henri Cohen, Fredrik Stromberg: Modular forms. A classical approach. AMS Graduate Studies of Mathematics 2017.
	Fred Diamond, Jerry Shurman: A first course in modular forms. Springer 2005.
	Max Koecher, Aloys Krieg: Elliptische Funktionen und Modulformen. Springer 2007.
	Toshitsune Miyake: Modular forms. Springer 1989.
	 Lloyd James Peter Kilford: Modular forms: A classical and computational introduction. Imperial College Press 2015.
	Deitmar Anton: Automorphic forms. Springer 2013.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Modular Forms' due to the large overlap
	in content.
Prerequisites	There are no further prerequisites, but basic knowledge of algebra and function theory is helpful.
Responsible Persons	Anton Deitmar
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-30	Module Title:Type of Module:Primes of the form $x^2 + ny^2 and Class Field Theory$ Compulsory Module with Choice							Choice		
ECTS-Points	3	3								
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	:		Self-St 120 h	udy:		
Duration	1 Semester									
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	ory can be described as her oretic structure of of a numb integers, through a deep un the central notions and conc tary motivating problem of welementary number theory to	Just as Gauss once described number theory as the queen of mathematics, Class Field Theory can be described as her Crown Jewel. Class field theory brings together the Galois theoretic structure of of a number field, and links it to the arithmetic structure of its the ring of integers, through a deep understanding of reciprocity laws. This course aims to introduce the central notions and concepts of Class Field theory naturally through solving the elementary motivating problem of what primes can be expressed as the form $x^2 + ny^2$, and bridge elementary number theory to modern number theory. • The theory of Quadratic forms and Genus theory.								
	Hilbert Class field.	 Generalisations of the law of quadratic reciprocity. Arithmetic in number fields and the Hilbert Class field. A statement of Artin Reciprocity, and a solution to the problem of describing primes of the form 								
Objectives	Students have learned the form of class field theory in the consumers on understanding of where the students are capable of narrassessing and explaining the critically analyse the current	ase onese ones	f qua conce nd p sente	adrat epts rovir ed co	ic fie come ng the onne	eld exte e from e esse ctions.	ensions of C through an ntial results Students v	Q, and have elementary of the lect	a very examp ure as	hands le. The well as
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Primes of the form $x^2 + ny^2 and Class Field Theory$	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	cided I	by the instru	uctor with a	pprova	l by the
Literature	Possible References : • David Cox: Prim Classfieldtheory.On • Jürgen Neukirch: Algebraic r	lineN	otes	2020			+ <i>ny</i> ² . <i>Wil</i>	ey 2013, Jar	nesMi	lne :

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	An understanding of Galois theory and other basic algebraic structures (groups and rings) is essential. Deep results taught in any first course in algebraic number theory will be clearly stated and introduced, but not proven, and will be treated as a black box.
Responsible Persons	Anton Deitmar

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-31	Module Title: Representation Theory of Fi	nite Gr	oups	6				of Module: ulsory Modul	le with	Choice		
ECTS-Points	6											
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:				
Duration	1 Semester						•					
Frequency	not regularly											
Term	1-3											
Language of Instruction	German	German										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS											
Content	 Groups and group actions. Representations, irreducibility, Schursch's lemma. Semisimplicity, Maschke's theorem. Characters, orthogonality relations. Isotypical decomposition, character tables. Representations of the symmetric group. Semi-simple Artinian algebras. 											
Objectives	In the lecture, students learn understanding for the interact are capable of naming and and explaining the presenter In the exercise classes they the terms, statements and non new problems, to analyse They are able to present the	ction of proving d conne have a nethods them a	georgeonection geometric g	metri e ess ns. ired a he le o wo	c and entia a cor cture rk on	d alget I result ofident, e. They or solution	oraic method ts of the led precise and have learn on strategies	ds. methods sture as well d independe ed to transfe s on their ow	. The solution . The sent har er the roll or in	students sessing adling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Representation Theory of Finite Groups	ш г Туре of Course	t Status	SMS 2	s ECTS	Coursework	Type of Exam or. o.	On: of Exam (min) 90-180 o. 20-30	a Grading	Weight for Grade		
	In this module an exercise c examination the coursework oral is decided by the instruc	must h	nave	beer	n acc	uired.	Whether the	e examination				

Literature	Possible References :								
	William Fulton, Joe Harris: Representation theory. Springer 1991.								
	Bertram Huppert: Character theory of finite groups. De Gruyter 1998.								
	Serge Lang: Algebra. Springer 2002.								
	Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.								
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. It can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Group Representations in Physics' due to the large overlap in content.								
Prerequisites	In terms of content, only basic knowledge of linear algebra is required.								
Responsible Persons	Victor Batyrev, Jürgen Hausen, Milena Wrobel								
Abbreviations: Grading System : g	=graded, ng=not graded								

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title:		Type of Module:									
MAT-45-40	Introduction to Combinatoria	al Birational Geometry	Compulsory Module with Choice									
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h									
Duration	1 Semester	<u> </u>										
Frequency	not regularly											
Term	1-3	-3										
Language of Instruction	German or English	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS										
Content	 map for surfaces <i>X</i> or divisors. The divisor of the divisor of the variety. Ample and veriety. The cone of curves or formations. Desingularization of racombinatorial construction. Cyclic quotient surfaction. Finite subgroups of <i>S</i> larization. Birational classification the Fine interior <i>F</i>(Δ) The Kodaira dimension els of nondegenerate 	over the complex numbers. The class group $Cl(X)$ and the bill bundles, invertible sheaves. But ample divisors. For aic curves in toric surfaces, we say a blow ups and blow down of a surface. The Zariski decorporate curves D on sucting minimal models of pair the singularities and their combination of nondegenerate surface. On of nondegenerate surface of their Newton polytopes Δ on of algebraic varieties. Compare C and of algebraic varieties.	mposition. Birational Cremona transmooth toric surfaces X via blow ups. $s(X,D)$ for normal toric surfaces X . Dinatorial minimal desingularization. In the gularities and their minimal desingularities and t									
Objectives	in order to analyse importate calculate complex algebro-gand deep classification problems and provexplaining the presented color in the exercise classes they the terms, statements and mon new problems, to analyse	ant classes of algebraic surfa- leometric constructions. The lem, the minimal models for a ring the essential results of ti- nections. have acquired a confident, parthods of the lecture. They he them and to work on solution	Its and methods of convex geometry aces. They learn to recognise and y are familiarised with an interesting algebraic surfaces. The students are he lecture as well as assessing and precise and independent handling of have learned to transfer the methods a strategies on their own or in a team.									

Requirements for Obtaining Credit, Grading, Weight if applicable	Title Introduction to Combinatorial Birational	П Type of Course	b Status	SMS 4	9 ECTS	Sework	Type of Exam o. o.	Dur. of Exam (min) 081-06	ص Grading	Weight for Grade
	Geometry E f 2 3 Or. 0. 20-30 In this module an exercise certificate is to be acquired as coursework. For participation examination the coursework must have been acquired. Whether the examination is written oral is decided by the instructor with approval by the Board of Examiners.									
Literature	 Possible References: Laurent Buse, Fabrizio History of Shapes. Spr Klaus Hulek: Elementa Tadao Oda: Convex Bo Toric Varieties. Springe Robin Hartshorne: Algo 	inger re Alq odies er 198	2023 gebra and 38.	3. aisch Alge	ne Ge braic	eometri Geom	e. Springer netry: An Int	2012.		
Transfer	The module belongs to the Stathe chosen personal Study State Advanced Knowledge in Matter strictive requirements of the results.	pecia nema	disat tics (ion, or <i>El</i>	it ca <i>lectiv</i>	n be ir	cluded in the	ne Sections	Study	Focus,
Prerequisites	Knowledge of commutative alg									e essen-
Responsible Persons	Victor Batyrev									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-45-41	Module Title: Introduction to Combinatoria	l Mirror Symmetry	Type of Module: Compulsory Module with Choice								
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h								
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS										
Content	Quintic 3-folds in proje	Quintic 3-folds in projective 4-space and their mirrors.									
		 Toric varieties associated with fans of rational polyhedral cones. Toric varieties associated with lattice polyhedra. Smoothness. 									
	Resolution of singular	rities. Cohomology rings of sm	nooth projective toric varieties.								
	 Construction of Calaborellexive polyhedra. 	 Construction of Calabi-Yau varieties as hypersurfaces in toric varieties associated with reflexive polyhedra. 									
	 A combinatorial form correspondence. 	ula for Hodge numbers of Ca	labi-Yau 3-folds. Monomial-divisor								
	Combinatorial mirror of Calabi-Yau varieties.	construction for Calabi-Yau cor	nplete intersections. Mirrors of rigid								
	Computation of period functions.	ds of Calabi-Yau hypersurfaces	s using generalized hypergeometric								
	Stringy Hodge number	ers of singular Calabi-Yau varie	eties.								
	 Moduli spaces. Bour secondary polytopes. 		s of Calabi-Yau hypersurfaces and								
	Computation of Grom	ov-Witten invariants of Calabi	-Yau complete intersections.								
	A combinatorial appro	oach to Berglund-Hübsch mirro	or symmetry.								
	based on polar duality in the the most famous examples of dodecahedron. In combinate considered reflexive polyhedra Δ^* below lattice of characters of an a one-parameter subgroups in is the theory of toric varieties symmetry discovered by physic N and from Δ to Δ^* . The aim of the module is to	e class of reflexive lattice poly of polar dual pairs of polyhedr orial mirror symmetry, an esse dra Δ are elements of the latt ng to the dual lattice N . The latting to the dual lattice N . The latting to the dual lattice N . The latting to the dual T . For this reason, the main is. Combinatorial mirror symmetricists for 3-dimensional Calater explain the connection between derstandable way possible at	atorial approach to mirror symmetry topes. The Platonic solids provide a: Cube-octahedron, icosahedronential fact is that the vertices of the ice M and the vertices of the dual attice M can be identified with the all lattice N becomes the lattice of tool of the combinatorial approachetry allows us to interpret the mirror abi-Yau manifolds by going from M ten reflexive polyhedra and Calabinat to inform students about further								

Objectives	Students are familiar with the complex issues of mirror symmetry, which establishes a duality between manifolds of symplectic and algebraic geometry and was first postulated by physicists. They have learnt how methods of toric geometry and discrete mathematics can be used for very important classes of Calabi-Yau varieties in order to calculate the mirrors of the manifolds and their invariants in concrete terms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Introduction to	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Combinatorial Mirror Symmetry	L E	f f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise cer examination the coursework noral is decided by the instructor	nust ł	nave	beer	n acc	uired.	Whether the	e examinatio		
Literature	Possible References :									
	 Victor Batyrev: Dual Polyhedra and Mirror Symmetry for Calabi-Yau Hypersurfaces in Toric Varieties. J. Alg. Geom. 3 (1994), no. 3, 493–535. 									
	 Victor Batyrev, Duco v curves on Calabi-Yau 168:3 (1995), 493–533 	comp								
	Victor Batyrev and Lev Calabi-Yau manifolds. Soc., Providence, RI (1	Mirro	r Sy	mme						
	David Cox, Sheldon K Surveys and Monograp						d Algebraic	Geometry.	Mathe	ematical
	Israil Gelfand, Mikhail k tidimensional Determin							nants, Resul	tants a	nd Mul-
	Masao Jinzenji: Class Band 29, 2018.	ical N	1irror	Syn	nmet	ry. Sp	ringerBriefs	in Mathem	atical F	Physics,
Transfer	The module belongs to the State chosen personal Study State Advanced Knowledge in Matastrictive requirements of the re	pecia hema	alisat <i>tics</i> (ion, or <i>El</i>	it ca <i>ectiv</i>	n be ir	icluded in th	ne Sections	Study	Focus,
Prerequisites	Knowledge of the modules Co	mmu	tativ	e Alg	ebra	and A	lgebraic Ge	ometry are	assum	ed.
Responsible Persons	Victor Batyrev									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-02	Module Title: Convex Geometry							of Module: ulsory Modu	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		lass	s:		Self-Si 180 h	tudy:			
Duration	1 Semester						·				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS										
Content	Cones, polytopes, polyhedra, fans, polyedral complexes.										
	Normal fans of polygons.										
	Triangulations, subdivisions, secondary fans, discriminants.										
Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Besides they enhance their geometric view and their spatial sense. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Convex Geometry	L	f	4	6	yes	wr. o.	90-180	0	100	
	Convex decinetry	Е	f	2	3	yes	or.	o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	have	bee	n acc	uired.	Whether th	e examinatio			
Literature	Possible References :										
	Günter M. Ziegler: Le	ctures	on F	Polyt	opes	. Sprin	ger 1998.				
Transfer	The module belongs to the state the chosen personal Study Advanced Knowledge in Mastrictive requirements of the	Specia Special sthema	alisat atics	ion, or <i>E</i>	it ca <i>lectiv</i>	n be ir	cluded in t	he Sections	Stud	/ Focus,	
Prerequisites	There are no further prerequ	isites.									
Responsible Persons	Hannah Markwig										

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-03	Module Title: Tropical Geometry							of Module: ulsory Modu	le with	Choice			
ECTS-Points	9												
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-Si 180 h	tudy:					
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German or English												
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS											
Objectives	 Tropical numbers and Tropical hypersurface Tropical toric varieties Matroid fans and tropical modifications Tropical modifications Tropical curves and linical curves and linical curves and linical curves Tropical modifications Tropical modifications Tropical modifications Tropical curves Tropical curves Tropical curves Tropical modifications Tropical modifications Tropical modifications Tropical curves Tropical modifications <li< th=""><th>s and values and value</th><th>strace interpretations of the strace the strace the strace the strace that the</th><th>subjections can</th><th>ects s lave i epts n nan ons. a cor</th><th>studied reache from cone and</th><th>from tropic d a deepen ombinatoric prove the c</th><th>al geometry d understan s can be ap entral result</th><th>ding of plied so of the ent har</th><th>convex uccess- e lecture</th></li<>	s and values and value	strace interpretations of the strace the strace the strace the strace that the	subjections can	ects s lave i epts n nan ons. a cor	studied reache from cone and	from tropic d a deepen ombinatoric prove the c	al geometry d understan s can be ap entral result	ding of plied so of the ent har	convex uccess- e lecture			
	the terms, statements and m to new problems, to analyse team. They are capable of p discourse.	e them	and	d to	work	on so	lution strate	egies on the	ir own	or in a			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Tropical Geometry	L E	f	4 2	6	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	bee	n acc	uired.	Whether th	e examinati					

Literature	Possible References: Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018. Diane Maclagan, Bernd Sturmfels: Introduction to tropical geometry. AMS 2015.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites. Knowledge from the modules Algebraic Geometry and Differential Geometry is helpful, however.
Responsible Persons	Hannah Markwig, Johannes Rau

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-04	Module Title: Tropical Enumerative Geome	etry						of Module: ulsory Modu	le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		Class	:		Self-St 180 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SW	S						
Objectives	Enumerative geometre Tropical enumerative Combinatorial method Correspondence theod Tropical and classic Good Real counts, Welsching Hurwitz numbers. Tropical correspondere Real Hurwitz numbers The students know basic teetext of tropical geometry meand limitations of the tropical they deepen their knowledge Gromov-Witten theory. The and they can explain their into the exercise classes they the terms, statements and means	probleds, floorems in a ces for sand 2 ces in the stude trinsic have a	ms a	and magrand magrand was a same as an action of the control of the	nultip ns an in th neory num mbe d me velop necti algeb ame ons. a cor	licities. Indicate and lattice Indicate an	e paths. e through g nus 0. nial invariar of enumera eper unders n more com ecometry tow rove the ce	ative geome tanding of the plex issues vards modulentral results d independe	he pos Furth ar spa of the	sibilities ermore, ces and e lecture
	to new problems, to analysteam. They are capable of p discourse.	e then	n and	d to	work	on so	lution strate	egies on the	eir own	or in a
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Tropical Enumerative Geometry	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	have	bee	n acc	uired.	Whether th	e examinati		

Literature	Possible References :
	Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Familiarity with the module Tropical Geometry is assumed.
Responsible Persons	Hannah Markwig, Johannes Rau

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, } \mbox{LE=lecture with integrated exercises, } \mbox{SL=seminar or lecture, } \mbox{E=exercise class, } \mbox{T=tutorial, P=project, S=seminar, IC=inverted classroom}$

Status : o=obligatory, f=facultative

Module Number: MAT-50-05	Module Title: Introduction to Tropical Enun	nerativ	e Ge	ome	try			of Module: ulsory Modu	le with	Choice
ECTS-Points	5									
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	in C	lass			Self-St 105 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS									
Content	 Enumerative geometry of algebraic curves, in particular in the plane. Tropical enumerative problems and multiplicities. Combinatorial methods, floor diagrams and lattice paths. Correspondence theorems for curves in the plane through given points. Real counts, Welschinger invariants and polynomial invariants. 									
Objectives	The students know basic te text of tropical geometry me and limitations of the tropical they deepen their knowledge students are capable of nar assessing and explaining the In the exercise classes they the terms, statements and m to new problems, to analyse team. They are capable of p discourse.	thods. I acce in the ning a prese have a ethods them	The ss in e fiel nd pentectangular recording to the state of the state	ey de con d of rovir l con ired a he le	velor necti alge ng th necti a cor cture work	o a dee on with braic ge e esse ions. nfident, e. They on so	eper unders n more com eometry tovential results precise an have learn lution strate	standing of the plex issues. Wards modules of the lected independent of the lected to transfer on the pegies on the standard independent of the pegies on the pegies of th	he posi Furth li space ure as ent har er the n	sibilities ermore, es. The well as adling of nethods or in a
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Tropical Enumerative Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise contains examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	s coursewo Whether th	ork. For part e examinatio		
Literature	Possible References :									
	Grigory Mikhalkin, Jol	nanne	s Ra	u: Tro	opica	al geom	etry. Manu	script 2018.		

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be counted alongside it.
Prerequisites	Familiarity with the module Tropical Geometry is helpful, but not necessary.
Responsible Persons	Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-06	Module Title: Type of Module: Compulsory Module with Choice									Choice		
ECTS-Points	5											
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	in C	lass	•		Self-St 105 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS											
Content	 Plane curves of higher genus. Multiplicities. Welschinger invariants. Lattice paths. Floor diagrams. Hurwitz numbers. Tropical moduli spaces. 											
Objectives	The students deepen their kr acquainted with various met merative problems which can name and prove the central tions. In the exercise classes they the terms, statements and m to new problems, to analyse team. They are capable of p discourse.	nods to be seemed	co en colvects of the acquision of the	ume d with he le ired a he le	rate h the cture a cor cture work	tropica e aid of e and the offident, e. They on so	I curves, as tropical ge ney can exp precise an have learn lution strate	s well as wit ometry. The blain their in d independe ed to transfe egies on the	h varion student student har the return the return the return the return town	ents can connec- adling of nethods or in a		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Tropical Enumerative Geometry - Part 2	E	f	1	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether th	e examination				
Literature	Possible References :											
	Diane Maclagan, Berr Grigory Mikhalkin, Joh							-	1 S 201	5.		

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be included alongside it.
Prerequisites	Familiarity with the module Tropical Enumerative Geometry is expected.
Responsible Persons	Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-10	Module Title: Geometry of Manifolds 1							of Module: ulsory Modu	le with	Choice		
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		lass			Self-Si 180 h	Self-Study: 180 h				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS											
Content	 Manifolds and submanifolds. Vector fields and flows. Metrics, foundations of Riemannian geometry. Complex structures. Theorem of Gauß-Bonnet on surfaces. 											
Objectives	The students know and under geometry and the basic techning especially of differential arematical concepts are nature proving the essential results connections. In the exercise classes they the terms, statements and more to new problems, to analysteam. They are capable of prediscourse.	niques and inte ally us of the have a ethoda e then	s for legral sed in lectured in lecture in l	nand calc geo ure a ired he le	ling tulus and the control of the co	hem. Tand hary. The las as a	They have down exempla extudents a students a sessing an precise and have learn lution strate	eepened the rily experient are capable described explaining dindependent described in the ransfergies on the	eir und ced ho of nam the prent har er the reir owr	erstand- w math- ning and esented adling of methods or in a		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Geometry of Manifolds 1	L	f	4	6	yes	wr. o.	90-180	g	100		
	-	E	f	2	3		or.	o. 20-30				
	In this module an exercise c examination the coursework oral is decided by the instruc	must l	have	bee	n acc	uired.	Whether th	e examination				

Literature	Possible References :										
	Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004.										
	John M. Lee: Introduction to Smooth Manifolds. Springer 2012.										
	Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996.										
	Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.										
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. Due to a significant overlap in contents the module cannot be taken together with either of the modules 'Geometry in Physics' and 'Introduction to Differential Geometry'.										
Prerequisites	There are no further prerequisites.										
Responsible Persons	Christoph Bohle, Frank Loose										
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio											

 $\label{eq:master} \mbox{Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio \\ \mbox{Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, }$

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-11	Module Title: Geometry of Manifolds 2							of Module: ulsory Modul	e with	Choice	
ECTS-Points	9	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-Study: 180 h				
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS										
Content	 Global aspects of Riemannian geometry Cohomology of manifolds. Analysis of differential operators on manifolds. Applications on Riemann surfaces (and complex manifolds). 										
Objectives	The students are familiar with geometry. They have deeped they have exemplarily expensive students are capable of narrassessing and explaining the line the exercise classes they the terms, statements and more to new problems, to analysic team. They are capable of prodiscourse.	ned the ienced ming a present the factorial mediane and the factorial	eir u d hov nd p ented acqui s of t	nder v loc rovir l con ired a he le	stand al an ng the necti a cor cture work	ding of ad glob e esse ons. ofident, e. They on so	methods in all aspects ntial results precise and have learn lution strate	differential in geometry of the lecture of the lecture dindependent dependent of the transference on the	geome intera ure as ent har er the r ir own	etry, and ct. The well as adling of nethods or in a	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	, ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Geometry of Manifolds 2	E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise or examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether the	e examinatio			

Literature	Possible References :
	 Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004.
	John M. Lee: Introduction to Smooth Manifolds. Springer 2012.
	Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996.
	 Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
	John Milnor: Morse Theory. PUP 1963.
	Donu Arapura: Algebraic Geometry over the Complex Numbers. Springer 2012.
	Sundararaman Ramanan: Global Calculus. AMS 2005.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules 'Geometry on Manifolds' or 'Geometry in Physics' is assumed.
Responsible Persons	Christoph Bohle, Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-12	Module Title: Information Geometry		Type of Module: Compulsory Module with Choice								
ECTS-Points	3						-				
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h					Self-St 60 h	Self-Study: 60 h			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content	 Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). Application to neural data processing (in particular supervised learning in artificial neural networks). 										
Objectives	Students have an elementary to problems in information the proving the essential results connections.	neory	and	stati	stics	. The	students ar	e capable o	of nam	ing and	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Information Geometry	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	cided	by the instru	uctor with a	pprova	I by the	
Literature	 Possible References: Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015. 										
Transfer	The module belongs to the ferential Geometry and Stock sation, it can be included in to r Elective Specialisation, in section.	<i>hastic</i> he Se	s. Ta ction	king s <i>Sti</i>	into i udy I	accour <i>Focus</i> ,	it the chose Advanced k	n personal S Knowledge i	Study S n Math	Speciali- ematics	

Prerequisites	Basic knowledge from differential geometry (Riemannian metrics, connections and curvature, geodesics) and from stochastics is assumed.
Responsible Persons	Christoph Bohle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-13	Module Title: Information Geometry and Ne		Type of Module: Compulsory Module with Choice									
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass			Self-St 60 h	Self-Study: 60 h				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	 Further basics of information geometry (e.g. dual flat structures for exponential families, Pythagoras' theorem and information projections, em algorithm). Application to neural data processing (in particular <i>Unsupervised Learning</i> in artificial neural networks, e.g. Boltzmann and Helmholtz machines). 											
Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Information Geometry and Neural Data Processing 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writter	or	oral	is de	cided	by the instru	uctor with a	pprova	I by the		
Literature	Possible References: Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.											
Transfer	The module belongs to the Signature Differential Geometry. Taking be included in the Sections Specialisation, in accordance	g into Study	Foc	ount us, A	the d Adva	chosen nced K	personal S <i>nowledge ii</i>	tudy Specia n <i>Mathemat</i>	lisation ics or	n, it can <i>Elective</i>		
Prerequisites	The module Information Geo	netry	and	Neu	ral D	ata Pro	cessing 1 is	s a prerequi	site.			

: o=obligatory, f=facultative

Status

Other

Responsible Persons	Christoph Bohle
Abbreviations:	
Grading System : g	=graded, ng=not graded
Examination Type : M	IT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	electure, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, etutorial, P=project, S=seminar, IC=inverted classroom

Module Number: MAT-50-14	Module Title: Mathematical Aspects of Neing 1		of Module: ulsory Modu	le with	Choice						
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h		lass	:		Self-St 60 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content	 Artificial neural netwo methods. Dynamic interpretation ics) and the change of simple neuroscientific. Recent work on the threachine learning. In a continuation of the dynamic origin, such a simple neuroscientific. 	n as the weight mode meoret	ne flo hts d els fo ical f	w of Juring r the ounc the	data g traii dyna latior	/activat ning (sl amics o	ions through ow dynamic of neural net eep learning	n the networes). tworks.	rk (fast ically p	dynam- olausible	
Objectives	The students have learned to ral networks and biologically systems as a possible frame dents can name and prove the connections.	more work	e pla for t	usibl heor	e alt etica	ernativ I and r	es. They a nathematica	re familiar v al investigati	vith dy ons.	namical The stu-	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Mathematical Aspects	Type of Course	J Status	SWS 2	ε ECTS	Coursework	Type of Exam	Dur. of Exam (min)	ص Grading	Weight for Grade	
	of Neuronal Information Processing 1 Whether the examination is Board of Examiners.							o. 20-30 uctor with a			

Literature	Possible References :
	Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016.
	 Anthony C. C. Coolen, Reimer Kühn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005.
	 Dmitry Krotov, John J. Hopfield: Unsupervised learning by competing hidden units. PNAS 2019.
	Guan-Horng Liu, Evangelos A. Theodorou: Deep Learning Theory Review - An Optimal Control and Dynamical Systems Perspective. arXiv:1908.10920.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	
Responsible Persons	Christoph Bohle
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio

 $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, } & \text{P=project, } & \text{S=seminar, } & \text{IC=inverted classroom} \\ \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-50-15	Module Title: Introduction to Riemann Sur	Module Title:Type of Module:Introduction to Riemann SurfacesCompulsory Module with Choice										
ECTS-Points	5											
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	in C	lass	:		Self-S 105 h	tudy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3	1-3										
Language of Instruction	German or English	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 1 SWS										
Content	Coverings and fundations	Coverings and fundamental groups.										
	Topological classifica	tion of	the s	surfa	ces.							
	Theorem of Riemann	-Hurwi	tz.									
	Differential forms and	integr	ation									
	Sheaves and cohomo	ology.										
	Theorem of Riemann	-Roch.										
	Serre duality.											
	Kobayashi metric.											
	Theorem of Picard.											
Objectives	Students develop an approad based on local-to-global rearigidity resulting from analyt damental questions naturally can ultimately be used to a terrelated and in many case proving the essential results connections. Students will be in the subject area.	soning ical proy lead nswer of the of the	. In the propert to industrial to industrial to the propert to the	he co creas tions depe ure a	once Usin singly s. The ender s we	pt of hong the standard the sta	olomorphy, sheaf concept concept the students as students as seessing ar	they grasp the ept, students tualisations a metry and a are capable and explaining	ne prin s see h and ho nalysis of nam the pr	ciples of now fun- w these s are in- ning and esented		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Introduction to Riemann Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise c examination the coursework oral is decided by the instructionally be offered by the course of the c	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1										

Literature	Possible References :
	Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992.
	Otto Forster: Riemannsche Flächen. Springer 1977.
	Klaus Lamotke: Riemannsche Flächen. Springer 2009.
	Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Riemann surfaces' due to the large overlap in content
Prerequisites	Knowledge from the lecture Introduction to Complex Analysis is required.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-16	Module Title: Type of Module: Riemannian Geometry Compulsory Module with Choice												
ECTS-Points	6						·						
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:					
Duration	1 Semester	1 Semester											
Frequency	not regularly	not regularly											
Term	1-3												
Language of Instruction	English												
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS												
Content	Riemannian manifolds	Riemannian manifolds.											
	Geodesics.												
	Curvature.												
	Geometry of submanif	Geometry of submanifolds.											
Objectives	manifolds from a classical p cussed. The students were e are sufficient to study their ronotions of curvature was deferential geometry was achie the essential statements and veloped in the lecture and to critically challenge the current Through homework assignment independent acquaintant lectures. They learn how to	The students have learned and understood definitions and main examples of Riemannian manifolds from a classical point of view. In addition, topics related to geodesics were discussed. The students were exposed to important geometric results involving geodesics which are sufficient to study their role in different areas of differential geometry. Intuition for various notions of curvature was developed by the students and familiarity with computations in differential geometry was achieved in the exercise sessions. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Riemannian Geometry	L	f	2	3	yes	wr. o.	90-180	<u> </u>	100			
	- Hemannan Geometry	Е	f	2	3	yes	or.	o. 20-30	g	100			
	In this module an exercise ce examination the coursework oral is decided by the instruct	must l	nave	bee	n acc	quired.	Whether th	e examination					
Literature	Possible References :												
	John M. Lee: Riemani	nian n	nanifo	olds:	An i	ntrodu	ction to curv	ature. Sprin	ger 19	97.			
	Barret O'Neill: Semi-F Press 1983.	Riema	nniaı	n ge	omet	try. Wi	th application	ons to relativ	rity. Ad	cademic			

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> , <i>Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Geometry in Physics is assumed.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-50-17	Module Title: Introduction to Integrable Sy Riemann Surfaces, and Spe	stems (Classical Mechanics, ctral Theory)	Type of Module: Compulsory Module with Choice							
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h							
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	class 2 SWS								
Content	group. The course will focus discrete counterparts. Origin during a famous horse ride a and the underlying theory in A fundamental idea for under as spectrum preserving deformation case symmetric matrices. We study an important class algebro-geometric) solutions of Riemann surface theory a ics, Riemann surface theory briefly touch upon an integral algebra. The KdV equation is related interpreted as a dynamical deeply related to the geomet in the lecture are related to the sequel to the lecture, it	on equations related to the Korally a mathematical model for along a canal, equations of Kd' volves various mathematical derstanding and solving KdV typermations of underlying auxiliants of explicit solutions that in a This class of solutions can and classical mechanics. The and spectral theory will be explicit systems interpretation of the discontinuous of the geometry of Riemann surfacts planed to explain how infinitions.	pe equations is their interpretation ry linear operators - in the simplest acludes solitons and finite gap (or be described using a combination relevant parts of classical mechan-caplained in the lecture. We will also the QR-algorithm of numerical linear erent ways: for example, it can be metrized curves in the plane; it is ups; the special solutions discussed							
Objectives	faces, mechanics, and spec were discovered mainly in the a branch of mathematics son dents can name and prove the connections. In the exercise classes they the terms, statements and me to new problems, to analyse	ctral theory – as well as other second half of the twentietle metimes called soliton theory he central results of the lecture have acquired a confident, protethods of the lecture. They have them and to work on soluti	n classical topics like Riemann sur- er branches of mathematics — that h century during the emergence of or integrable mathematics. The stu- e and they can explain their intrinsic recise and independent handling of ave learned to transfer the methods on strategies on their own or in a applicable to argue for it in a critical							

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Integrable Systems	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise cerexamination the coursework noral is decided by the instructor	nust ł	nave	beer	n acc	uired.	Whether the	e examination		
Literature	Possible References: Olivier Babelon, Denis tems. CUP 2004. Leonid A. Dickey: Solit Alan C. Newell: Soliton Sergei P. Novikov, Sergof Solitons - The Inversions.	on ed s in r gei V.	luation nathe Man	ons a emat	and Hics a	lamilto and phy	nian system sics. SIAM aevskii, Vlad	is. World Sc 1985. dimir E. Zak	ientific	2003.
Transfer	The module belongs to the St ential Geometry and Mathem Specialisation, it can be include ematics or Elective Specialist spective section.	<i>atica</i> led in	<i>Phy</i> the	<i>sics.</i> Sect	. Tak ions	king int <i>Study</i>	o account th <i>Focus</i> , <i>Adva</i>	ne chosen p anced Know	ersona <i>ledge i</i>	al Study in Math-
Prerequisites	The module Introduction to Co Basic knowledge of differenti necessary.									
Responsible Persons	Christoph Bohle, Frank Loose									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, P=project, S=seminar, IC=inverted classroom} \\ \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-50-18	Module Title: Integrable Systems (and Inf bras)	inite [Dime	nsior	nal L	ie Alg		of Module: ulsory Modul	le with	Choice		
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS										
Content	group. The course will focus of discrete counterparts. Original during a famous horse ride at and the underlying theory involved A fundamental idea for under as spectrum preserving deformation case symmetric matrices. This lecture is the continuation sical Mechanics, Riemann S	This lecture is the continuation of the lecture called Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory). This continuation will investigate integrable equations using sl(2,C)–loop algebras. In particular, we will study explicit solutions										
Objectives	The students have aquired a algebra of sl(2,C). The studer can explain their intrinsic con In the exercise classes they the terms, statements and m to new problems, to analyse team. They are capable of prediscourse.	nts car nection have a ethods ethem	n nan ns. acqui s of t	ne ar red a he le	nd pr a cor cture work	ove the fident, e. They on so	precise an have learn lution strate	ults of the le d independe ed to transfe egies on the	ecture a ent har er the r eir own	and they ndling of nethods or in a		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Integrable Systems (and Infinite Dimensional Lie Algebras)	п Гуре of Course	t Status	SMS 4	e θ ECTS	Sework	Type of Exam o. o					
	In this module an exercise ce examination the coursework oral is decided by the instruct	must h	nave	beer	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	 Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004.
	Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003.
	Alan C. Newell: Solitons in mathematics and physics. SIAM 1985.
	 Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).
Transfer	The module belongs to the Study Specialisations Algebra and Geometry, Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge from the module Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory) is assumed.
Responsible Persons	Christoph Bohle, Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-19	Module Title: Mathematical Aspects of Neing 2	urona	l Info	rma	tion I	Proces		Type of Module: Compulsory Module with Choice				
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: Time in Class: 30 h							Self-Study: 60 h				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German	German										
Forms of Teaching and Learning	Lecture 2 SWS											
Content	data. • Dynamic interpretatio through the network (namics). • Simple neuroscientific	 Dynamic interpretation of neural data processing methods as flow of data/activations through the network (fast dynamics) and change of weights during training (slow dy- 										
Objectives	Students have learnt the bas biologically more plausible al framework for theoretical and ing and proving the essentia presented connections.	ternat d math	ives. nema	The tical	y are inve	familia stigatio	ar with dyna ns. The stu	mic systems dents are ca	s as a apable	possible of nam-		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Mathematical Aspects of Neuronal Information Processing 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instru	uctor with a	pprova	al by the		
Literature	Anthony C. C. Cooler cessing Systems. OU											
Transfer	The module belongs to the Differential Geometry. Takin be included in the Sections Specialisation, in accordance	g into Study	acco Foc	ount <i>us</i> , <i>i</i>	the o	chosen nced K	personal S <i>(nowledge ii</i>	tudy Specia n <i>Mathemat</i>	alisatio tics or	n, it can <i>Elective</i>		

Prerequisites	In terms of content, the module Dynamic Systems and information processing 1 module is a prerequisite.
Responsible Persons	Christoph Bohle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-20	Module Title:							of Module:	lo with	Chaina		
ECTS-Points	Topology						Сопр	ilsory iviodui	e with	Choice		
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	lass 2	SWS	3								
Objectives	Review of metric sp spaces, compactness Set-theoretic topology aration axioms. Spaces of continuous pactification, the theocompactness in space. Baire's spaces and a theorems. Outlook on algebraic Students have familiarised theoretical topology and hav nomena in different areas of ferent areas of mathematics.	topological topological functions of functio	logical construction of the construction of th	urys Urys ons. of Ba with od th ics.	aces sohn' Weier the c at thi In th are c	etric sp , conting s lemmestraß, theory entral s theory capable	nuity conver	gence, compositions, Stoconvergence unction class esults and modes and proving and proving	ne-Cece in full ses, extended the manage of the extended the exten	ch com- nctions, distence		
	results of the lecture as well In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	have a ethods them a	acqui s of the	ired a he le o wo	a cor cture rk on	nfident, e. They solution	precise an have learn on strategies	d independe ed to transfe s on their ow	ent han er the n n or in	nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Topology	L E	f f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	beei	n acc	uired.	Whether the	e examinatio				

Literature	Possible References :
	Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914.
	Boto von Querenburg: Mengentheoretische Topologie. Springer 2001.
	Volker Runde: A Taste of Topology. Springer 2005.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-21	Module Title: Algebraic Topology 1							of Module: ulsory Modul	le with	Choice	
ECTS-Points	9						'				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-St 180 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German	German									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS									
Objectives	Basic concepts of cate The fundamental group Theory of covering spaces, into a precise theory how abstract concepts, e.g. ways of speaking that enables tudents are capable of nate.	 Set theoretical topology. Basic concepts of category theory. The fundamental group of a punctured topological space. Theory of covering spaces. Basic concepts of singular homology theory. Applications. The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented. The									
	assessing and explaining the In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	e prese have a lethods them a	ented acqui s of t and t	l con ired : he le o wo	necti a cor cture rk on	ions. nfident, e. They n solution	precise an have learn on strategies	d independe ed to transfe s on their ow	ent har er the r n or in	ndling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Algebraic Topology	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise of examination the coursework oral is decided by the instruc	must h	nave	bee	n acc	uired.	Whether th	e examination	icipation on is w	on in the ritten or	

Literature	Possible References :
	Allen Hatcher: Algebraic topology. Cambridge University Press 2009.
	Horst Schubert: Topologie. Teubner 1971.
	Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966.
	Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar, Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-22	Module Title: Algebraic Topology 2							of Module: ulsory Modul	e with	Choice			
ECTS-Points	9												
Workload - Time in Class - Self-Study	Workload: 270 h												
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German												
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2 S	SWS	3									
Content	Further development	of singu	ılar l	homo	ology	theor	/.	·					
	Simplicial complexes	and the	eir sii	mplic	cial h	omolo	gy.						
	CW spaces and their	cellular	hon	nolo	gy.								
	Axiomatic homology.												
	Homological algebra.												
	Cohomology.												
	Homology and Cohon	nology	with	coef	ficie	nts.							
	Product structures in	homolo	gy a	ınd c	ohor	nology							
	The Poincaré duality t	heoren	n for	topo	logic	al mar	ifolds.						
Objectives	The students extend their abstructions. They deepen the even technically very challer essential results of the lectur In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	ir knowiging ta e as we have a ethods them a	ledg sks. Il as cqui of th	ge in The assered and he le	abstessires cor essires cor cture rk on	tract madents and	athematical re capable explaining to precise and have learn on strategies	I disciplines of naming a he presented independed to transfes on their ow	to accord provided connections that the nection and the nection are to accord to according to accord to according to acc	omplish ving the ections. Idling of nethods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Type of Course Status SWS SWS ECTS Coursework Type of Exam Type of Exam Grading Grading									Weight for Grade			
	Algebraic Topology 2	L E	f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	ave	beer	acc	uired.	Whether th	e examinatio	icipatio on is w	n in the ritten or			

Literature	Possible References :
	Allen Hatcher: Algebraic topology. Cambridge University Press 2009.
	Horst Schubert: Topologie. Teubner 1971.
	Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966.
	Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Content-wise, the module Algebraic Topology 1 is a prerequisite for participating in this module.
Responsible Persons	Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-23	Module Title: Algebraic Topology 3							of Module: ulsory Modu	le with	Choice
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass			Self-St 60 h	udy:		
Duration	1 Semester									
Frequency	ot regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	 Basic concepts of hom Homotopy group of spl Spectral sequences; K-theory; 									
Objectives	With the in-depth knowledge introduced to current areas or which can lead to a Master's possible doctorate in algebrai essential results of the lecture	f rese s thes c topo	arch sis, fo ology	and or ex . The	they amp	tackle le. The dents a	a small res ey will also re capable	search proje lay the fou of naming a	ct ther indatio	nselves, ns for a ving the
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Topology 3	L	f	2	3	no	Р		g	100
	Specifics on the portfolio will I	oe ex	plain	ed b	y the	exami	ner at the b	eginning of	the co	urse.
Literature	Possible References:	oundle D. S	es ar tashe	nd K- eff: C	theo Chara	ry. Mar acteristi	nuskript 200 c classes.	09. Princeton U		

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the modules Algebraic Topology 1 and 2 are prerequisite for participation in this module.
Responsible Persons	Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-50-24	Module Title: Introduction to K-theory							of Module: ulsory Modu	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 60 h										
Duration	1 Semester						·				
Frequency	ot regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content		 Topological K-theory. Künneth formula and Bott periodicity. Characteristic classes. Chern character. Algebraic K-theory 									
Objectives	The students have learnt an algebra and number theory. different areas. They can und gorical K-groups and apply the capable of naming and pand explaining the presented	They I dersta nem. proving	nave ind a They g the	lear nd u hav ess	nt to se te ve le	recognerms su arnt to	ise and use ich as vecto think in lar	the connector or fibre bugge contexts.	ctions bundles The s	oetween or cate- students	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to K-theory	L	f	2	3	no	wr. o. or.	o. 20-30	g	100	
Literature	Possible References :	1	1	1	1	1	I	1	l	' 	
	 Michael Atiyah: K-theory Max Karoubi: K-theory Emilio Lluis-Puebla, J Higher algebraic K-the 	. Spri ean-L	nger ouis	200 Loda	8. ay, ⊦	lenri G	illet, Christo	ophe Soule,	Victor	Snaith:	
Transfer	The module belongs to the Differential Geometry. Taking be included in the Sections Specialisation, in accordance	g into Study	Foc	ount <i>us</i> , <i>A</i>	the o	chosen nced K	personal S <i>nowledge i</i>	tudy Specia n Mathemat	ılisatioi <i>ics</i> or	n, it can <i>Elective</i>	

Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-25	Module Title: Applied topology 1							f Module: Ilsory Modul	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content	Simplicial complexes a	Simplicial complexes and their homology.									
	Persistent homology.										
	Basic notions from top	Basic notions from topological data analysis.									
Objectives	The students are familiar with basic concepts of algebraic topology and their application in the context of topological data analysis. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Applied topology 1	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writter	n or	oral	is de	cided	by the instru	uctor with a	pprova	I by the	
Literature	Possible References :										
	Herbert Edelsbrunner	John	L. H	arer:	Cor	nputati	onal Topolo	gy. AMS 20	10.		
	Robert Ghrist: Elemen	ntary A	Applie	ed To	polo	gy. Cre	eate Space	2014.			
	Sergey V. Matveev: Le	ecture	s on	Alge	braic	Topolo	ogy. EMS 20	006.			
Transfer	The module belongs to the single Geometry and Stochastics. Tions into account, the modu Knowledge Mathematics or E	Taking Ie can	the a	perso assig	onal Ined	special to the	isation and	the restriction	ons of	the sec-	
Prerequisites	There are no further prerequ	sites.									
Responsible Persons	Christoph Bohle										

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-26	Module Title: Applied Topology 2							of Module: ulsory Modul	le with	Choice		
ECTS-Points	3	3										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 60 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German	German										
Forms of Teaching and Learning	Lecture 2 SWS	ecture 2 SWS										
Content	Advanced aspects of	Advanced aspects of persistent homology (e.g. stability).										
	Applied Morse theory.											
	Applied sheaf theory.	Applied sheaf theory.										
Objectives	The students are familiar with advanced concepts of applied topology and topological data analysis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Applied Topology 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writter	n or	oral	is de	ecided	by the instri	uctor with a	pprova	l by the		
Literature	Possible References :											
	Herbert Edelsbrunner	, John	L. H	arer	: Coi	mputati	onal Topolo	gy. AMS 20	10.			
	Robert Ghrist: Elemei	ntary A	Appli	ed To	polo	ogy. Cre	eate Space	2014.				
	Sergey V. Matveev: Le	ecture	s on	Alge	braid	c Topolo	ogy. EMS 20	006.				
Transfer	The module belongs to the State chosen personal Study Advanced Knowledge in Mastrictive requirements of the	Specia thema	alisat <i>tics</i>	ion, or <i>E</i>	it ca <i>lecti</i> v	ın be ir	ncluded in t	he Sections	Study	/ Focus,		
Prerequisites	The contents of the module from differential geometry is			opolo	ogy 1	l' are a	ssumed. N	loreover, ba	sic kno	owledge		
Responsible Persons	Christoph Bohle											

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-27	Module Title: Topological Vector Spaces a	Module Title: Topological Vector Spaces and Distributions						of Module: ulsory Modul	le with	Choice	
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:			
Duration	1 Semester						-				
Frequency	not regularly	not regularly									
Term	1-3	1-3									
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 2 SWS									
Content	A selection of the following t	opics v	vill be	e cov	ered	:					
	Locally convex topolo	gical v	ecto	r spa	ces,	Freche	et spaces, L	F spaces an	nd LB s	paces.	
	Duality: Hahn-Banaci	h theor	em,	dual	spac	e, topo	ologies on th	ne dual spac	e.		
	Generalised functions	s, Rado	on m	easu	res a	and dis	tributions.				
	Properties of distribution	ions ar	nd op	oerat	ions	on the	space of di	stributions.			
	Applications and example	mples.									
Objectives	Students master the basic portion and understand how to ap Schwartz. Students are also which classical questions or capable of naming and provexplaining the presented correctly in the exercise classes they the terms, statements and more on new problems, to analyse They are able to present the	ply this so able f mathe ring the nnectio have a nethods them a	s to e to emat e ess ns. acqu s of t	the name ical pention ical pent	theore the ohysical research	ry of g e main ics can sults of nfident, e. They n solution	peneralised applications be treated the lecture precise and have learn on strategies	functions as of the the with it. The as well as a d independent on their ow	ccordir eory ar e stude assess ent har er the r	ng to L. and show ents are sing and adding of methods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable Title Title Title Title Title Title Title									Grading	Weight for Grade	
Topological Vector Spaces L f 2 3 yes wr. o. 90-180 or. 0. 20-30										100	
	In this module an exercise c examination the coursework oral is decided by the instruc	must h	nave	beei	n acc	uired.	as coursewo Whether th	rk. For part e examinatio			

Literature	Possible References :
	Gerald Folland: Real Analysis. Wiley 1999.
	Helmut H. Schäfer: Topological Vector Spaces. Springer 1999.
	Laurant Schwartz: Theorie des Distributions. Hermann 1998.
	Laurant Schwartz: Mathematics for the Physical Sciences. Dover 2008.
	Francois Trèves: Topological Vector Spaces, Distributions and Kernel. Dover 1967.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Functional Analysis and basic knowledge of set-theoretical topology is assumed.
Responsible Persons	Ulrich Groh, Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom \\$

Status : o=obligatory, f=facultative

Module Number: MAT-50-28	Module Title: Uniformisation of Riemann Surfaces							f Module: Ilsory Modul	e with	Choice
ECTS-Points	5									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study 150 h 45 h 105 h									
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 1 SWS								
Content	Uniformisation of Rien	nann s	surfa	ces						
Objectives	The students have learnt how to determine the simply connected Riemann surfaces by successively solving suitable differential equations. They are then able to classify Riemann surfaces under suitable conditions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Uniformisation of Riemann Surfaces	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruc exceptionally be offered by t points will be awarded for the	must l tor wit he led	nave h ap ture	beer prova with	n acc al by nout	quired. the Bo exercis	Whether the ard of Exan	e examination niners. – Th	on is w e mod	ritten or ule may
Literature	Possible References :									
	Hershel M. Farkas, Irv	vin Kra	a: Rie	emar	ın Sı	urfaces	. Springer 1	992.		
Transfer	The module belongs to the Differential Geometry. Taking be included in the Sections Specialisation, in accordance	g into <i>Study</i>	Foc	ount i us, <i>P</i>	the c I <i>dvai</i>	chosen nced K	personal S <i>nowledge ii</i>	tudy Specia n <i>Mathemat</i>	lisation <i>ics</i> or	n, it can <i>Elective</i>
Prerequisites	The module Introduction to Bachelor of Science Mathem								1odule:	s of the
Responsible Persons	Reiner Schätzle									

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-29	Module Title: Algebraic Curves and Riema	ann Surfaces	Type of Module: Compulsory Module with Choice						
ECTS-Points	9								
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h						
Duration	1 Semester								
Frequency	not regularly								
Term	1-3	1-3							
Language of Instruction	German or English								
Forms of Teaching and Learning	Lecture 4 SWS								
Content	Compact Riemann su	urfaces.							
	Normalisation of plan	e curves.							
	Topological genus.								
	Coverings.								
	Forms and integration	٦.							
	Sheaves and cohomo	ology.							
	Hodge theory.								
	Arithmetic and geome	etric genus.							
	Abel's theorem.								
	Riemann-Roch theore	em.							
	Serre duality.								
	 Jacobian and Abelian 								
	Riemann bilinear rela								
	Jacobi inverse proble								
	Elliptic curves and fur	nctions.							
	• j-Invariant.								
	Uniformisation.	naat Diamann aurfaan							
	Topology of non-comp	pact Riemann sunaces.							
Objectives	based on local-to-global reast rigidity resulting from analyti see how fundamental questi how these can ultimately be are interrelated and in many and proving the essential re	soning. In the concept of hold ical properties. By example on an aturally lead to increasing used to answer questions. The cases mutually dependent. It is sults of the lecture as well as the will be able to reflect and of the lecture.	understand classification techniques omorphy, they grasp the principles of of the concept of sheaves, students ngly abstract conceptualisations and ley learn how geometry and analysis. The students are capable of naming is assessing and explaining the precritically analyse the current state of						

Status

Other

: o=obligatory, f=facultative

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Algebraic Curves and Riemann Surfaces	L ü	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.											
Literature	Possible References :											
	Frederice Mangolte: Relationships	eal A	lgebr	aic \	/ariet	ies. Sp	oringer 2020					
	Robert Silhol: Real Alg	ebrai	c Su	rface	s. S	oringer	1989.					
	Riccardo Benedetti, Jo Editions Herrmann 199		acqu	es F	Risler	: Rea	l Algebraic	and Semi-a	ılgebra	ic Sets.		
	Alex Degtyarev, Viatch eties: du côté de chez	eslav						ties of real	algebra	aic vari-		
Transfer	The module belongs to the Significant Differential Geometry. Taking be included in the Sections Significant Specialisation, in accordance The module cannot be taken to the large overlap in content	into Study with togetl	acco Foc the re	ount <i>us, A</i> estric	the c A <i>dvai</i> ctive	chosen nced K require	personal S nowledge in ements of th	tudy Špecia n <i>Mathemat</i> e respective	lisation ics or esection	n, it can <i>Elective</i> on.		
Prerequisites	In terms of content, the course to function theory of ordinary							as well as tl	ne intro	duction		
Responsible Persons	Ivo Radloff											
Examination Type: No Teaching Format: L	=graded, ng=not graded /T=master's thesis, or.=oral exa =lecture, LE=lecture with inte =tutorial, P=project, S=seminar	grate	d ex	ercis	ses,	SL=se						

Module Number: MAT-50-30	Module Title: Geometric Group Theory							of Module: ulsory Modu	le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	•		Self-S 180 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS								
Content	 Group actions on graph Quasi isometries. Growth types. Hyperbolic groups. Ends.	ohs, fre	ee gr	oups	S.					
Objectives	Students learn to explore profrom the Cayley graph of the of the Cayley graphs with the tothe underlying group. Students about groups. The statements about groups are sults of the lecture as well in the exercise classes they the terms, statements and mon new problems, to analysteam. They are capable of produced the cayley are cayley are cayley are capable of produced the cayley are capable of produced the cayley are	e groune help udents the in The stas assate thave a there	ip. To of a und terfactuden session acquires of to another terms.	They analy erstace of the angle and the lead to	are tical and h f alg re ca nd ex a con ecture work	able to methon now algebra and apable oplaining offident, e. They	o investigate das and to gebra and a geometro of naming ag the prese are have learrollution strate.	e the geome work out the analysis can by that leads and proving ented connected independent of transferegies on the	etric preir consister work for the interest of the extended to the interest of	operties nections together eresting essential adling of nethods or in a
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometric Group Theory	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.								
Literature	Possible References: • Clara Löh: Geometric • Thorsten Camps, Volk binatorische und die g	mar G	iroße	Reb	oel, G	Gerhard	l Rosenber	ger: Einführu		

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.	
Prerequisites	There are no further prerequisites.	
Responsible Persons	Hannah Markwig	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-40	Module Title: Gromov-Witten Theory Type of Module: Compulsory Module with Choice									Choice			
ECTS-Points	6												
Workload - Time in Class - Self-Study	Workload: 180 h	The state of the s											
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German or English												
Forms of Teaching and Learning	Lecture 3 SWS + Exercise C	Lecture 3 SWS + Exercise Class 1 SWS											
Content	Enumerative geometr	ту,											
	Moduli spaces of state	ole curv	ves,										
	Moduli spaces of state	ole map	os,										
	 Universal families, 												
	 Forgetful maps, 												
	Gluing maps,												
	Gromov-Witten invariant	ants,											
	Computation of Grom	ov-Wit	ten i	nvari	ants,								
	 Divisor equations, 												
	Kontsevich's formula.												
Objectives	Students are based on their research field of Gromov-W understand important exam them as cut products on mo Gromov-Witten invariants. results of the lecture as well in the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	itten the ple cland claim in the structure as asserted as the claim in the market as t	sses aces uder essi acqu s of t	and and a soft a	enumident re cand exact correcture rk on	merative erative s mast upable plaining	ve geometral invariants er the basic of naming g the prese precise and have learn on strategie	y. The stude and know he algorithms and proving inted connect d independent ed to transfers s on their ownem in critical	ents kr low to for cal the ections. ent har er the r	ow and present culating ssential adling of nethods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Gromov-Witten Theory	L E	f	3	4,5 1,5	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise of tion is written or oral is decided.	ertifica led by	te is the i	to be	e acq	uired a	as coursewo proval by th	ork. Whethe ne Board of I	r the e Examir	xamina- ners.			

Literature	Possible References :
	 Joachim Kock, Israel Vainsencher: An invitation to quantum cohomology: Kontsevich's formula for rational plane curves. Birkhäuser 2007.
	 Ravi Vakil: The moduli space of curves and Gromov-Witten theory. Enumerative invariants in algebraic geometry and string theory. Lecture Notes in Mathematics, 1947. Springer 2008.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Algebraic Geometry is required.
Responsible Persons	Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-50-50	Module Title: Hyperbolic Geometry: Axion Algebraic	natic,	Refle	ectio	n Ge	ometrio		of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h										
Duration	1 Semester										
Frequency	not regularly	not regularly									
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS									
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.										
Objectives	lute and hyperbolic planes) in particular, they have learn etry, which rarely appears in also deepened their knowled are able to name and prove the relationships presented. In the exercise classes they the terms, statements and manufactured in the second in the exercise classes they	In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
		must l	nave	bee	n acc	uired.	Whether th	e examination			
Literature	examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. Possible References: • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.										

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Geometry is helpful but not required.
Responsible Persons	Hermann Hähl, Hannah Markwig

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-01	Module Title:Type of Module:Functional AnalysisCompulsory Module with Choice												
ECTS-Points	9												
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-St 180 h	Self-Study: 180 h					
Duration	1 Semester												
Frequency	regularly												
Term	1-3												
Language of Instruction	German or English												
Forms of Teaching and Learning	ecture 4 SWS + Exercise Class 2 SWS												
Content	Hahn-Banach theorer Closed graph theoren	 Normed spaces, Banach spaces, Dual spaces. Hahn-Banach theorem, uniform boundedness principle. Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. Compact Operators, normal operators, spectral theorems. 											
Objectives	The students are aquainted dimensional spaces and car stand the complex of proble analytical problems. The stu the lecture as well as assess In the exercise classes they the terms, statements and m on new problems, to analys team. They are capable of p discourse.	applyms of dents ing an have a ethods e then	ther spectare of the spectare	n to ctral capal plain ired a he le de	probitheorole of the correction of the correctio	lems in and finaming the presention of the present	n analysis a can use its ng and prov sented conn precise an have learn dution strate	nd geometry results for ing the esse ections. d independe ed to transfe egies on the	y. They the sole that re ent har er the re eir own	y under- lution of esults of adling of nethods or in a			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Functional Analysis	L E	f f	4 2	6 3	yes	K o. mP o. H	90-180 o. 20-30	g	100			
	In this module an exercise co examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether the	e examination					

	B
Literature	Possible References :
	Nicolas Bourbaki: Topological vector spaces. Springer 1987.
	 Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014.
	Harro Heuser: Funktionalanalysis. Teubner 2006.
	Markus Haase: Functional analysis. American Mathematical Society 2014.
	Peter D. Lax: Functional analysis. Wiley 2002.
	Gert Kjaergaard Pedersen: Analysis now. Springer 1995.
	Walter Rudin: Functional analysis. McGraw-Hill 1991.
	Dirk Werner: Funktionalanalysis. Springer 2011.
	Kosaku Yosida: Functional analysis. Springer 1995.
	Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus Numerical Mathematics and Optimisation.
Prerequisites	There are no prerequisites.
Responsible Persons	Carla Cederbaum, Anton Deitmar, Gerhard Huisken, Reiner Schätzle
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-02	Module Title: Non-Linear Functional Analy	rsis						of Module:	le with	Choice		
ECTS-Points	9							<u> </u>				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:				
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3	1-3										
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS											
Content	Differentiation and integration in Banach spaces.											
	Compact, coercive, p	Compact, coercive, proper mappings and gradient mappings.										
	Fredholm mappings.											
	Continuity method.											
	Degree of mapping.											
	Fixed point theorems											
	Variational inequalitie	S.										
	Monotone operators.											
Objectives	Students master the differentional analytical methods for students are capable of national assessing and explaining the In the exercise classes, stumethods they have learnt and to present their problem solves.	solvin ming a e prese dents l d can a	g not ind p ented have apply	n-line rovir I con acq ther	ear	quation e esse ons. confic epend	ns in infinite ntial results lence in the ently to othe	dimensiona of the lect technical her problems. n problems	al spacure as andling They	es. The well as g of the are able		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Non-Linear Functional Analysis	ü	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise c examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	Melvyn Berger: Nonlinearity in Functional Analysis. Elsevier 1977.
	Klaus Deimling: Nonlinear Functional Analysis. Springer 1985.
	 Eberhard Zeidler: Nonlinear Functional Analysis and its Applications I. Fixed-Point Theorems. Springer 1986.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	The Integration and Measurement Theory module and the Functional Analysis module must have been successfully completed.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-03	Module Title: Operator Theory Type of Module: Compulsory Module with Choice											
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: 270 h							tudy:				
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS											
Content	Operator semigroups	Operator semigroups and abstract Cauchy problems.										
	Theorem of Hille-Yosi	da.										
	Applications of concre	ete evo	lutio	n eq	uatio	ns.						
	Spectral theory of ser	nigrou	os ar	nd th	eir ge	enerato	ors.					
	Asymptotic of semigroups	oups.										
	Applications:											
	Semigroups ofSemigroups of		-	•		differer	tial equatio	ns;				
	Semigroups of		-		1115,							
Objectives	Students have understood the able to deal with concrete everyosedness using the Hille-You The students are capable of assessing and explaining the In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	olution osida the naming present the first the second t	equal heory g and ented acqui s of t and t	ation: em a d pro l con ired a he le o wo	s in the contraction of the cont	nis absiscuss the essons. ons. ofident, e. They	tract form. The qualitate sential resurprecise and have learn an strategies.	They are abletive behaviour lts of the lec dindepende ed to transfers on their owners in critical transfers.	e to prour of so ture as ent har er the r	ove well- olutions. s well as adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Operator Theory	L	f	4	6	yes	wr. o.	90-180 o. 20-30	g	100		
		ü	f	2	3		or.	0. 20-30				
	In this module an exercise or examination the coursework oral is decided by the instruc	must h	nave	beei	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	Bruce Blackadar: Operator algebras. Springer 2006.
	 Klaus Jochen Engel, Rainer Nagel: One-parameter semigroups for linear evolution equations. Springer 2000.
	Klaus Jochen Engel, Rainer Nagel: A short course on operator semigroups. Springer 2006.
	Gert Pedersen: Analysis now. Springer 1995.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Anton Deitmar, Rainer Nagel, Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-04	Module Title: Operator Algebras		Type of Module: Compulsory Module with Choice							
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h							
Duration	1 Semester									
Frequency	not regularly	ot regularly								
Term	1-3									
Language of Instruction	German or English	erman or English								
Forms of Teaching and Learning	Lecture 4 SWS									
Content	 Spectral theory in Ba Commutative Banac Gelfand-Naimark. The spectral theorem Operator topologies a Kaplansky's density t Von Neumann algebr struction of examples The axiomatics of C* 	spaces and their spectral nach algebras. h algebras and the reference for normal operators of and von Neumann's bicor heorem. as and their classification is - and W^* -algebras, the tand the representation the	presentation theorem of Gelfand and a Hilbert space.							
Objectives	the theory of operator algebroady using the example of volume recognise how taking a high different questions to be denaming and proving the essented connections. In the exercise classes they the terms, statements and non new problems, to analyse	oras. They have learnt the non-Neumann algebras and the point of view, i.e. the alt with and solved simult sential results of the lecture have acquired a confidenthods of the lecture. The them and to work on solution Neumand Neuma	central concepts, results and methods of the interplay between algebra and topoloid their classification. The students also axiomatic nature of the problem, allows taneously. The students are capable of the ure as well as assessing and explaining ant, precise and independent handling of the they have learned to transfer the methods attion strategies on their own or in a team. Stary, defend them in critical discourse.							

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Algebras	L	f	4	6	yes	wr. o.	90-180 o. 20-30	g	100
		ü	f	2	3		Oi.	0. 20-30		
	In this module an exercise cer examination the coursework noral is decided by the instructor	nust l	nave	beei	n acc	uired.	Whether the	e examinatio		
Literature	Possible References :									
	Bruce Blackadar: Operator algebras. Springer 2006.									
	Ola Bratelli, Derek Rob	inson	: Op	erato	or Alg	gebras	and Quantu	m Physics.	Springe	er 1997.
	Richard Kadison, John IV. AMS 1997.	Ring	rose	: Fur	ndam	entals	of the Theo	ry of Operat	tor Alge	ebras I -
	Gert Pedersen: Analys	is no	w. Sp	oring	er 19	95.				
	• Shoichiro Sakai: C^* - a	nd W	*-Al	gebra	as. S	pringe	r 1998.			
	Masamichi Takesaki: T	heor	y of C	Oper	ator .	Algebra	as I - II. Spri	nger 2002.		
Transfer	The module belongs to the Stuematical Physics. Taking into included in the Sections Studicialisation, in accordance with	acc Foo	ount <i>us, A</i>	the I <i>dva</i>	chos nced	en per Knowl	sonal Study <i>edge in Ma</i> i	/ Specialisa thematics o	tion, it <i>Electi</i>	can be
Prerequisites	The content of the Functional	Anal	ysis r	nodı	ıle is	prerec	quisite for pa	articipation i	n this n	nodule.
Responsible Persons	Ulrich Groh, Rainer Nagel									
Abbreviations: Grading System : g	graded, ng=not graded									

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-05	Module Title: Ergodic Theory							of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:			
Duration	1 Semester	1 Semester									
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS										
Content	Topological and measure-theoretical dynamical systems.										
	Recurrence and mixing properties.										
	Ergodic theorems of \(\)	on Ne	uma	nn a	nd Bi	rkhoff.					
	Spectral theory of the	Koopr	man	oper	ator.						
	Operators with discre	te spe	ctrun	n (Ha	almos	s-von N	leumann)				
	Applications in stocha	stics a	and n	umb	er th	eory.					
Objectives	The students have familiaris of ergodic theory. They have and topology using the example analytical perspective makes ously. The students are capa well as assessing and explais In the exercise classes they the terms, statements and more on new problems, to analyse They are able to present the	re exposed policy in exposed policy of the contraction of the contract	erier dyn ssible nam ne pr acqu s of t	nced amic e to ning esen ired he le	the part of the pa	orofour ems a with a proving connec onfident e. They a solution	nd interplay nd their class nd solve va the essentitions. precise and have learn on strategies	between messification. Trious probletial results of dindependent of transfers on their own.	reasure The fur The sir The le The le The har The r The r The r	e theory nctional- nultane- cture as ndling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Ergodic Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module on suspice -	E	f to in	2 to b	3	u iro d			ioins*:	n in the	
	In this module an exercise of examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	uired.	Whether th	e examination			

Literature	Possible References :
	 Manfred Einsiedler, Thomas Ward: Ergodic Theory with a View Towards Number Theory. Springer 2011.
	 Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015.
	Paul Halmos: Lectures on Ergodic Theory. Martino Fine Books 2013.
	Marcelo Viana, Krerley Oliveira: Foundations of Ergodic Theory. CUP 2016.
Transfer	The module belongs to the <i>Study specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is Prerequisite for participation in this module.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-06	Module Title: Control Theory							of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	S							
Content	 Introduction to finite-dimensional linear control systems with examples from mechanics. Controllability, observability, stabilisability. Kalman criterion. Feedback systems. Stabilisability through feedback. Examples. Introduction to infinite-dimensional control theory. Mathematical framework and examples. 										
Objectives	They are able to use the the are capable of naming and and explaining the presented in the exercise classes they the terms, statements and mon new problems, to analyse	The students learn important foundations of finite- and infinite-dimensional control theory. They are able to use the theory in areas of application such as mechanics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Control Theory	L E	f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature	Possible References :
	Hans W. Knobloch: Lineare Kontrolltheorie. Springer 1985.
	 Hans W. Knobloch, Alberto Isidori, Dietrich Flockerzi: Topics in control theory. Birkhäuser 1993.
	Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992.
	Rurth F. Curtain, Hans Zwart: An Introduction to Infinite Dimensional Systems Theory. Springer 1995.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Rainer Nagel
Abbreviations:	Laraded na-net graded

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title: Linear Control Theory							of Module:	a with	Choice
ECTS-Points	Linear Control Theory Compulsory Module with Choice 6									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h 120 h									
Duration	1 Semester						'			
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	ass 2	SWS	3						
Content	and processes. The underlyi but also, in its abstract form, this lecture, finite-dimensiona analysis and linear algebra is resulting criteria for stabilisab	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.								
Objectives	rienced and understood the ir analysis and their benefits for proving the essential results of connections. Students will be in the subject area. In the exercise classes they in the terms, statements and me	In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Linear Control Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruct	rtifica nust h	te is	to be beer	acc acc	quired.	Whether the	e examination		
Literature	Possible References :									
	Hans Wilhelm Knobloo	h, Hu	ibert	Kwa	kerr	naak: L	ineare Kont	rolltheorie. S	Springe	er 1985.
	Jerzy Zabczyk: Mathe	matica	al Co	ntrol	The	ory. Bi	rkhäuser 19	92.		
	Ruth F. Curtain, Hans Springer 1995.	Zwar	t: An	Intro	oduc	tion to	Infinite-Dim	ensional Sy	stems	Theory.

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.	
Prerequisites	Knowledge of the modules Analysis and Linear Algbra is sufficient.	
Responsible Persons	Rainer Nagel	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-08	Module Title: Spectral Theory of Positive Operators Type of Module: Compulsory Module with Choice								Choice	
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: Time in Class: 60 h						Self-St 120 h	udy:		
Duration	1 Semester						·			
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	ass 2	SWS	3						
Content	matrices, positive linear map properties are analysed. The powers and means, can ther	Starting from the classical theorems of Perron and Frobenius on the spectrum of positive matrices, positive linear mappings to C^* - and W^* -algebras and their spectral and algebraic properties are analysed. The ergodic properties of these operators, i.e. the convergence of powers and means, can then be derived from these. We then discuss the generalisation to operator semigroups. Applications of the theory can be found in mathematical physics, among others.								
Objectives	Students learn the basic spectral properties of positive operators on C^* - and W^* -algebras and the connections with non-commutative ergodic theory. In the seminar following the lecture, students can work on topics that lead to a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Spectral Theory of Positive Operators	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners. – The ercise classes; in this case, 5.	writter modul	e ma	ау ех	cept	ionally	be offered	by the lectu	rer witl	hout ex-
Literature	Possible References :									
	Tanja Eisner, Markus Theory. Springer 2015		e, Ra	ainer	Nag	gel : O	perator The	eoretic Aspe	cts of	Ergodic
	Ulrich Groh: Spectral Preprint.	Theo	ry of	Con	nplet	ely Pos	sitive Maps	on C^* - and	W^* -A	lgebras.
Transfer	The module belongs to the Steematical Physics. Taking intincluded in the Sections Studies cialisation, in accordance with	o acc ly Foc	ount us, A	the A <i>dva</i>	chos nced	en per Know	sonal Stud ledge in Ma	y Specialisa <i>thematics</i> o	tion, it Elect	can be
Prerequisites	Knowledge from functional ar	nalysis	s and	l ope	rato	algeb	ras is assur	ned.		

Responsible Persons

Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

T=tutorial, P=project, S=seminar, IC=inverted classroom

Module Number: MAT-55-09	Module Title: Non-Commutative Ergodic Theory							of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h					Self-Si 180 h	Self-Study: 180 h			
Duration	1 Semester						'				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS										
Content	discussed. Then, based on confined. With the help of the so- dynamical systems can be cl	Firstly, the essential basic concepts and properties of C* and W* algebras are introduced and iscussed. Then, based on commutative theory, non-commutative dynamical systems are dened. With the help of the so-called cross products it is then shown how such non-commutative lynamical systems can be characterised with the help of the group representation. The significance in mathematical physics is always emphasised.									
Objectives	The students have learnt the central concepts, results and methods of non-commutative ergodic theory, i.e. of dynamical systems on operator algebras. They have experienced the fascinating interplay between the structure of von Neumann algebras and the (asymptotic and spectral-theoretical) behaviour of operators on these algebras. The students realised how an axiomatic and structural point of view makes it possible to treat and solve different problems simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	am (min)				Weight for Grade	
	Non-Commutative Ergodic Theory	Lü	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruct	nust h	ave	bee	n acc	uired.	Whether th	ie examinatio			
Literature	Possible References :										
	 Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015. Bruce Blackadar: Operator Algebras. Springer 2006. Alai Guichardet: Systèmes dynamiques non commutatifs. Astérisque 13-14 1974. Dirk Werner: Funktionalanalysis. Springer 1995. Volker Runde: A Taste of Topology. Springer 2005. 										

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Good knowledge of functional analysis and basic knowledge of topology. Interest in mathematical quantum mechanics.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-10	Module Title: Pseudo Differential Operators							of Module:	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 120 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS									
Content	Fourier transform and	Fourier transform and Sobolev spaces.									
	Pseudodifferential operation	Pseudodifferential operators on manifolds.									
	Finite propagation vel	Finite propagation velocity.									
	Fredholm operators a	Fredholm operators and elliptic complexes .									
	The heat conduction I	kernel	and	the l	ocal i	index tl	neorem.				
	The Atiyah-Bott-Patoo										
	Von Neumann algebra		l repi	eser	ntatio	ns.					
	The L2 index theorem	1.									
Objectives	geometry. They will understated how both merge into the most transition from one to the other will be able to use theoretical will learn to use modern apport The students are capable of as assessing and explaining	Students learn basic techniques in the theory of elliptic differential operators and spectral geometry. They will understand the connection between differential and integral operators and how both merge into the more general calculus of pseudo-differential operators and how the transition from one to the other results in solution techniques for differential equations. They will be able to use theoretical approaches to solve specific problems in concrete cases. They will learn to use modern approaches of L2 theory to prove deep group theoretical statements. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Pseudo Differential Operators	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instru	uctor with a	pprova	l by the	

Literature	Possible References :
	 Peter B. Gilkey: Invariance theory, the heat equation, and the Atiyah-Singer index theorem. Publish or Perish 1984.
	 Wolfgang Lück: L2-invariants: theory and applications to geometry and K-theory. Springer 2002.
	Michael Taylor: Pseudo differential operators. Springer 1974.
	Man-Wah Wong: An introduction to pseudo-differential operators. World Scientific Publishing 2014.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Functional Analysis is assumed.
Responsible Persons	Anton Deitmar
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title: Introduction to Harmonic An	alvsis						of Module: ulsory Modu	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Self-St	rudy:								
Duration	1 Semester										
Frequency	not regularly	not regularly									
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SW	S							
Content	Fourier series and Fourier	urier tr	ransf	orma	ition.						
	Plancherel theorem a	nd inve	erse	theo	rems	i.					
	Poisson summation for	ormula	١.								
	tempered distribution	S.									
		Additionaly a selection of the following topics will be covered:									
	LCA groups; general Fourier	tranef	orms	ation:							
	general Fourier transformation;non-abelian groups and representations;										
	- Sobolev-space	-		•		,					
	 Singular integral 	als;									
	 Poisson integra 	ls.									
Objectives	Students can combine algebraic and analytical methods and apply them to solve problems. They recognise the interplay between the properties of functions and their Fourier transforms and can apply the knowledge gained from this to questions in physics, analysis and number theory. They understand the interaction of group theory and analysis and gain deep insights into various function spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Harmonic Analysis	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise c examination the coursework oral is decided by the instruc	must I	have	bee	n acc	uired.	Whether th	e examination			

Literature	Possible References :
	Anton Deitmar: A first course in harmonic analysis. Springer 2005.
	 Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press, 1970.
	Elias M. Stein, Guido Weiss: Introduction to fourier analysis on euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-12	Module Title: Harmonic Analysis in Euclidean Space Type of Module: Compulsory Module with Choic								Choice		
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SW	S							
Content	 Fourier transformation Covering-, decomposi Singular integrals, Poi Hardy- and BMO-space 	tion- a sson i	nteg	rals.				lley theory.			
Objectives	The students got to know the euclidean space. The student lecture as well as assessing In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	its are and ea have a ethoda them	capa xplain acqu s of t and t	able ning ired a he le o wo	of na the p a cor cture rk on	ming a resent ofident, e. They o solution	and proving ed connecti precise an have learn on strategie	the essentia ons. d independe ed to transfe s on their ow	I result ent han er the n n or in	dling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	rpe of Course atus WS CTS oursework rpe of Exam rading rading									
	Harmonic Analysis in										
	Euclidean Space	E	f	2	3	,	or.	o. 20-30			
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	have	beei	n acc	uired.	Whether th	e examination			

Literature	Possible References :
	Charles L. Feffermann, Elias M. Stein: H^p spaces of several variables. Acta Mathematica 129, pp. 137-193, 1972.
	 Christopher D. Sogge: Fourier integrals in classical analysis. Cambridge University Press 2017.
	 Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press 1970.
	Elias M. Stein: Harmonic analysis. Princeton University Press 1993.
	Elias M. Stein, Guido Weiss: Introduction to Fourier analysis on Euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the modules functional analysis and Introduction to Harmonic Analysis are a prerequisite for participation in this module.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, P=project, S=seminar, IC=inverted classroom} \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-13	Module Title: Harmonic Analysis on Abelia	n Gro	ups					of Module: ulsory Modul	le with	Choice		
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h											
Duration	1 Semester											
Frequency	not regularly											
Term	1-3	1-3										
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise CI	Lecture 4 SWS + Exercise Class 2 SWS										
Content	Convolution algebras,	 Locally compact groups, existence and uniqueness of Haar measures. Convolution algebras, Banach algebras, the Gelfand-Neumark theorem. LCA groups, Pontryagin duality, Plancherel theorem, structure theory of LCA groups. 										
Objectives	monic analysis and know how topological/analytical/geomet braic structures such as C^* -cories. The students are capa well as assessing and explair In the exercise classes they be the terms, statements and me	The students have become familiar with the central concepts and methods of abstract harmonic analysis and know how to use them. They have understood the connection between topological/analytical/geometric concepts such as LCA groups and their expression in algebraic structures such as C^* -algebras and are able to apply this way of thinking to other theories. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Harmonic Analysis on Abelian Groups	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise ce examination the coursework oral is decided by the instruct	nust ł	nave	beer	n acc	uired.	Whether th	e examination				
Literature	Possible References :											
	Anton Deitmar: A first	cours	e in I	Harm	nonic	Analys	sis. Springe	er 2005.				
	Anton Deitmar, Siegfrig	ed Ec	hterh	off:	Princ	iples o	f Harmonic	Analysis. S	pringe	r 2008.		
	Edwin Hewitt, Kenneth	Ross	: Ab	strac	t har	monic	analysis. V	ol. I. Springe	er 1979	Э.		
	Walter Rudin: Fourier	analys	sis oı	n gro	ups.	John \	Wiley 1990.					

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Functional Analysis is a prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-14	Module Title: Harmonic Analysis on Gener	al Gro	ups					of Module: ulsory Modul	le with	Choice			
ECTS-Points	9						'						
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 180 h												
Duration	1 Semester	1 Semester											
Frequency	not regularly												
Term	1-3												
Language of Instruction	German	German											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS											
Content	Representation theory	Representation theory of compact groups, Peter-Weyl theorem.											
	Representation theory	Representation theory of general groups.											
	• trace formula and applications to the Heisenberg group and $SL_2(\mathbb{R}).$												
Objectives	harmonic analysis and know understand its far-reaching i are capable of naming and p and explaining the presented In the exercise classes they the terms, statements and m on new problems, to analyse	The students have familiarised themselves with the deeper concepts and methods of abstract harmonic analysis and know how to use them. They have mastered the trace formula and understand its far-reaching implications, also for other areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Harmonic Analysis on General Groups	L E	f	4		yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise context examination the coursework oral is decided by the instruction	must	have	bee	en acc	quired.	Whether th	e examination					
Literature	Possible References :												
	Anton Deitmar, Siegfr	ed Ed	hterh	noff	Princ	ciples o	f Harmonic	Analysis. S	pringe	r 2008.			
	Gerald B. Folland: A cematics. Boca Raton		in a	bstr	act ha	armonio	analysis. S	Studies in Ad	dvance	ed Math-			
	Michael E. Taylor: No.		nutai	tive	Harm	ionic Ai	nalysis. AM	S 1986.					
Transfer	The module belongs to the sinto account the chosen per Study Focus, Advanced Knowith the restrictive requirements	rsonal <i>wledg</i>	Stu e in	dy \$ <i>Mat</i>	Specia <i>hema</i>	alisation <i>tics</i> or	n, it can be <i>Elective Sp</i> e	included in	the S	Sections			

Prerequisites	In terms of content, the module Harmonic Analysis on abelian groups is a prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-15	Module Title: Selected Chapters from Ope	Module Title:Type of Module:Selected Chapters from Operator TheoryCompulsory Module with Choice								Choice		
ECTS-Points	9						·					
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h Self-Study: 180 h										
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	S								
Content	Spectral theory of resculus.	Spectral theory of restricted and unrestricted linear operators, especially spectral calculus.										
	Spectral theory of pos	sitive o	pera	tors	– Pe	rron-Fr	obenius the	ory.				
	Spectral theory for op	Spectral theory for operators of ergodic theory.										
Objectives	Students master the concept lus. They can then apply this behaviour. They are also all such as stochastics, ergodic and proving the essential resented connections. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	to cor ole to theor sults of have a ethode	recory or the acquision of the acquisions and t	e ope gnis num le lect ired he le o wo	erato e cro aber t ture a cor ecture ork or	rs and oss-con theory. as well offident, e. They osolution	discuss pro nections to The stude as assessi precise an have learn on strategie	perties such other math hits are capang and explained independed to transfers on their ow	as asy ematicable of aining ent har er the r	mptotic al fields naming the pre- adling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Selected Chapters from	L	f	4	6	yes	wr. o.	90-180	g	100		
	Operator Theory	E	f	2	3	,55	or.	o. 20-30	9			
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	have	bee	n acc	quired.	Whether th	e examination				
Literature	Possible References :											
	Klaus Jochen Engel, equations. Springer 2		er Na	agel:	One	e-parar	neter semiç	groups for li	near e	volution		
	Markus Haase: The F		nal (Calc	ulus f	or Sect	orial Opera	tors. Birkhä	user 20	006.		
Transfer	The module belongs to the Mathematical Physics. Takin be included in the Sections Specialisation, in accordance	ng into <i>Study</i>	acc Foc	ount <i>us</i> , <i>i</i>	the o	chosen <i>nced K</i>	personal S nowledge i	Study Specia n Mathemat	ılisatioı <i>ics</i> or	n, it can <i>Elective</i>		

Prerequisites	Solid knowledge of operator theory, in particular Hille-Yosida theory for operator semigroups is a prerequisite.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-21	Module Title:	ntial Fa	au ati	000				of Module:	طائند ما	Chaina			
ECTS-Points	Introduction to Partial Differe	muai Ec	quali	OHS			Compu	lisory Modul	ie with	Choice			
Workload - Time in Class - Self-Study	9 Workload: 270 h	Workload: Time in Class: Self-Study:											
Duration	1 Semester												
Frequency	regularly												
Term	1-3												
Language of Instruction	English												
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	cture 4 SWS + Exercise Class 2 SWS											
Objectives	 Harmonic functions. Maximum principles. Sobolev spaces. L² theory. Important examples (Fundamental solution Weak solutions of elli The students got to know a mental for many fields, like strong connections to geom central terms, results and m these methods in advanced central results of the lecture In the exercise classes they the terms, statements and m on new problems, to analysteam. They are capable of p discourse. 	central numeric etry, ar ethods course as well have a ethods e them	brancs or lires. Tas a acquire of the and	ch of stocking of the stocking red at the left of the stocking red at the	f anachas of the part stude ssing a cor cture work	alysis, tics. A le lecturial differents and entificient, and entificient, on so	whose term lso evolution re. The studerential equal e capable of explaining the precise and have learned	s and methonary equations are actions and actions and actions and actions and actions are presented independed to transfergies on the	ons, wo cquain are able nd pro- conne ent har er the r eir own	ho have ted with e to use ving the ections. adling of methods or in a			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Introduction to Partial Differential Equations	Little Type of Course Coursework Coursework Coursework Coursework Coursework Coursework Coursework Coursework Coursework Weight for Grading											
	In this module an exercise or examination the coursework oral is decided by the instruc	must h	ave I	beer	acq	uired.	Whether the	e examinatio					

Literature	Possible References :
	 Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010.
	 David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001.
	 Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus Numerical Mathematics and Optimisation.
Prerequisites	There are no further prerequisits.
Responsible Persons	Gerhard Huisken, Reiner Schätzle
	graded, ng=not graded IT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-22	Module Title: Type of Module: Partial Differential Equations Compulsory Module with Cho								Choice						
ECTS-Points	9														
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:							
Duration	1 Semester	1 Semester													
Frequency	not regularly														
Term	1-3	1-3													
Language of Instruction	German or English	German or English													
Forms of Teaching and Learning	Lecture 4 SWS														
Content	 Schauder estimates. Calderon-Zygmund externack's inequality. Hölder regularity. Viscosity solutions. Existence of solutions. Evans-Krylov theoren 	accor		to P	erron	1.									
Objectives	After the students have lear ferential Equations, this kno to current research question results of the lecture as well In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	wledges. The as ass have a thods them a	e is o stuc essi acqu s of t and t	deepolents Ing ar Ired a Ing le Ing wo	ened are nd ex a cor cture rk on	I. Stud capabl cplainin nfident, e. They n solution	lents are property of naming the prese and have learn on strategies.	epared for a and proving nted conned d independe ed to transfe s on their ow	and into g the e ctions. ent har er the r	roduced essential adling of nethods a team.					
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Partial Differential Equations	г Type of Course	4 Status	SMS 4	9 ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade					
	In this module an exercise or examination the coursework oral is decided by the instruc	ü ertifica must h	nave	beei	n acc	uired a	Whether the	e examination	icipatic	n in the					

Literature	Possible References :
	 Luis Angel Caffarelli, Xavier Cabre: Fully nonlinear elliptic equations. American Mathematical Society 1995.
	 Michael G. Crandall, Hitoshi Ishii, Pierre-Louis Lions: User's Guide to Viscosity Solutions of second Order Partial Differential Equations. Bulletin of the American Mathematical Society 27, No. 1, pp. 1-67, 1992.
	 David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001.
	 Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear elliptic equations.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Instationary Differential Equations, it can be included in the specialisation Numerical Mathematics and Optimisation.
Prerequisites	The content of the module Introduction to Partial Differential Equations is a prerequisite for the participation in this module.
Responsible Persons	Gerhard Huisken, Reiner Schätzle
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-24	Module Title: Nonlinear Elliptic Partial Diffe Surface Theory	erentia	l Equ	uatio	ns in	Minima		of Module: ulsory Modu	le with	Choice		
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-S	tudy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS										
Content	The course will consider PDE aspects of minimal surfaces, beginning with the existence theory for minimal graphs with prescribed boundary data. Emphasis will be placed on the De Giorgi-Nash estimate, which is one of the key achievements of 20th Century mathematics, and is foundational in the study of quasilinear elliptic and parabolic equations. We will also explore connections between minimal surfaces and the Allen-Cahn equation, which is a semilinear equation arising in the theory of phase transitions. Here the focus will be on rigidity results for entire solutions (namely the Bernstein problem and closely related De Giorgi conjecture) and their use in proving regularity via rescaling.											
Objectives	The students obtain an adva understanding of connection will acquire an array of new on objects governed by nonlapplications of Sobolev theorem Moser iteration, and the us and when these techniques prove the essential statement developed in the lecture and critically challenge the currer in the exercise classes they the terms, statements and mon new problems, to analysteam. They are capable of prediscourse.	s betwood technine are aputs and to put to the theorem that a technology is between the technical technology is between the technology is between the technical	reen nique differ caling conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica conciplica concipli	this es for ential and to a cepts to a esea ired a de to	theory estail equal to a strong large arching correcture work	ry and ablishing actions a pactner produced by the learn frame on the soft control of the contro	profound page quantitated are the case arguments. Students or oblem. The cture as well are cific area precise are have learn lution strate.	roblems in g cive and qua chniques incl nts, Stampa s will be able hey are able ell as to expl y are able to t. Id independent and independent and independent and independent and independent and independent and independent	eometalitative and commentalitative and commentalit	ry. They control dvanced teration, ssess if me and context ribe and adding of methods or in a		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Nonlinear Elliptic Partial Differential Equations in	Type of Course	+ Status	SMS 4	e ECTS	Coursework	Type of Exam o.	Dur. of Exam (min)	Grading	Weight for Grade		
	Minimal Surface Theory	Е	f	2	3	yes	or.	o. 20-30	g	100		
	In this module an exercise or examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	Lawrence C. Evans: Partial Differential Equations. AMS 2010.
	 David Gilbarg, Neil Trudinger: Elliptic partial differential equations of second order. Springer 1083.
	David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications. Siam 2000.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basics of linear elliptic PDE (Schauder theory, existence for Dirichlet problem) are desirable but not completely necessary.
Responsible Persons	Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom$

Status : o=obligatory, f=facultative

Module Number: MAT-55-25	Module Title: Introduction to Partial Differe	ntial E	quat	ions	– Pa	rt 1		of Module: ulsory Modul	e with	Choice	
ECTS-Points	5										
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	_	lass	:		Self-St 105 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	lass 1	SW	S							
Content Objectives	 Harmonic functions. Maximum principles. Sobolev spaces. The students have familiaris	Maximum principles. Sobolev spaces.									
	as numerics and stochastic methods of linear partial diff the more advanced courses results of the lecture as well in the exercise classes they the terms, statements and mon new problems, to analyse	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								ults and thods in ssential adling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Partial Differential Equations – Part 1	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	nave	bee	n acc	uired.	Whether the	e examinatio			
Literature	Possible References: Lawrence C. Evans: 2010. David Gilbarg, Neil S. Springer 2001. Olga A. Ladyzenskaja ear equations of para	Trudi	nger /olod	: Elli A. S	ptic p	oartial o	differential e	equations of	secon	d order.	

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. In combination with one of the modules 'Numerics of Stationary Differential Equations' or 'Numerics of Nonstationary Differential Equations', it can be included in the Specialisation Numerical Mathematics and Optimisation. The module is part of the module Introduction to Partial Differential Equations and cannot be taken together with this module.
Prerequisites	There are no further prerequisites.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-26	Module Title: Partial Differential Equations Yamabe Problem	in Co	nforn	nal G	ieom	etry: th		of Module: ulsory Modu	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	_	lass	:		Self-St 120 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	English	English									
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS									
Content	conformally equivalent to one with constant sectional curvature. The appropriate higher-differential problems as a consentational curvature. The appropriate higher-differential problems are sectional curvature. The appropriate higher-differential section in 1960. The flaws in his proof were fixed only in 1984, by combining the works of Trudinger, Aubin, Schoen, and others. This course provides an overview of the complete proof of the Yamabe problem, which is extremely rich in techniques of calculus of variation, geometric analysis, and elliptic partial differential equations. It also employs crucial results of mathematical relativity, such as the Positive Mass Theorem, which is motivated by physics but possesses powerful analytical consequences. Moreover, basic results for non-compact manifolds or manifolds with boundary and the parabolic analogue, the Yamabe flow, will be discussed.										
Objectives	The students can state the dents are familiar with basic (sub-critical) Yamabe energy can study the conformal Grelem. The students are also frenowned Positive Mass The can provide the main ideas of ter elements of the minimal statements and concepts frolecture and to put it into a lar the current state of research Through homework assignmand independent acquaintar lectures. They learn how to to develop solution strategie solutions and to stand for the	methor function funct	ods of ional unction with still proof ee the lecture special and e the three incomes and e the three incomes and e the three incomes and e the incomes and e	of the san on a the linki of the cory. Ure a cork. Cific a core e no e se own a	e calced the nd rendered notice Poor The second tions are all the next method with the next next next next next next next nex	culus of associate it on of m to the sitive N are all as to y are all asses; , statemods to within a	variation, parated elliption to the solvantes in mathematics of the solution o	particularly of contents of the semi-linear street of the yamabe and, in page and prove context defibe and critical velop a contents of methods exems, to ana	concer or PDE Yamal lativity proble particul e the e evelope cally cl fident, kplaine lyse th	ning the s. They be proband the em; they ar, massessential ed in the nallenge precise, d in the em and	
Requirements for Obtaining Credit, Grading, Weight if applicable Title Solutions and to stand for them in a critical discourse if necessary. Conumber 1							Grading	Weight for Grade			
	Partial Differential Equations in Conformal Geometry: the Yamabe Problem		f	2	3	yes	Wr. O. Or.	o. 20-30	g		
	Whether the examination is Board of Examiners.	writte	n or	orai	is de	ciaea	by the instri	uctor with a	pprova	u by the	

Literature	Possible References :
	John M. Lee, Thomas H. Parker: The Yamabe problem. Bull. Am. Math. Soc., New Ser. 17, 37–91, 1987.
	 Richard Schoen, Shing-Tung Yau: Lectures on differential geometry. International Press 1994.
	Thierry Aubin: Some nonlinear problems in Riemannian geometry. Springer 1998.
	 Michael Struwe: Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems. Springer 2008.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Geometry in Physics and Introduction to Partial Differential Equations is assumed.
Responsible Persons	Carla Cederbaum

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-27	Module Title: Type of Module: Fully Non-Linear Elliptic Equations Compulsory Module with									Choice	
ECTS-Points	5						'				
Workload - Time in Class - Self-Study	Workload: 150 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 1 SWS									
Content	 Solution of general fully non-linear equations with elliptic equations. Solution of the Monge-Ampere equation. 										
Objectives	The students learn the techniques to successively estimate the supremum, the gradient and the second derivatives of a given solution of a fully non-linear elliptic equation. The students learn how the modulus of continuity of the second derivatives is then estimated using the Evans-Krylov theorem and learn the continuity method, which leads to the existence of a solution. In particular, they are able to apply the methods to general uniformly elliptic equations and to the special, non-uniformly elliptic Monge-Ampere equation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.								students sing the nce of a quations capable plaining		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	Coursework Type of Exam Dur. of Exam (min)					Weight for Grade	
	Fully Non-Linear Elliptic Equations	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruct exceptionally be offered by the points will be awarded for the	must I tor wit he led	have th ap	beei prova with	n acc al by nout	quired. the Bo exercis	Whether the ard of Exar	e examination miners. – Th	on is w	ritten or lule may	
Literature	Possible References :										
	 Luis A. Caffarelli, Jo second-order elliptic e Pure and Applied Mat Luis A. Caffarelli, Jose nonlinear second-orde elliptic, equations. In 209-252. David Gilbarg, Neil S. Springer 1998. 	equation hemate ph Ko er ellip : Con	ons. tics 3 ohn, l otic e nmur	I. M 7,3 p _uis quat nicati	onge op. 3 Nirer ions. ons	e-Ampe 69-402 nberg, II. Co on Pur	ere equatior d. Joel Spruck Implex Mon e and App	n. In: Comn The Dirich ge-Ampere, lied Mathem	let pro and unatics	blem for niformly 38,2 pp.	

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module MAT-60-36 'Fully nonlinear elliptic and parabolic partial differential equations' due to the large overlap in content.
Prerequisites	For participation, the modules Introduction to Partial Differential Equations and Partial Differential Equations are required.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-28	Module Title:Type of Module:Morse TheoryCompulsory Module with Choice									Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	•		Self-St 60 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS	ecture 2 SWS									
Content	Riemannian metrics o Dynamic systems on o Homotopy type of diffe Main approaches of M	 Topology of differentiable manifolds. Riemannian metrics on differentiable manifolds. Dynamic systems on differentiable manifolds. Homotopy type of differentiable manifolds. Main approaches of Morse theory. Outlook on Morse homology. 									
Objectives	Students learn how to analysis using the tools of analysis, in learn how the level surfaces used to obtain statements at to algebraic topology, which The students are capable of assessing and explaining the	n parti of nor bout t analy namin	cula n-deq he h ses g an	r the gene omot the t d pro	theorate topy opoloving	ory of d function type of ogy (of the ess	ynamical syns, so-called manifolds. manifolds)	rstems. In postems. In postems. In postems of the state o	ctions, build a braic m	lar, they can be a bridge nethods.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Morse Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writter	n or	oral	is de	ecided k	by the instru	uctor with a	pprova	I by the	
Literature	Possible References: • John Milnor: Morse Theorems 1961. • Morris W. Hirsch: Differences 1988.										

Transfer	The module belongs to the Study Specialisation Studienschwerpunkt Analysis and Differential Geometry and Algebra and Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of differentiable manifolds and of dynamic systems is helpful.
Responsible Persons	Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-32	Module Title: Selected Chapters from Dyna	amical	Sys	tems	The	ory		f Module: Ilsory Modu	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	•		Self-St 60 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS	ecture 2 SWS									
Content	A selection of the following to	selection of the following topics will be covered:									
	dynamical systems as	dynamical systems as solution flows of ordinary and partial differential equations;									
	isomorphic invariants	of dyn	amic	al sy	/sten	ns, esp	ecially the d	iscrete spec	ctrum;		
	linear skew-product flo	ws;									
	applications in number	theo	ry, co	mbi	nator	rics and	l stochastics	S.			
Objectives	Students are familiar with qu tial equations and the methor functional analysis, operator applicability of abstract mather proving the essential results connections.	ds us theor emat	ed to y an ical o	o an d er conc	alyse godic epts.	them. theor	On the bay, they have students ar	asis of solid e experience e capable o	knowled the of nam	ledge of diverse ing and	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Selected Chapters from Dynamical Systems Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writter	n or	oral	is de	cided I	by the instru	uctor with a	pprova	I by the	
Literature	Possible References :										
	Tanja Eisner, Balint Fa of ergodic theory. Spri				Haas	e, Rair	ner Nagel: (Operator the	oretic	aspects	
	Manfred Einsiedler, Theory. Springer 2011.	nomas	s Wa	rd: E	rgod	lic theo	ry: with a v	iew towards	Numb	per The-	
	David Kerr, Hanfeng 2016.	Li: Er	godi	c the	eory:	indep	endence ar	nd dichotom	nies. S	Springer	

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. It can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Dynamic Systems Prerequisite for participation in this module.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-33	Module Title: Abstract Dynamical Systems							of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	•		Self-St 180 h	tudy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS									
Content	(topological) ergodicity are plied to category theoretical deLeeuw-Glicksberg decomp Furstenberg-Zimmer structuresearch topics are address	The central properties of topological dynamic systems such as minimality, recurrence and topological) ergodicity are repeated. Subsequently, the statements proven there are applied to category theoretical foundations. Important structural results such as the Jacobs-leLeeuw-Glicksberg decomposition, the theorem of Halmos-von Neumann's theorem and the furstenberg-Zimmer structure theory are discussed and generalised. In this context, current esearch topics are addressed and a category-theoretical perspective is developed. Among other things, the application of ergodic theory to number theory and combinatorics is preented.									
Objectives	The students have learnt ho can be developed and further can apply the techniques of theoretical or ergodic-theoret ples of the usefulness of abstand proving the essential resented connections. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	er absing evelope tical) per tract nesults of the tract and	tracted in the contract of the	ed from the ems. emative lectricians in the lectric	om comese a corecture or	oncrete areas, e stude heories as well nfident, e. They n solution	e questions to deal with this have the stude as assessing precise an have learn on strategie	(in number of concrete (bus learnt iments are capang and explaid independent of transfers on their ow	theory (e.g. raportant able of aining ent hare er the ray or in	number- t exam- naming the pre- ndling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Abstract Dynamical Systems	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise continuous examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	quired.	Whether th	e examination			
Literature	Possible References: Tanja Eisner et al.: Op Jan de Vries: Topolog tinuous mappings. De Saunders Mac Lane: Helmut H. Schaefer: I	ical dy Gruy Categ	ynam ter 2 ories	nical 014. for t	syste	ems. A	n introduction	on to the dy	namics er 1998	of con-	

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Dynamical Systems' due to the large overlap in content.
Prerequisites	Solid knowledge of Topology, Functional Analysis and Operator Theory, in particular spectral theory of positive operators is required. operators are assumed. Fundamentals of Ergodic Theory and Category Theory are also very useful, but not strictly necessary.
Responsible Persons	Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-34	Module Title:Type of Module:Introduction to Dynamical SystemsCompulsory Module with Choice											
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: 90 h											
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English	German or English										
Forms of Teaching and Learning	Lecture 2 SWS	ecture 2 SWS										
Content	 Kepler's laws. Equilibrium positions. Stability. Predator-prey model. Poincaré-Bendixson the Limit sets. Periodic orbits. Celestial mechanics. 	 Equilibrium positions. Stability. Predator-prey model. Poincaré-Bendixson theorem. Limit sets. Periodic orbits. 										
Objectives	The students can ask and exferential equations, like e.g.: states or periodic orbits? The lecture as well as assessing	How le	ong o	do ex able	ist m of na	nathem Iming a	atical solution at the street	ons? Are the the character.	ere equ	uilibrium		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Introducation to Dynamical Systems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
Literature	Possible References: Morris W. Hirsch, Step algebra. Academic Pro Vladimir I. Arnold: Ma Carl Ludwig Siegel, Jü	ess 19	974. atical	metl	nods	of clas	ssical mecha	ınics. Sprinç	ger 20 ⁻	10.		

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Dynamical Systems' due to the large overlap in content.
Prerequisites	There are no further prerequisites.
Responsible Persons	Frank Loose

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-41	Module Title: Type of Module: Introduction to Geometric Measure Theory Compulsory Module with Choice											
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German or English	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS										
Content	Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities.											
	Isodiametric inequalit	y.										
	Rademacher's theore	m and	Whi	tney	s em	nbeddir	ng theorem.					
	Surface- and Cosurfa	ce forn	nula.									
	Countable rectifiable	sets, re	ectifia	able	varifo	olds.						
Objectives	Students have familiarised analysis and geometry and various problems. They ha methods of geometric meas courses. The students are cas well as assessing and ex In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	whose ye fame the sure the sapable plaining have a nethods	e co iliaris eory of r g the acqu s of t and t	ncepsed fand and amire president in the left of the le	ts and them can are are corrected as correct	nd meiselves succes ad proved conrections. They a solution	thods can be with the basefully applying the essentions. precise any have learn on strategies	pe successfi asic concept these methential results d independential transfer d to transfers on their ow	ully apts, res nods ir s of the ent har er the r	plied to ults and a further e lecture adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course Status SWS ECTS Coursework Type of Exam Dur. of Exam (min)					Dur. of Exam (min)	Grading	Weight for Grade			
	Introduction to Geometric L f 4 6 Weasure Theory E f 2 3 yes wr. o. 90-180 o. 20-30 g								100			
	In this module an exercise c examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	quired.	Whether th	e examination				

Literature	Possible References :
	 Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992.
	Herbert Federer: Geometric measure theory. Springer 1969.
	Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-42	Module Title: Geometric Measure Theory							of Module: ulsory Modu	le with	Choice	
ECTS-Points	9						Oompt	alsory wodu	ie with	Onoice	
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St	tudy:			
Duration	1 Semester						'				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS									
Content	First and second varia	ation fo	r var	ifold	S.						
	Monotonicity formula.										
	Allard's integral comp	actnes	s the	eorer	n.						
	Lipschitz approximation	on.									
	tilt-excess descent.										
	Allard's regularity thee	orem.									
	General and rectifible	flows.									
	Deformation theorem.										
	Surface minimizing flo	WS.									
Objectives	After having learned the basi this knowledge is deepened recent research. The studen lecture as well as assessing In the exercise classes they the terms, statements and mon new problems, to analysteam. They are capable of p discourse.	I. The ts are cand ex have a tethods e them	stud capa cplair acqui of the	lents able of ning fred he le	will of na the p a cor cture work	be preming a present of the premium	epared for a nd proving ed connecti precise an have learn lution strate	and guided the essential ons. d independed to transfeegies on the	to prob Il resul ent har er the r eir own	olems of ts of the adling of methods or in a	
Requirements for Obtaining Source (min)								Grading	Weight for Grade		
	Geometric Measure Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise contains examination the coursework oral is decided by the instruc	must h	nave	bee	n acc	uired.	s coursewo Whether th	ork. For part e examinatio			

Literature	Possible References :
	Herbert Federer: Geometric measure theory. Springer 1969.
	Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984.
	Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Regarding to content the module Introduction to Geometric Measure Theory is a prerequisite for participation in the module Geometric Measure Theory.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-43	Module Title: Type of Module: Area Minimising Flows Compulsory Module with Choice									Choice		
ECTS-Points	5											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 150 h 105 h											
Duration	1 Semester	1 Semester										
Frequency	not regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	lass 1	SWS	3								
Content	·	 Compactness theorem for integral flows. Regularity of area minimising flows. 										
Objectives	After having learned the essential notions and methods of geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Area Minimising Flows	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise ce tion is written or oral is decid	rtifica	te is	to be	acc							
Literature	Possible References:	surfa	ces a	and f	uncti	ons of	bounded va	ariation. Birk				
Transfer	The module belongs to the sinto account the chosen per Study Focus, Advanced Knowith the restrictive requirements	sonal wledg	Stud e in i	dy S <i>Math</i>	pecia <i>ema</i> :	alisation <i>tics</i> or	n, it can be Elective Sp	included in	n the S	Sections		
Prerequisites	Knowledge from the modules sure Theory is expected.	Intro	ductio	on to	Geo	metric	Measure Tl	neory and G	eomet	ric Mea-		

Responsible Persons	Reiner Schätzle
Abbreviations:	
Grading System : g	=graded, ng=not graded
Examination Type: M	IT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, =tutorial P=project S=seminar IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-44	Module Title: Introduction to Geometric Methods	/leasu	re TI	neory	/ — I	Measur		of Module: ulsory Modul	e with	Choice	
ECTS-Points	5										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 105 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 1 SWS									
Content	Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities.										
	Isodiametric inequalit	y.									
	Rademacher's theore	Rademacher's theorem and Whitney's embedding theorem.									
Objectives	Students have familiarised analysis and geometry and various problems. They have methods of geometric meas courses. The students are cas well as assessing and explinithe exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	whos ve fam ure th apable blainin have a ethod them	e co illiaris eory e of r g the acqu s of t and t	ncepsed to and amir presented a tender to the second a tender to the	ts and them the cand	nd met selves succes nd provi ed conn nfident, e. They n solution	hods can be with the basefully applying the essections. precise an have learn an strategies	pe successfi asic concept these methential results d independe ed to transfe s on their ow	ully ap is, resinods in s of the ent har er the r	plied to ults and a further e lecture adling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Geometric Measure Theory – Measure Theoretic Methods	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise or examination the coursework oral is decided by the instruc	must l	nave	beei	n acc	uired.	Whether th	e examination			
Literature	Possible References: • Lawrence C. Evans, tions. CRC Press 199 • Herbert Federer: Geo • Leon Simon: Lecture 1984.	02. ometric	mea	asure	the	ory. Sp	ringer 1969				

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	There are no further prerequisites.
Responsible Persons	Reiner Schätzle

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-45	Module Title: Introduction to Geometric Me	asure	The	ory -	- Var	ifolds		of Module: ulsory Modul	le with	Choice		
ECTS-Points	5						'					
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 150 h 45 h 105 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	ecture 2 SWS + Exercise Class 1 SWS										
Content		Surface- and Cosurface formula. Countable rectifiable sets, rectifiable varifolds.										
Objectives	analysis and geometry and w ious problems. They have fa of geometric measure theory The students are capable of assessing and explaining the In the exercise classes they the terms, statements and mon new problems, to analyse	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Introduction to Geometric Measure Theory – Varifolds	п Туре of Course	t Status	SMS 2	S ECTS	Coursework	Type of Exam o. o.	Dur. of Exam (min) 90-180 o. 20-30	Grading	Weight for Grade		
	In this module an exercise ce examination the coursework oral is decided by the instruct	rtificat must h	e is	to be	acc acc	quired.	Whether th	e examinatio	icipation on is w	on in the ritten or		
Literature	Possible References: • Lawrence C. Evans, Fitions. CRC Press 1999 • Herbert Federer: Geo. • Leon Simon: Lecture: 1984.	2. metric	mea	ısure	the	ory. Sp	ringer 1969					

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	The module Integration and Measure Theory from the B.Sc. Mathematics or an equivalent module must have been successfully completed during the course of studies.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-46	Module Title: Elastic Curves							f Module: Ilsory Modul	e with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: Time in Class: 30 h							Self-Study: 60 h			
Duration	1 Semester										
Frequency	not regularly	not regularly									
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content	Order reduction of the Qualitative behaviour	 Classification of elastic curves according to Langer and Singer. Order reduction of the Euler-Lagrange equation of the elastic energy of a curve. Qualitative behaviour of an elastic curve. Solving the Willmore equation under axial symmetry with variational methods. 									
Objectives	Students learn how to deal with a geometrically relevant functional and its critical points using the example of the elastic energy of a curve. This gives them an insight into the theory of fourth-order elliptic differential equations where familiar techniques, such as the maximum principle, can no longer be used. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Elastic Curves	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or	ora	l is de	ecided	by the instru	uctor with a	oprova	l by the	
Literature	Possible References :										
	Filippo Gazzola, Han Value Problems, Sprin			h (Gruna	u, Gui	do Sweers:	Polyharmo	nic B	oundary	
	David Gilbarg, Neil S. Springer 1998.	Trudi	nger	: El	liptic p	oartial	differential e	quations of	secon	d order.	
	Joel Langer, David A. ential Geom. Band 20							of closed cu	rves, c	J. Differ-	
	John M. Lee: Introduc	tion to	smo	oth	n mani	ifolds.	Springer 20	13.			
	Michael Struwe: Varia	tional	Meth	nod	s. Spr	inger 2	2008.				

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the module Introduction to Partial Differential equations and basic knowledge of differential geometry are assumed.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-47	Module Title: Type of Module: Geometric Measure Theory – Varifolds Compulsory Module with Choice											
ECTS-Points	5											
Workload - Time in Class - Self-Study	Workload: 150 h											
Duration	1 Semester											
Frequency	not regularly											
Term	1-3	1-3										
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	Lecture 2 SWS + Exercise Class 1 SWS										
Content	First and second varia	First and second variation for varifolds.										
	Monotonicity formula.											
	Allard's integral compa	actnes	s the	eorer	n.							
	Lipschitz approximatio	n.										
	tilt-excess descent.											
	Allard's regularity theo	rem.										
Objectives	After the students have learn Measure Theory, this knowled prepared for and introduced ting and proving the essential presented connections. In the exercise classes they leave the terms, statements and meaning on new problems, to analyse they are able to present their	edge i o curr I resu nave a ethods them a	rent rent related of the control of	eperesea f the ired a he le	ned varch (lection control) a control cture rk or	with a valuestion ure as only of the state o	view to vari ns. The stu well as ass precise an have learn on strategie	iabilities. Standents are casessing and dindependent to transfess on their ow	explaid explaid ent hare er the r	s will be of nam- ning the adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Geometric Measure Theory	L	f	2	3	yes	wr. o.	90-180	g	100		
	- Varifolds	E	f	1	2	,	or.	o. 20-30				
	In this module an exercise ce examination the coursework oral is decided by the instruct	must l	nave	beer	n acc	quired.	Whether th	e examination				
Literature	Possible References :											
	Herbert Federer: Georgian	metric	mea	asure	the	ory. Sp	ringer 1969).				
	Enrico Giusti: Minimal	surfa	ces a	and f	uncti	ions of	bounded va	ariation. Birk	häuse	r 1984.		
	• Leon Simon: Lecture: 1984.	s on (geom	netric	: me	asure t	heory. Aus	stralian Natio	onal U	niversity		

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	In terms of content, the module Introduction to Geometric Dimension Theory is a is a prerequisite for participation.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-48	Module Title: Geometric Measure Theory – Flows Type of Module: Compulsory Module with Choice									Choice		
ECTS-Points	5											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 150 h 105 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German or English	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	Lecture 2 SWS + Exercise Class 1 SWS										
Content	General and rectifiable	General and rectifiable flows.										
	Deformation theorem.											
	Surface minimizing flo	Surface minimizing flows.										
Objectives	After the students have learn Measure Theory, this knowled prepared for and introduced to ing and proving the essential presented connections. In the exercise classes they the terms, statements and more on new problems, to analyse They are able to present their	edge i o curr I resu nave a ethods them a	s de ent r Its o acqu s of t and t	eper esea f the ired a he le o wo	ned varch of lection a correcture or the lection a correct or the lection and lection and lection a correct or the lection and lection	with a valuestion ure as a solution of the sol	view to varins. The stuwell as ass precise an have learn on strategie	iabilities. St dents are ca sessing and d independe ed to transfe s on their ow	udents apable explai ent har er the r	s will be of nam- ning the adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Geometric Measure Theory – Flows	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise ce examination the coursework oral is decided by the instruct	must l	nave	beei	n acc	quired.	Whether th	e examination				
Literature	Possible References :											
	Herbert Federer: Geo	metric	mea	asure	the	ory. Sp	ringer 1969).				
	Enrico Giusti: Minimal	surfa	ces a	and f	uncti	ons of	bounded va	ariation. Birk	häuse	r 1984.		
	Leon Simon: Lecture: 1984.	s on (geom	etric	mea	asure t	heory. Aus	stralian Natio	onal U	niversity		

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	In terms of content, the module Introduction to Geometric Dimension Theory is a is a prerequisite for participation.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-49	Module Title: Calculus of Variations							of Module: ulsory Modul	le with	Choice			
ECTS-Points	5												
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 150 h 105 h												
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German or English												
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 1 SWS											
Content	Direct method of calc	Direct method of calculus of variations.											
	Euler-Lagrange equa	tions.											
	Palais-Smale condition	n.											
	Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.												
Objectives	In the first part of the course which is primarily used to probut also has applications in a basics from functional analy different context, e.g. geomabout a so-called mountain in the existence of solution naming and proving the essible presented connections. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	ove the e.g. differs and etric a -pass of pass ential have a nethods them a	exist feren d par nalys lemn artial resul acqui s of t	tial control of the left of th	e of water of the office of th	reak so letry. The ential electore its her all equilecture infident, e. They a solution	lutions of pathey have all equations and part of the part of the part of the pathey care ations. The as well as precise and have learn on strategie	artial differer so acquired nd can also he course, so analyse no estudents as assessing and independent to transfers on their ownem in critical	ntial eq the ne use th student on-unic are cap and ex ent har er the r	uations, cessary ese in a ts learnt queness pable of plaining adding of methods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Calculus of Variations	L	f	2	3	yes	wr. o.	90-180	g	100			
	- Calodido of Variations	Е	f	1	2	,555	or.	o. 20-30	9				
	In this module an exercise c examination the coursework oral is decided by the instruc	must h	nave	beer	n acc	uired.	Whether th	e examinatio					

Literature	Possible References :
	Michael Struwe: Variational Methods, Springer 2008.
	 David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998.
	Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the modules Introduction to Partial Differential Equations and Functional Analysis is an advantage, but not essential.
Responsible Persons	Reiner Schätzle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-51	Module Title: Lie Groups Type of Module: Compulsory Module with Choice												
ECTS-Points	9						Оотпри	noory wooda	- WILLI	0110100			
Workload - Time in Class - Self-Study	Workload: 270 h	,											
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German or English	German or English											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS											
Content	Manifolds and Lie groups,												
	Lie algebras and exposition												
	Covering spaces and	classii	ricati	on of	Lie	groups	by their Lie	algebras,					
	Classical Lie groups, Operations of Lie gro	ups an	d ho	mog	eneo	us spa	ces.						
Objectives	Lie groups lie at the interfact describing the symmetries of differential equations, in par learn from a prominent examathematics can work toge developed that can precisely capable of naming and provexplaining the presented correctly in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	of geonaticular mple hather early descring the nection have a sethods them a	netrion if the low of the low of the low of the lower	c objese s differency a vari sential ired a the le	ects, symment description desc	but all netries lisciplinessfull of symbol sults of mfident, a solution but a sol	so algebraic form a con nes of math y and how metry phen the lecture precise and have learn on strategies	c equations tinuous set. ematics can a convincing omena. The as well as a d independent to transfers on their ow	or solu The s discip g form e stude assess ent har er the n n or in	ations of students blines of alism is ents are ing and adling of nethods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Lie Groups	L E	f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise c examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	uired.	Whether th	e examinatio					

Literature	Possible References :						
	Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991.						
	Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965.						
	Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.						
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.						
Prerequisites	There are no further prerequisites.						
Responsible Anton Deitmar, Frank Loose Persons							

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-52	Module Title: SL2(R)							f Module: Ilsory Modul	le with	Choice		
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	:		Self-St 60 h	udy:				
Duration	1 Semester						·					
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	Deutsch und Englisch											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	• Structure theory of the Lie group $SL_2(\mathbb{R}).$											
	Introduction to the rep	• Introduction to the representation theory of $SL_2(\mathbb{R}).$										
	Computation of the ur	nitary o	dual.									
	Proof of the explicit Pl	Proof of the explicit Plancherel formula.										
Objectives	have become familiar with the with the basics of hyperbolic classify representations. More Lie groups, and they have gaunderstand the analysis lying The students are capable of	With the $SL_2(\mathbb{R})$ the students have studied an important Lie group in detail. That way they have become familiar with the basics of the representation theory of Lie groups as well as with the basics of hyperbolic geometry. The students have learned to construct, to split and to classify representations. Moreover, they are able to transfer their knowledge to analyse other Lie groups, and they have gathered a deeper understanding of the theory of Lie groups. They understand the analysis lying underneath the Theorem of Plancherel and apply it successfully. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	$SL_2(\mathbb{R})$	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
Literature	Possible References :				ory of	f semis	imple group	s. PUP 200	1.			
Transfer	The module belongs to the Differential Geometry. Takin be included in the Sections Specialisation, in accordance	g into Study	Foc	ount <i>us</i> , <i>i</i>	the o A <i>dva</i>	chosen nced k	personal S <i>(nowledge ii</i>	tudy Specia n <i>Mathemat</i>	lisatio <i>ics</i> or	n, it can <i>Elective</i>		
Prerequisites	There are no further prerequ	isites.										
Responsible Persons	Anton Deitmar											

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-53	Module Title: Automorphic Forms							f Module: Ilsory Modu	le with	Choice	
ECTS-Points	5										
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	in C	lass:			Self-St 105 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	lass 1	SWS	3							
Content	Modular forms for the	modu	le gr	oup a	and i	ts cong	ruence sub	groups.			
	Examples: Eisenstein	series	s, Ra	manı	ujan	delta fı	unction, the	ta series.			
	Modular curves.										
	Arithmetic applications and conjectures.										
	Hecke operators.										
	The L-function of a magnetic forms of the L-function of a magnetic forms of the function	odular	form	and	its c	onnect	ions with el	liptic curves			
Objectives	Students have familiarised the of automorphic forms in exaconnection between modular are capable of naming and and explaining the presented in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	amples r, real proving conn have a ethods them	s and represent the section acquired to the section ac	d are senta essenta es	ation entia cor cture	e to us theory I result infident, e. They in solution	se them. To and adelic is of the lector precise and have learned attacked in strategies.	They have under the L-functions between as well and independent to transfers on their own.	inderst . The s I as as ent har er the r n or in	ood the students ssessing adling of methods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Automorphic Forms	L	f	2	3	yes	K o. mP	90-180	<u> </u>	100	
	Automorphic Forms	Е	f	1	2	yes	o. H	o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	nave	beer	acc	quired.	Whether the	e examination			
Literature	Possible References :										
	Deitmar, Anton: Autor	norphi	c Fo	rms.	Spri	nger 20)12.				
	Goldfeld, Dorian: Auto University Press 2015		nic fo	rms a	and I	L-functi	ons for the (group GL(n,	R). Ca	mbridge	
	Serre, Jean-Pierre: A	cours	e in a	arithn	netic	. Sprin	ger 1973.				

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module is part of the module Introduction to Geometric Measurement Theory and cannot be taken together with this module.
Prerequisites	Basic knowledge of function theory is assumed.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-60	Module Title: Introduction to Mathematical	Logic						f Module: Ilsory Modul	le with	Choice								
ECTS-Points	3																	
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h		lass	:		Self-St 60 h	udy:										
Duration	1 Semester	1 Semester																
Frequency	not regularly																	
Term	1-3																	
Language of Instruction	German																	
Forms of Teaching and Learning	Lecture 2 SWS																	
Content	Languages of the first	 Incompleteness of arithmetic: First and second incompleteness theorem. 																
Objectives	Students are able to underst ematical logic. They underst the difference between truth mathematical content. The sof the lecture as well as asse	stand and p studen	the li roval ts ar	mits pility e cap	of p and pable	ossible can ap of nar	mathemati ply basic the ming and pr	cal knowled eoretical mo oving the es	dge, re odel thi	cognise nking to								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade								
	Introduction to the Mathe- matical Logic	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100								
Literature	2008.							Logik. Vie	Possible References : • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner									

Transfer	The module is not assigned to a specialisation. It can be included in the Sections <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-61	Module Title: Cohomology and Sheaves							of Module: ulsory Modul	e with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study		Time 60 h	in C	lass:			Self-Si 120 h	tudy:			
Duration	1 Semester						•				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Cla	ass 2	SWS	3							
Content	It is shown how different cohomology theories (singular, de Rham, Cech) can all be understood as derivatives of the section functor from sheaf theory and thus their equality (after coefficient expansion) can be shown very easily: • Introduction to category theory. • Presentation of the current cohomology theories. • Sheaves, derived functors, sheaf cohomology. • Comparison of cohomology theories.										
Objectives	The students see and unders ries. They understand mecha They have learned to abstract preciate cohomology theory as generalisations of function naming and proving the esset the presented connections. State of research in the subjections.	inisms ot arbi as a g i spac ential i Studer	tha trary ener es fo resul	t con mat al ob or top ts of	nbine them ostac oolog the	e algeb atical t le theo lical qu lecture	raic, geome heories usi ry in applic lestions. Ti as well as	etric and ana ng category ations and to ne students assessing a	theory theory use sare ca	nethods. y, to ap- sheaves pable of plaining	
Requirements for Obtaining		ourse				ork	Exam	Dur. of Exam (min)		Grade	
Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of E	Dur. of E	Grading	Weight for Grade	
Credit, Grading, Weight if		Type of C	4 Status	SMS 4	e ECTS		Wr. o.	90-180			
Credit, Grading, Weight if	Title Cohomology and Sheaves	1				Sex	Type of		Grading	Weight for	
Credit, Grading, Weight if		L E writter	f f or or o	4 2 oral i	6 3 is de	yes cided l	wr. o. or.	90-180 o. 20-30 uctor with a y the lecture	g pprova	100	
Credit, Grading, Weight if	Cohomology and Sheaves Whether the examination is v Board of Examiners. – The m	L E writter	f f or or o	4 2 oral i	6 3 is de	yes cided l	wr. o. or.	90-180 o. 20-30 uctor with a y the lecture	g pprova	100	
Credit, Grading, Weight if applicable	Cohomology and Sheaves Whether the examination is v Board of Examiners. – The m cises; in this case, only 6 cred	L E writter nodule dit poi	f f or or or or may nts w	4 2 oral i / exc vill be	6 3 is de eptic e awa	yes ecided l onally b arded f	wr. o. or. by the instruction of the model	90-180 o. 20-30 uctor with a y the lecture ule instead o	g pprova r witho f 9.	100	
Credit, Grading, Weight if applicable	Cohomology and Sheaves Whether the examination is v Board of Examiners. – The m cises; in this case, only 6 cred Possible References:	E writter nodule dit poin	f f or or or may nts w	4 2 oral i / exc vill be	6 3 is deseption e awa	yes cided bonally barded f	wr. o. or. by the instruction of the modern mathematic	90-180 o. 20-30 uctor with a y the lecture ule instead o	g pprova r witho f 9.	100	
Credit, Grading, Weight if applicable	Cohomology and Sheaves Whether the examination is was Board of Examiners. — The macises; in this case, only 6 cred Possible References: • Saunders Mac Lane: Compared to the control of	L E writter nodule dit poin Catego	f f or or or may nts w	4 2 oral i / exc vill be for th	6 3 is deseptions awarene we	yes ecided lonally barded f	wr. o. or. by the instruction of the modern mathematic niversity Pr	90-180 o. 20-30 uctor with a y the lecture ule instead o	g pprova r witho f 9.	100	

Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.	
Prerequisites	In terms of content, only basic knowledge from the analysis and linear algebra is required.	
Responsible Persons	Anton Deitmar	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-62	Module Title:Type of Module:Consistency ProofsCompulsory Module with Choice											
ECTS-Points	6											
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass:	:		Self-St 120 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	lass 2	SWS	3								
Content	 Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes). 											
	Philosophical foundational programs (logicism; formalism; intuitionism).											
	The Hilbert program a	and Gö	del's	thed	orem	S.						
	Gentzen's transfinite	consist	ency	pro	of for	numb	er theory.					
	Alternative approache	es to co	nsis	tenc	y (ind	cluding	Gödel's T).					
	Current situation of co	onsiste	ncy p	oroof	s.							
Objectives	Students learn about the hist formal mathematical theories ing this question mathematic in mathematics both historica ematical tools to be able to to a certain extent, to carry proving the essential results connections. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	s arose ally. The ally and compression of the have a sethods them a	e, as ney a phile hen but the lecture is of the	well are all losor demonstrated and the memoral are are less to the less to th	as the ble to blicate corselves well a corselves rk on	ne releved categorally. In a categoral categor	vant moderr orise the pro- addition, the ading proofs students a sessing and precise an- have learn on strategies	n techniques oblem of nor ey have acque of non-con re capable of d explaining d independe ed to transfe s on their ow	for invalue of the formal of the property of the property of the property of the representation of the property of the representation of the property of the p	restigat- adiction e math- on and, ing and esented adling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Consistency Proofs	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	ave	beer	n acc	uired.	Whether th	e examinatio				

Literature	Possible References :
	 Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931).
	 Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936).
	 Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015.
	Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Transfer	The module belongs to the Study Specialisation Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic mathematical knowledge to the extent of the basic lectures is assumed. Previous knowledge of mathematical logic is helpful, but not necessary.
Responsible Persons	Reinhard Kahle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, } & \text{P=project, } & \text{S=seminar, } & \text{IC=inverted classroom} \\ \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-55-63	Module Title: Introduction to set theory							f Module: Ilsory Modul	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass			Self-St 60 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 2 SWS										
Content	Content: •										
Objectives	- The students are able to view mathematical theorems and theories in the context of mathematical logic. They understand the boundaries of the mathematically knowable, the difference between truth and provability, and are able to apply model-theoretic considerations to mathematical problems. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to set theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
Literature	Possible References :	1				ı					
Transfer											
Prerequisites	There are no further prerequi	sites.									
Responsible Persons	Frank Loose										
Examination Type : M Teaching Format : L T Status : 0	Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative										

Module Number: MAT-55-64	Module Title: Theory of Mathematical Prod	ıfs						of Module: ulsory Modu	le with	Choice	
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German	German									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS										
Content	 Axiomatic theories, incompleteness. Gentzen's proof of consistency for arithmetic. Ordinal number analysis. Provable recursive functions. Predicative analysis. Theories of inductive definitions. 										
Objectives	Students are familiar with the calculations for mathematical are capable of naming and prexplaining the presented concurrent state of research in the line the exercise classes they the terms, statements and monnew problems, to analyse They are able to present their	I theo oving nection sub have a ethods	ries a the e ns. S iject acqu s of t and t	and tesser Stude area ired a he le o wo	their ntial rents v a con ecture rk or	metam results of will be a nfident, e. They n solution	athematica of the lectur able to reflect precise an have learn on strategie	I properties. e as well as ct and critica d independe ed to transfe s on their ow	The sassessally ana ent har er the ron or in	students sing and lyse the adling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Theory of Mathematical Proofs	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	nave	beei	n acc	quired.	Whether th	e examination			
Literature	Possible References : • Wolfram Pohlers. Prod	of The	ory.	Sprir	nger	2009.					
Transfer	The module belongs to the Differential Geometry. Takin be included in the Sections Specialisation, in accordance	g into <i>Study</i>	Foc	ount <i>us</i> , <i>A</i>	the d Adva	chosen nced K	personal S nowledge i	tudy Specia n Mathemat	lisation ics or	n, it can <i>Elective</i>	

Prerequisites	Basic mathematical knowledge to the extent of the basic lectures is assumed. Previous knowledge of mathematical logic is helpful, but not necessary.
Responsible Persons	Reinhard Kahle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-65	Module Title: Explicit Mathematics							of Module: ulsory Modu	le with	Choice
ECTS-Points	6						·			
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h		lass	:		Self-St 120 h	udy:		
Duration	1 Semester	1 Semester								
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German	German								
Forms of Teaching and Learning	ecture 2 SWS + Exercise Class 2 SWS									
Content	 Applicative theories. Explicit mathematics. Universes in explicit mathematics. Applications in proof theory. 									
Objectives	systems of analysis and are students are capable of nam sessing and explaining the pranalyse the current state of r In the exercise classes they the terms, statements and m on new problems, to analyse	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Explicit Mathematics	L E	f	2	3	yes	or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework offered by the lecturer without the module instead of 6.	must	have	bee	n ac	quired	- The mo	dule may ex	ceptio	nally be

Literature	Possible References :
	 Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975.
	 Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium ?78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979.
	 Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999.
	 Reinhard Kahle: The applicative realm. Textos de Matematica, 40, Departamento de Matemática, Universidade de Coimbra, 2007.
	 Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3),141-162, 2001.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of mathematical logic.
Responsible Persons	Reinhard Kahle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-70	Module Title: Selected Chapters from Fun	7.								Choice
ECTS-Points	6							<u> </u>		
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass			Self-St 120 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English	German or English								
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 2 SWS								
Content	A selection of the following to	selection of the following topics will be covered:								
	Topological vector spa	aces a	nd dı	uality	theo	ory.				
	(LB) and (LF) spaces and distributions.									
	Compactness concepts (Eberlein's theorem, Banach-Alaoglu, Krein-Milman, Smulian).									
	Theorems from topology (Tietze, Urysohn, Stone-Cech) and their applications in functional analysis.									
	Uniform spaces.									
Objectives	Students are familiar with the to apply their methods and resuch as the theory of distribution connections to other parts of are capable of naming and pexplaining the presented concurrent state of research in the exercise classes they the terms, statements and monnew problems, to analyse They are able to present the	esults putions mather roving inection he sub have a tethods them a	to co The emati the e ns. S ject a acqui s of the	ey had concretely had concretely be concretely be concretely by the concretely be conc	ete exave ruch a tial ruch a tial ruch a tial ruch a corecture rk on	ecognias mea esults vill be a nfident, solution	s from the issed and unasure theory of the lecturable to reflect precise and have learn on strategies.	field of funct derstood the or topology e as well as ct and critical d independe ed to transfe s on their ow nem in critical	ional are many and the many assess ally ana ent hare refer the refer to or in	analysis, y cross- students sing and lyse the adling of nethods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Selected Chapters from Functional Analysis	L E	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	beer	n acc	uired.	Whether th	e examination	icipatic on is w	on in the ritten or

Literature	Possible References :
	Gerald Folland: Real Analysis. Wiley 1999.
	Helmut H. Schäfer: Topological Vector Spaces. Springer 1999.
	Volker Runde: A Taste of Topology. Springer 2005.
	Gert K. Pedersen: Analysis Now. Springer 1989.
	Paul R. Halmos: Measure Theory. Springer 1950.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the Functional Analysis module is assumed.
Responsible Persons	Ulrich Groh, Rainer Nagel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-55-71	Module Title: Operator Algebras and the Mechanics	Operator Algebras and their Applications to Statistical Compulsory Module with Choice								
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h		Class	:		Self-St 120 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 3 SWS + Exercise C	ecture 3 SWS + Exercise Class 1 SWS								
Content	 Basics on operator algebras (C*-algebras, algebraic states, inductive limits); Introduction to algebraic deformation quantization (general set-up, coherent states, examples); Applications to the classical limit of quantum mechanics and statistical mechanics including asymptotic emergence (phase transitions, large deviations (entropy), spontaneous symmetry breaking). 									
Objectives	theory with an emphasis on classical limit of quantum metechniques in order to develor They are familiar with technicoded by algebraic states, exthey understand the physical equilibrium thermodynamics. They are able to describe the name and prove the central tions. In the exercise classes they the terms, statements and meto new problems, to analyse	The students have obtained deepend knowledge in selections questions in algebraic quantum theory with an emphasis on algebraic deformation quantisation and their applications to the classical limit of quantum mechanics and statistical mechanics. They have learned algebraic techniques in order to develop abstract structures encoding the features of a physical theory. They are familiar with techniques to prove existence results of limits of sequences/nets encoded by algebraic states, examine these, and put them into a general perspective. Moreover, they understand the physical relevance of the results and are able to relate them to features of equilibrium thermodynamics, such as phase transitions and spontaneous symmetry breaking. They are able to describe the current state of research in the specific area. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Operator Algebras and their Applications to Statistical Mechanics	п Туре of Course	t Status	SWS 3	SL 4,5	yes	Type of Exam o. o. or.	Onr. of Exam (min) 0.20-30	ص Grading	Weight for Grade
	In this module an exercise context examination the coursework oral is decided by the instruction	ertifica must l	te is	to be	e acq	uired a	Whether th	e examination		

Literature	Possible References :
	Klaas Landsman: Foundations of Quantum Theory, From Classical Concepts to Operator Algebras. Springer 2017.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry, Mathematical Physics</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of C*-algebras and functional analysis as well as in thermodynamics are assumed.
Responsible Persons	Andreas Prohl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-01	Module Title: Geometric Evolution Equation	ns						f Module: Isory Modu	le with	Choice
ECTS-Points	3						1			
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h		lass	:		Self-Sti 60 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	 Examples of geometric evolution equations such as Mean curvature flow, Ricci flow, Inverse mean curvature flow. Parabolic maximum principles. Regularity theory for parabolic evolution equations. Rescaling techniques and description of singularities. Asymptotic behavior of solutions. 									
Objectives	Students learn to combine their knowledge of differential geometry and partial differential equations and apply it to specific problems in selected geometric evolution equations. They learn techniques for checking solutions of non-linear evolution equations, which enables them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.					s. They es them h a view				
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometric Evolution Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	cided	by the instru	uctor with a	pprova	ll by the
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Nonstationary Differential Equations or Numerics of Differential Equations on Surfaces, it can be included in the subject specialisation <i>Numerical Mathematics and Optimisation</i> .									
Prerequisites	Knowledge from the module of differential geometry are re			n to	Partia	al Differ	ential Equat	tions and ba	ısic kn	owledge
Responsible Persons	Gerhard Huisken									

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-02	Module Title: Geometric Variation Problems							ype of Module: compulsory Module with Choice			
ECTS-Points	3										
Workload - Time in Class - Self-Study		Time 30 h	in C	lass	:		Self-St 60 h	Self-Study: 60 h			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS	ecture 2 SWS									
Content	 Examples of geometric variation problems such as minimal surfaces, capillary surfaces, harmonic mappings and associated boundary value problems. Direct methods of calculating variations. Regularity theory for solutions of variational problems. Relationship between variational problems and partial differential equations. Stability properties of solutions. 										
Objectives	it to specific problems in sele proving solutions to various va which provide a basis for inde	Students learn to combine their knowledge of differential geometry and analysis and apply it to specific problems in selected geometric variational problems. They learn techniques for proving solutions to various variational problems and for analysing the properties of solutions, which provide a basis for independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Geometric Variation Prob- lems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is v Board of Examiners.	vritter	or or	oral	is de	ecided	by the instru	uctor with a	pprova	al by the	
Transfer	into account the chosen pers	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the module In of differential geometry are re			n to I	Partia	al Diffe	rential Equa	tions and ba	sic kn	owledge	
Responsible Persons	Carla Cederbaum, Gerhard H	uiske	n								

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-03	Module Title: Topics in Mathematical Relativity							f Module: Ilsory Modul	le with	Choice
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h						udy:		
Duration	1 Semester									
Frequency	regularly									
Term	1-3	1-3								
Language of Instruction	English	English								
Forms of Teaching and Learning	ecture 2 SWS									
Content	metrics, physical invamass. • Geometric and analyti	 A selection of concrete models of Mathematical Relativity, such as black holes, static metrics, physical invariants of isolated systems, positivity estimates for energy and mass. Geometric and analytical structure of the models, existence and properties of concrete models as solutions to Einstein's equations. 								
Objectives	Students acquire in-depth knowledge of selected issues in the mathematical theory of relativity. They learn analytical and geometric techniques for proving and investigating solutions to Einstein's equations and are able to categorise the physical relevance of the mathematical results. The lecture introduces students to their first independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Topics in Mathematical Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided I	by the instru	uctor with a	pprova	I by the
Transfer	ematical Physics. Taking infinituded in the Sections Stud	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.								
Prerequisites	In terms of content, the modu Equations are assumed.	les M	ather	natio	al R	elativity	and Introdu	uction to Par	tial Dif	ferential
Responsible Persons	Carla Cederbaum, Gerhard I	Huiske	n							

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-04	Module Title: Space-Like Hypersurfaces in		of Module: ulsory Modu	le with	Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	_	lass	:		Self-Si 120 h	tudy:		
Duration	1 Semester									
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 2 SWS								
Content	Space-like hypersurfaces of a Lorentzian manifold play a crucial role in the study of solutions to Einstein's equations modelling some phenomenon in General Relativity. The course explores how geometric choices of space-like hypersurfaces such as maximal surfaces, constant mean curvature surfaces or solutions of mean curvature flow and inverse mean curvature flow can be used to achieve a splitting of space and time that is suitable for the study of both isolated gravitating systems and cosmological spacetimes.									
Objectives	They learn analytic and geo stein equations and to exam of the mathematical solution and concepts from the lectu to put it into a larger framew state of research in the spec Through homework assignmand independent acquaintar lectures. They learn how to	The students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Space-Like Hypersurfaces in Lorentzian Manifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature	Possible References :
	Barrett O'Neill: Semi-Riemannian Geometry - With applications to Relativity. Academic Press 1983.
	 Andrejs E. Treibergs: Entire space-like hypersurfaces of constant mean curvature in Minkowski space. Inventiones Math. 66, (1982) 39–56.
	 Klaus Ecker, Gerhard Huisken: Parabolic methods for the construction of spacelike slices of prescribed mean curvature in cosmological spacetimes. Comm. Math. Phys. 135 (1991), 595–613.
	 Helmut Friedrich, Alan Rendall: The Cauchy Problem for the Einstein Equations. In: Schmidt B.G. (eds) Einstein's Field Equations and Their Physical Implications. Lecture Notes in Physics, vol 540. Springer 1999.
	Hans Ringström: The Cauchy Problem in General Relativity. European Math. Society 2009.
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Mathematical Relativity is expected.
Responsible Persons	Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-05	Module Title: Limits of Spaces							of Module: ulsory Modu	le with	Choice
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3	1-3								
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS									
Content	 Basic concepts of metric geometry, e.g. geodesics, doubling property and Hausdorff measure. Generalized curvature conditions in the sense of Alexandrov and Busemann. Gromov-Hausdorff and ultra convergence. Gromov's Precompactness Theorem and stability theorems. 									
Objectives	Students generalise their knowledge in analysis and know how to apply the methods to particular problems in metric geometry. They get to know different convergence notions and learn which properties are stable in the limiting process. In addition, the students are familiar with synthetic and concrete curvature notions, which help to better understand curvature notions in differential geometry and general relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Limits of Spaces	ш Г Туре of Course	t Status	SMS 3	SL 4,5 1,5	yes	Type of Exam or.	Onr. of Exam (min) 90-180 0. 20-30	ص Grading	Weight for Grade
	In this module an exercise ce tion is written or oral is decide									
Literature	 Possible References: Jeff Cheeger, David Ebin: Comparison Theorems in Riemannian Geometry. AMS 1975. Dimitri Burago, Yuri Burago, Sergei Ivano: A Course in Metric Geometry. AMS 2001. Mikhail Gromov: Metric Structures for Riemannian and Non-Riemannian Spaces. Springer 2007. 									

Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge in analysis and measure theory is assumed.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-06							of Module: ulsory Modul	le with	Choice		
ECTS-Points	6						'				
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h					Self-St 120 h	Self-Study: 120 h			
Duration	1 Semester						·				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS									
Content	The lecture introduces the basic properties of the Ricci flow and develops the necessary techniques, e.g. tensor maximum principles and regularity estimation. The long-term existence of solutions and resulting classifications for metrics of positive curvature are presented. Finally, the monotonicity of functionals according to Perelman is derived and used for the classification of possible singularities, with an outlook on the surgery methods of Hamilton and Perelman, which have led to the proof of the Poincaré and geometrisation conjectures.										
Objectives	The students have learnt basic methods for the treatment of geometric evolution equations in Riemannian geometry. At the same time, they have experienced the interplay of local geometric assumptions on the curvature properties of a metric with analytic techniques for the study of parabolic equations and have learnt and understood how the local assumptions have global consequences for the geometry and topology of the underlying manifolds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	The Ricci Flow of	L	f	2	3	no	wr. o.	90-180	g	100	
	Riemannian Metrics	ü	0	2	3	110	or.	o. 20-30	9	100	
	Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.										
Literature	Possible References :										
	Simon Brendle: Ricci-	flow a	nd th	e sp	here	theore	m. AMS 20	10.			
	Peter Topping: Lecture	es on t	the F	Ricci-	Flow	. Lectu	re Notes 20	006.			
	Richard Hamilton: Rie 17, 1982.	manni	an 3	-mar	nifold	s with p	oositive Rice	ci curvature.	J. Diff.	Geom.	
Transfer	The module belongs to the Si ematical Physics. Taking int included in the Sections Stuccialisation, in accordance wit	o acc	ount us, A	the A <i>dva</i>	chos nced	en per Knowi	sonal Studgedge in Ma	y Specialisa thematics o	tion, it <i>Electi</i>	can be	

Prerequisites	Knowledge from the module Introduction to Partial Differential Equations as well as fundamental knowledge in differential geometry is required.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-07	Module Title: Special Relativity							f Module: Isory Modul	le with	Choice		
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h					Self-St 60 h	Self-Study: 60 h				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	 Derivation of the Minkowski metric from basic physical assumptions. Physical consequences of relativity such as length contraction, time dilation and some popular paradoxes. 											
Objectives	Students have learnt and understood the derivation of the special theory of relativity and important concepts such as length contraction and time dilation. They are familiar with important paradoxes that arise. Students have developed an intuition for various aspects of the theory of relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Special Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writte	n or (oral i	s de	cided I	by the instru	uctor with a	pprova	ll by the		
Literature	Possible References :											
	Albert Einstein: Relati	vity: tl	ne sp	ecia	and	gener	al theory. Po	ublic domair	า 1920			
	Thomas A. Moore: Six	idea	s that	t sha	ped	physics	s: unit R. Mo	Graw-Hill 2	003.			
	Robert Resnick: Introd	ductio	n to S	Spec	ial R	elativity	. Wiley 200	7.				
	Bernard Schutz: A First Course in General Relativity. Cambridge University Press 2009.											
Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.											
Prerequisites	Knowledge from the module of differential geometry are re			n to F	Partia	al Differ	ential Equa	tions and ba	ısic kn	owledge		

Other

Responsible Persons	Carla Cederbaum, Gerhard Huisken
Abbreviations:	
, ,	=graded, ng=not graded
1	//T=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, =tutorial, P=project, S=seminar, IC=inverted classroom
Status : c	e-obligatory, f=facultative

Module Number: MAT-60-08	Module Title: Null Geometry in General Relativity						of Module: ulsory Modul	le with	Choice	
ECTS-Points	5									
Workload - Time in Class - Self-Study	Workload: 150 h	Time 45 h	in C	lass			Self-St 105 h	tudy:		
Duration	1 Semester						·			
Frequency	not regularly									
Term	1-3	1-3								
Language of Instruction	English	English								
Forms of Teaching and Learning	Lecture 2 SWS	Lecture 2 SWS								
Content	This module provides an introduction to null geometry. Topics include the properties of light-like vector fields and curves, as well as the geometry of light-like hypersurfaces that carry a degenerate induced metric. Another major topic is the extrinsic curvature of space-like surfaces in higher codimension, which are considered in particular along light-like hypersurfaces. Optionally, geometric flows along light-like hypersurfaces can also be treated.									
Objectives	Students know and understand the concepts and methods mentioned and can use them to analyse known and new questions from null geometry. Furthermore, they link physical problems in cosmology and astrophysics and their mathematical modelling using differential geometric methods and are able to question the relevance and adequacy of mathematical modelling and the mathematical results derived from it. In particular, they expand on the methods learnt in the MAT-65-11 module and connect their methodological and specialist knowledge. They are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Null Geometry in General Relativity	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit								ritten or ule may	
Literature	points will be awarded for the module instead of 5. Possible References: Barrett O'Neill: Semi-Riemannian Geometry. Academic Press 1983. Johannes Sauter: Foliations of Null hypersurfaces and the Penrose Inequality. Dissertation (ETH Zürich), url: https://www.research-collection.ethz.ch/handle/20.500.11850/150826.									

Transfer	The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.	
Prerequisites	The content of the Geometry in Physics module is a prerequisite.	
Responsible Persons	Carla Cederbaum	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-09	Module Title: The Einstein Constraint Equ	ations	Type of Module: Compulsory Module with Choice								
ECTS-Points	6		,								
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h								
Duration	1 Semester		,								
Frequency	not regularly	ot regularly									
Term	1-3										
Language of Instruction	English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 2 SWS									
Content	- The Einstein ed - The Cauchy pro • The constraint equation - The conformal of - Overview of the - Classification of • Asymptotically Euclided - AE manifolds as	e elliptic theory on closed man f constant mean curvature on	ns. d: ifolds;								
Objectives	liptic partial differential equal equations and analyse prop nections between the theory problem and the Yambe pronian geometry, geometric at The students are capable of as assessing and explaining critically analyse the current In the exercise classes they the terms, statements and monnew problems, to analyse	tions and thus describe parts erties of the associated solut and questions of geometric are ablem and are familiar with the nalysis and physics for answer for naming and proving the essente presented connections. State of research in the subject have acquired a confident, projecthods of the lecture. They have them and to work on solution	in's constraints into a system of el- of the solution spaces of Einstein's ions. They have learnt about con- nalysis such as the scalar curvature e interplay of methods of Rieman- ering questions of general relativity. ential results of the lecture as well Students will be able to reflect and ct area. recise and independent handling of ave learned to transfer the methods strategies on their own or in a team. defend them in critical discourse.								

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	The Einstein Constraint	L	f	2	3	yes	wr. o.	90-180	g	100
	Equations	Е	f	2	3	, , ,	or.	o. 20-30	9	
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. — The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.									
Transfer	The module belongs to the same mathematical Physics. Taking be included in the Sections Same specialisation, in accordance	into Study	acco Foci	ount us, A	the d I <i>dval</i>	chosen nced K	personal S nowledge in	tudy Specia <i>Mathemat</i>	lisatior <i>ics</i> or	n, it can <i>Elective</i>
Prerequisites	Basic knowledge of differenti- knowledge of partial differenti- edge of general relativity is a lecture.	al equ	ıatioı	าร is	an a	ıdvanta	ge, but not e	essential. P	revious	knowl-
Responsible Persons	Carla Cederbaum									
Abbreviations:										

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-10	Module Title: Special Topics in Evolution (with Exercise Class)	Equat	ions	for S	Subm	nanifolo		of Module: ulsory Modul	le with	Choice
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS									
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.									
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Special Topics in Evolution Equations for Submanifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	beer	n acc	uired.	s coursewo Whether th	ork. For part e examinatio		
Literature	Possible References :									
	Klaus Ecker: Regulari	ty the	ory fo	or me	ean c	urvatu	re flow. Birk	khäuser 2004	4.	
Transfer	The module belongs to the Sematical Physics. Taking initincluded in the Sections Studies included in the Sections Studies in accordance with	o acc dy Fod	ount <i>us</i> , A	the A <i>dva</i>	chos nced	en per Knowl	sonal Stud edge in Ma	y Specialisa <i>thematics</i> o	tion, it Elect	can be
Prerequisites	Knowledge from the module of differential geometry is rec			n to F	Partia	al Differ	ential Equa	tions and ba	sic kno	owledge
Responsible Persons	Gerhard Huisken									

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-11	Module Title: Special Topics in Evolution (without Exercise Classes)	Equat	ions	for :	Subn	nanifolo		Type of Module: Compulsory Module with Choice				
ECTS-Points	3											
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:		Self-St 120 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.											
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Special Topics in Evolution Equations for Submanifolds	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	cided	by the instri	uctor with a	pprova	I by the		
Literature	Possible References :											
	Klaus Ecker: Regulari	ty the	ory fo	or m	ean c	curvatu	re flow. Birk	häuser 2004	4.			
Transfer	The module belongs to the <i>S</i> ematical Physics. Taking in included in the Sections Stuccialisation, in accordance with	o acc dy Fod	ount cus, A	the A <i>dva</i>	chos ncea	en pei Know	sonal Study <i>ledge in Ma</i>	y Specialisa thematics o	tion, it r <i>Elect</i>	can be		
Prerequisites	Knowledge from the module of differential geometry is rec			n to	Partia	al Diffe	ential Equa	tions and ba	sic kn	owledge		
Responsible Persons	Gerhard Huisken											

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

	0		Module Title: Gravitational Collapse and Singularities in General Relativity									
M/- 111	3											
Ti i Ol	Workload: 90 h	Time 30 h	in C	lass	:		Self-St 60 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 2 SWS											
	The course is divided into the in general relativity, the cause we will study singularities and And finally we will study Perholes, the phenomenon of gaingularities, and some examicosmic censorship conjecture. • Causality theory: — Time orientation. • Singularities: — Raychoudhuri's. • Black holes: — Cosmic censors.	sal hid the trose's gravitation ples of the trose, cause equate	erarc celel s cos tiona of gr con	thy a corate smic captured to the corate capture captu	ind ved sir censillaps tiona is as chy, q	rarious ngularit sorship se, which al collap follows global r	theorems of theorems of theorems of theorems of the theorems o	elated to ca by Penrose some properason for the parently doe	ausality and H erties e form s not c	y. Then lawking. of black ation of		
	Students have acquired in-d general relativity. They will le ing singularity theorems. The naked singularities. They are well as categorise and explai and critically scrutinise the cu	arn to by will able on the i	app also to n elati	ly to get ame onsi	oolog an ov and nips p	gical me verview prove present	ethods in ca of cosmic the main st ed. Student	usality theo censorship o atements of s will be abl	ry and conject the le e to re	in prov- ture and cture as		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	b ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Gravitational Collapse and Singularities in General Relativity Whether the examination is	L writte	f n or	2 oral	3 is de	no ecided I	wr. o. or.	o. 20-30	g pprova	100		

Literature	Possible References :
	Robert M. Wald: General Relativity. The University of Chicago Press 1984.
	 Stephen W. Hawking and George F. R. Ellis: The large scale structure of spacetime. Cambridge Monographs on Mathematical Physics 1973.
	 Pankaj S. Joshi: Gravitational collapse and spacetime singularities. Cambridge University Press 2007.
	Barret O'Neill: Semi-Riemannian Geometry with applications to relativity. Academic Press 1983.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of relativity is required to follow the course.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-35	Module Title: Non-Linear Elliptic and Page Equations	araboli	ic P	artia	l Dit	fferenti		of Module: ulsory Modu	le with	Choice		
ECTS-Points	6											
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass			Self-Si 120 h	tudy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3	1-3										
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 2 SWS										
Content	 Minimum surface ope Parabolic geometric e Hölder continuity accordinates 	 Semilinear and quasilinear elliptic and parabolic partial differential equations; Minimum surface operator and surfaces of prescribed mean curvature; Parabolic geometric equations, e.g. flow along the mean curvature; Hölder continuity according to De Giorgi and Nash; Inner regularity and boundary regularity of solutions. 										
Objectives	Students have learnt analytic differential equations of secramples of partial differential techniques were learnt to property. The students are capable of assessing and explaining the ln the exercise classes, students they have learnt and to present their problem sol research.	ond or equatione the naming presents in dents in	der dions e exi g and ented nave apply	of the from isten d pro l con acqu ther	e elli mat ce a ving necti uired n inc	ptic an thematind register the estimate in the	d parabolic cal physics ularity of so sential resu ence in the ently to othe	type. Using and different plutions to so lits of the lecter technical her problems.	g cond ntial ge uch eq ture as nandlin They	rete ex- eometry, uations. s well as g of the are able		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Non-Linear Elliptic and Parabolic Partial Differential Equations	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	beer	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	 Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptic PDEs. AMS 1998.
	Gary Lieberman: Second order parabolic differential equations. World Scientific 1996.
	Fritz John: Introduction to Partial Differential Equations. Springer 1982.
	Jürgen Jost: Partielle Differentialgleichungen. Springer 1998.
	 David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications, Pure and Applied Mathematics, Vol. 88. Academic Press 1980.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Introduction to Partial Differential Equations is a prerequisite.
Responsible Persons	Gerhard Huisken

Grading System : g=graded, ng=not graded

 $\label{thm:master} \textbf{Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolious exam, Pr=presentation, Pr=presen$ Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-60-36	Module Title: Fully Non-Linear Elliptic and Equations	Parab	olic	Parti	al Di	fferenti		Type of Module: Compulsory Module with Choice				
ECTS-Points	3						·					
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	:		Self-St 60 h	tudy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3	1-3										
Language of Instruction	English											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	The lecture investigates fully non-linear elliptic and parabolic partial differential equations of second order. Classical examples are the Monge-Ampère equation, the equation of prescribed Gaussian curvature or, more general, equations of prescribed other scalar invariants of curvature together with their parabolic analogues. They also arise in problems of stochastic control and optimal transport. The course establishes basic techniques for solving Dirichlet- and Neumann boundary value problems for such equations, in particular techniques for deriving the necessary a priori estimates for solutions.											
Objectives	The students have learnt and linear partial differential equation of such differential equation of solutions of such equation apply the methods they have The students are capable of assessing and explaining the In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	ations s, tecl ns and e learn namin prese have a ethod them	of the niquest to g and entection of the acquest of	e ellipies vassiothed problem of the legarithms of the legarithms. The legarithms of	ptic avere ociater proporting nected acousting rk or	and par learnt ed bou bblems the es ions. nfident, e. They n solution	abolic type, to prove the ndary value and related sential resurprecise and have learn on strategie	. Using conce existence e problems. If equations alts of the lected independent on their own their own.	erete ex and re Stude indepe iture as ent har er the r	camples egularity ents can indently. It is well as indling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Fully Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instr	uctor with a	pprova	I by the		

Literature	Possible References :
	 David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001.
	 Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptc PDEs. AMS 1998.
	Gary Lieberman: Second order parabolic differential equations. World Scientific 1996.
	 Ilya J. Bakelman: Convex functions and nonlinear geometric elliptic equations. Springer 1994.
	Luis Caffarelli, Xavier Cabrè: Fully nonlinear elliptic equations. AMS 1995.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'MAT-55-27 Fully Nonlinear Elliptic Equations' due to the large overlap in content.
Prerequisites	At least one course on partial differential equations, basic concepts of differential geometry.
Responsible Persons	Gerhard Huisken
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-05	Module Title: Groups and Representations	s	Type of Module: Compulsory Module with Choice								
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h								
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	English	English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	ecture 4 SWS + Exercise Class 2 SWS									
Content		 Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. 									
		 Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. 									
	Applications: symmetry	Applications: symmetries and degeneracies in quantum mechanics, selection rules.									
	Representations of fi potents.	nite groups: group algebra, re	egular representation, ideals, idem-								
	Symmetric groups: Yes	oung tableaux, Young operato	rs, dimensions and characters.								
	Applications: identica	I particles in quantum theories	S.								
	Lie groups: Haar mea	asure, representations, Lie alg	ebras.								
	Tensor representation	ns of classical groups: symme	try classes, Young tableaux.								
	Applications: SU(2) a	and SU(3) in particle physics (s	spin, isospin, flavour)								
	- Applications: no	esentations of the Lorentz and otion of particles in quantum the	• .								
Objectives	apply these abstract algebra developed a deepend under The students are familiar w sentation theory of groups i essential results of the lectur In the exercise classes they the terms, statements and m on new problems, to analyse	The studens know the basic concepts of group and representation theory. They are able apply these abstract algebraic concepts in the context of theoretical physics and have, the developed a deepend understanding for the connections between mathematics and physic. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connection. In the exercise classes they have acquired a confident, precise and independent handling the terms, statements and methods of the lecture. They have learned to transfer the method on new problems, to analyse them and to work on solution strategies on their own or in a teat. They are able to present their solutions and, if necessary, defend them in critical discourse.									

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	- Status	SMS 4	e ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Groups and Representations	E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise cer examination the coursework n oral is decided by the instructe	nust ł	nave	beei	n acc	uired.	Whether the	e examinatio		
Literature	Irene Verona Schenste Mechanics. NEO Press Barry Simon: Represe	 Possible References: Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. Barry Simon: Representations of Finite and Compact Groups. AMS 1996. Wu-Ki Tung: Group Theory and Physics. World Scientific 1985. 								
Transfer	The module belongs to the S Physics. Taking into account the Sections Study Focus, Ad accordance with the restrictive The module cannot be take Groups' due to the large overl	the clean contract the request of the contract	hose ed Kr uirem ethe	n pe nowle nents r wit	rson edge of th	al Stud in Mat ne resp	ly Specialisa hematics or pective secti	ation, it can <i>Elective Sp</i> on.	be inc ecialis	luded in ation, in
Prerequisites	There are no further prerequis	sites.								
Responsible Persons	Stefan Keppeler	Stefan Keppeler								
Abbreviations:	_aradod_na_not_aradod									

Grading System : g=graded, ng=not graded

 $\label{eq:master} \mbox{Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio \\ \mbox{Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, }$

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-31	Module Title: Mathematical Methods for Co	onden	sed I	Matte	er Ph	ysics		of Module: ulsory Modul	le with	Choice	
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass	:	Self-Study: 120 h					
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	English										
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 2 SWS									
Content	 The course provides an introduction, with an analytic perspective, to the basic mathematical tools necessary to have a deeper understanding of the mathematical theories of topological insulators. In particular, the course will cover the following topics: Direct integrals on Hilbert spaces. Stability theorems for relatively bounded perturbations. Bloch-Floquet transformations and their application to the periodic Schrödinger operator. Introduction to the theory of vector bundles and Chern classes. Definition of the Bloch bundle. 										
Objectives	The students know, understathey have developed a deep a natural way in solid state essential results of the lectur. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present their	er uno physic e as w have a ethods them a	derstes. The ell as acquires of the ell as acquires of the ell and	andi he s ass ired he le o wo	ng o stude essir a cor ecture erk or	f how rents are ng and nfident, e. They	mathematica e capable of explaining to precise and have learn on strategies	al concepts of naming are the presente d independe ed to transfe s on their ow	are apend produced the design of the design	oplied in ving the ections. Indling of methods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Mathematical Methods for Condensed Matter Physics	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	quired.	Whether the	e examination			
Transfer	The module belongs to the Mathematical Physics. Takin be included in the Sections Specialisation, in accordance	g into Study	acco Foc	ount <i>us, 1</i>	the o	chosen nced K	personal S Inowledge in	itudy Specia n Mathemat	alisatio <i>ics</i> or	n, it can <i>Elective</i>	
Prerequisites	In terms of content, only kno Mathematik are required.	wledg	e fro	m th	e ba	sic cou	rses of the	first two yea	rs in th	ne B.Sc.	

Responsible Persons	Stefan Teufel
Abbreviations: Grading System :	g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-32	Module Title:Type of Module:Mathematical Aspects of the Quantum Hall EffectCompulsory Module with Choice								Choice	
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	_	lass	•		Self-St 120 h	udy:		
Duration	1 Semester						·			
Frequency	not regularly	not regularly								
Term	1-3									
Language of Instruction	English	inglish								
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 2 SWS								
Objectives	The course is focused on the In particular, the course will of the Particular, the course will of the Review of the classical Analysis of the Landar Linear response theor Wannier functions and Magnetic perturbation The students have learned, the lectures. In particular, the lectures. In particular, the and concepts from the lecture to put it into a larger framework attention of the special put it into a larger framework assignment and independent acquaintant and independent acquaintant in the special particular in the special particular independent acquaintant independent in	Hall of Hall o	he for effect and the formal terms of the form	t and ian a vatio ian a vatio ions da fo	ng to history indo	ppics: prical ir f the go the Kub e Hall of a. come for a dee to nar ain the descri	eometry of to formula. conductivity amiliar with p understaine and provion context de be and critistudents dements, and	the conception of the essen veloped in totally challer velop a continued by the essen welcomes and the essen veloped in the essen velop	ts explained the lecting the lecting the fident, explained	ained in ematical tements ure and current precise, d in the
	lectures. They learn how to to develop solution strategies solutions and to stand for the	transi on th	fer th neir c	iese wn a	meth and w	nods to vithin a	new proble group. The	ems, to ana	lyse th	em and
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Mathematical Aspects of the Quantum Hall Effect	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	uired.	Whether the	e examination		
Transfer	The module belongs to the matical Physics. Taking the account, the module can be a Mathematics or Elective Spe	perso assign	nal s	speci the	alisa	tion an	d the restri	ctions of the	e section	ons into

Prerequisites	It is strongly recommended that the students have attended the course mathematical methods for condensed matter physics.
Responsible Persons	Stefan Teufel

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

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Status : o=obligatory, f=facultative

Module Number: MAT-65-33	Module Title: Wave Equations of Relativistic Quantum Mechanics							of Module: ulsory Modu	le with	Choice	
ECTS-Points	6						<u> </u>				
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 120 h										
Duration	1 Semester										
Frequency	not regularly	not regularly									
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	ecture 2 SWS + Exercise Class 2 SWS									
Content	Klein-Gordon equatio	Klein-Gordon equation.									
	Dirac equation.	Dirac equation.									
	Representation Theory of the Lorentz Group.										
	Relativistic Many-Par	ticle S	yster	ns (N	/lulti-	Time F	ormalism).				
Objectives	The students obtain knowled mechanics. They learn and solutions of the Klein-Gordotheir properties. The studen results. They are able to not lecture as well as to explain framework. They are able to the specific area. Through homework assignment and independent acquaintal lectures. They learn how to develop solution strategies solutions and to stand for the	alytical on equits are ame an the condescription description transfer on the condescription and the condescription are the condescription and the condescription are the condescription and the condescription are also are also and the condescription are also are	tech ation able and p onte ibe a ind e th th fer th	nniquato a rove xt de and control xercine no nese own a	the ssess the ceveloperities se clations methand v	or provous pro	equation, a hysical relevant statement the lecture llenge the control students dements, and a new problem group. The	stence and as well as for vance of the other and concerned and to put current state evelop a commethods exems, to analey are able to	unique or inves mathe cepts f it into of res fident, cplaine lyse th	emess of stigating ematical rom the a larger earch in precise, d in the em and	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Wave Equations of Relativistic Quantum	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	Mechanics In this module an exercise of tion is written or oral is decided.						as coursew	ork. Whethe			

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Literature	Possible References :
	Bernd Thaller: The Dirac equation. Springer 1992.
	 Silvan S. Schweber: An introduction to relativistic quantum field theory, Chap. 2-4. Dover Books 2005.
	Paul R. Garabedian: Partial differential equations. AMS 1998.
	Erich Zauderer: Partial differential equations of applied mathematics. Wiley 2006.
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge in Quantum Mechanics and Special Relativity Theory is expected. Moreover, basic knowledge of Functional Analysis and Partial Differential Equations would be helpful but is not mandatory.
Responsible Persons	Roderich Tumulka

Grading System : g=graded, ng=not graded

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T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-36	Module Title: Quantum Information Theory		of Module: ulsory Modul	e with	Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass:			Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS	Lecture 4 SWS								
Objectives	 Basic notions on the universal quantum computer: Quantum gates, quantum circuits, universality and measurements. Quantum algorithms: Deutsch-Jozsa, Shor and Grover. Quantum communication: No-cloning theorem, quantum teleportation and superdense coding. Quantum key distribution. Physical realizations: DiVincenzo criteria, Cirac Zoller quantum computer, Circuit QED. Decoherence and open quantum systems. Quantum error correction. Fault tolerant quantum computing. Alternative quantum computing models: Adiabatic quantum computation. Introduction to the theory of entanglement: Definition, criteria and measurement of entanglement, multipartite entanglement. Students are familiar with the basic concepts and theoretical tools of quantum information processing. They understand the concept of quantum algorithms and quantum circuits and have learnt how to program a quantum computer. They understand how important quantum algorithms work and can describe quantum channels. They know the principles of quantum error correction and entanglement theory and also understand the most advanced concepts of physical realisations of quantum computers. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connec-									
	In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present their	ethods them a	of th nd to	ne le o wo	cture rk on	e. They solution	have learn on strategies	ed to transfe s on their ow	er the n n or in	nethods a team.
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
							90-180	g	100	

Literature	Possible References :			
	 Michael A. Nielsen, Isaac L. Chuang: Quantum Computation and Quantum Information. http://mmrc.amss.cas.cn/tlb/201702/W020170224608149940643.pdf 			
	 Ronald de Wolf: Quantum Computing: Lecture Notes. https://homepages.cwi.nl/ ~rdewolf/qcnotes.pdf 			
	• John Preskill: Quantum Computation. Lecture Notes. http://theory.caltech.edu/~preskill/ph219/index.html			
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.			
Prerequisites	There are no prerequisites.			
Responsible Persons	Angela Capel Cuevas			
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio				

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-37	Module Title: Matrix Analysis and Applications Type of Module: Compulsory Module with Che								Choice	
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time i 60 h	in Cla	ass:			Self-St 120 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 3 SWS + Exercise C	Lecture 3 SWS + Exercise Class 1 SWS								
Content	Foundations of opera	tors and	d mat	trice	s: So	quare i	matrices and	d tensor pro	ducts.	
	Mappings and algebra	as.								
	Positive matrices.	Positive matrices.								
	Functional calculus and derivations.									
	Matrix monotone fund	tions ar	nd co	nve	xity.					
	Matrix means and ine	qualitie	S.							
	Applications in quantu	ım infor	matio	on th	neory	y.				
Objectives	tional analysis. They have I matrices, including topics su entropies, quantum Markov tions of matrix analysis in quand proving the essential resented connections. In the exercise classes they the terms, statements and mon new problems, to analyse	Students have acquired in-depth knowledge of matrix analysis from the perspective of functional analysis. They have become familiar with some aspects of analysis in the context of matrices, including topics such as monotone matrix functions, matrix averages, majorisation, entropies, quantum Markov triplets, etc. They are also familiar with several typical applications of matrix analysis in quantum information theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Marix Analysis and	Type of Course	J Status	SWS a	ECTS 4,5	Coursework	Type of Exam	Dur. of Exam (min) 081-06	Grading	Weight for Grade
	Applications	Е	f	1	1,5	yes	or.	o. 20-30	g	100
	In this module an exercise or examination the coursework oral is decided by the instruc	must ha	ave b	oeen	acq	uired.	Whether the	e examinatio		

Literature	Possible References :
	 Fumio Hiai, Denes Petz: Introduction to Matrix Analysis and Applications. https://math.bme.hu/~petz/matrixPD.pdf
	 Denes Petz: Matrix Analysis with some Applications. https://math.bme.hu/~petz/matbme.pdf
	Rajendra Bhatia: Matrix Analysis. Springer 1997.
	Rajendra Bhatia, Positive Definite Matrices. Princeton University Press 2007.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of functional analysis is desirable.
Responsible Persons	Angela Capel Cuevas
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Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-65-38	Module Title:Type of Module:Hamiltonian SystemsCompulsory Module with Cho									Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h						Self-Study: 180 h		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	5						
Content	mechanics. This builds a bi ometry and dynamical syste are:	The module provides an introduction to the theory of Hamiltonian systems as used in classical nechanics. This builds a bridge between the fields of differential geometry, symplectic geometry and dynamical systems as well as theoretical physics. The main points of the lecture are: • Symplectic manifolds and the canonical 1-form of the cotangent bundle.								
	Darboux-Moser theorem.									
	 Lagrangian and Hami 	Lagrangian and Hamiltonian systems.								
	Integrable systems ar	nd Arno	old-L	iouvi	lle th	eorem				
	Moment mappings.									
	Symplectic reduction.									
	Symplectic manifolds	and to	ric e	ffects	S.					
Objectives	methods of symplectic geom of different areas of mathen theoretical physics. The study the lecture as well as assess In the exercise classes they the terms, statements and mon new problems, to analyse	The students are familiar with the theory of Hamiltonian systems and their investigation using methods of symplectic geometry. They are familiar with the interplay of methods and questions of different areas of mathematics (differential geometry, geometry, dynamical systems) and theoretical physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Hamiltonian Systems	L E	f f	2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	ave	beei	n acc	uired.	Whether th	e examination		

Literature	Possible References :
	Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 1989.
	Ana Cannas da Silva: Lectures on symplectic geometry. Springer 2001.
Transfer	The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the Geometry in Physics module is assumed.
Responsible Persons	Carla Cederbaum

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, } \mbox{LE=lecture with integrated exercises, } \mbox{SL=seminar or lecture, } \mbox{E=exercise class, } \mbox{T=tutorial, P=project, S=seminar, IC=inverted classroom}$

Status : o=obligatory, f=facultative

Module Number: MAT-65-39	Module Title: Propagation of Chaos							of Module: ulsory Modul	le with	Choice
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	udy:		
Duration	1 Semester									
Frequency	regularly	regularly								
Term	1-3	1-3								
Language of Instruction	German	German								
Forms of Teaching and Learning	Lecture 4 SWS									
Content	 Interacting many body systems (quantum and classical), importance of correlations. Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). Explicit treatment of correlations. Large deviations from the expected value. 									
Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Propagation of Chaos	L ?	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	uired.	Whether th	e examination		
Literature	Possible References: Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.									
Transfer	The module belongs to the S ing into account the chosen Study Focus, Advanced Kno with the restrictive requireme	persor wledg	nal S e in l	tudy <i>Math</i>	Sped emai	cialisati tics or i	on, it can b Elective Sp	e included i	n the S	Sections

Prerequisites	In addition to the basics of analysis and linear algebra, the content of the Stochastics module is a prerequisite.
Responsible Persons	Peter Pickl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-01	Module Title: Algorithms of Numerical Mat		of Module: ulsory Modul	le with	Choice						
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: Self-Study: 180 h									
Duration	1 Semester										
Frequency	regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS									
Content	 Advanced, important algorithms of numerics (without differential equations) such as: Fast Fourier transformation; QR algorithms for the calulation of eigenvalues; Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; Simplex method and interior point methods in linear optimisation. 										
Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Algorithms of Numerical Mathematics	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	nave	bee	n acc	uired.	Whether th	e examination			
Literature	oral is decided by the instructor with approval by the Board of Examiners. Possible References: Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.										
Transfer	The module belongs to the Taking into account the chos tions Study Focus, Advanced dance with the restrictive requirements.	en pe d <i>Kno</i> v	rson: vledg	al St ge in	udy S <i>Matl</i>	Special hematic	isation, it ca s or <i>Electi</i> v	an be includ	led in t	he Sec-	

Prerequisites	There are no further prerequisites.
Responsible Persons	Christian Lubich, Andreas Prohl
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-02	Module Title: Numerics of Stationary Differential Equations							Type of Module: Compulsory Module with Choice				
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass:			Self-St 180 h	Self-Study: 180 h				
Duration	1 Semester											
Frequency	regularly											
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	Lecture 4 SWS	Lecture 4 SWS										
Content		Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.										
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.						pable of plaining adling of methods a team.					
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Numerics of Stationary	L	f	4	6	yes	wr. o.	90-180	g	100		
	In this module an exercise certificate is to be acquired as coursework. For partic examination the coursework must have been acquired. Whether the examination oral is decided by the instructor with approval by the Board of Examiners.						ticipation in the					
Literature	Possible References :											
	 Dietrich Braess: Finite Elemente. Springer Spektrum 2013. Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986. 						n. Teub-					
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.											
Prerequisites	Knowledge of the numerical	algorit	hms	mod	ule is	s helpfı	ul, but not n	nandatory.				
Responsible Persons	Christian Lubich											

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-03	Module Title: Numerics of Instationary Differential Equations							Type of Module: Compulsory Module with Choice			
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h						Self-Study: 180 h			
Duration	1 Semester										
Frequency	regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS										
Content	Numerical treatment of transient (i.e. time-dependent) differential equations, such as: stiff ordinary differential equations, stochastic differential equations, parabolic and hyperbolic partial differential equations.										
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of instationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Numerics of Instationary Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	quired.	Whether th	ie examinatio			
Literature	Possible References: • Ernst Hairer, Gerhard Wanner: Solving Ordinary Differential Equations II. Stiff Problems. Springer 1996. • Vidar Thomee: Galerkin Finite Element Methods for Parabolic Problems. Springer 1997.										
Transfer	The module belongs to the Study Specialisation Numerical Mathematics and Optimisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.										
Prerequisites	Knowledge from the module helpful, but not absolutely ne			l Ma	then	natics (of Stationar	ry Differentia	ıl Equa	ations is	
Responsible Persons	Christian Lubich										

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-04	Module Title: Ordinary Differential Equatio		of Module: ulsory Modu	le with	Choice						
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-St 180 h	Self-Study: 180 h			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	S							
Content	 Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. Numerical approximation: linear multi-step processes, adaptive processes, geometric integration. 										
Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Ordinary Differential Equations - Analysis and Numerics	L E	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										
Literature	Possible References: • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.										
Transfer	The module belongs to the S merical Mathematics and Opticalisation, it can be included ics or Elective Specialisation section.	otimisa in the	<i>tion.</i> Sect	Tak ions	ing ir <i>Stuc</i>	nto acc <i>ly Focu</i>	ount the ch s, <i>Advance</i>	osen persor d Knowledg	nal Stu e in Ma	idy Spe- athemat-	

Prerequisites	Basic knowledge of the theory of the ordinary differential equations are required, such as those taught in the module Algorithms of Numerical Mathematics.
Responsible Persons	Andreas Prohl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-05	Module Title: Optimal Control Theory with tions	Optimal Control Theory with Ordinary Differential Equa-												
ECTS-Points	5													
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	e in C	Class	:		Self-St 180 h	udy:						
Duration	1 Semester						·							
Frequency	not regularly													
Term	1-3													
Language of Instruction	German	rman												
Forms of Teaching and Learning	Lecture 2 SWS	cture 2 SWS												
Content	Brief overview of exis	Brief overview of existence and uniqueness theory for ODEs.												
	Numerical solutions to	ODE	s.											
	Introduction to optima	l cont	rol pr	oble	ns w	ith OD	Es.							
	Existence and unique problems).	eness	thec	ory fo	r lin	ear qu	adratic opti	mal control	proble	ms (LQ				
	Pontryagin's maximur	n prin	ciple											
	Numerical approxima	tion of	LQ	orobl	ems.									
Objectives	Students are familiar with the tions and various approaches statements on unambiguous essential results of the lectur. In the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	es to solvate as whave them	olvin bility ell as acqu s of t	g the The s assired in the letter the second in the letter the let	e pro estudessir a cor ecture rk or	blem. dents and	They are all re capable explaining to precise and have learn on strategies	so familiar of naming a he presented independe ed to transfes on their owner in critical	with quand produced p	alitative ving the ections. Idling of nethods a team.				
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade				
	Optimal Control Theory with Ordinary Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100				
	In this module an exercise co	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.												
Literature	Possible References :													
	Matthias Gerdts: Opti	mal C	ontro	ol of (DDE	s and D	AEs. De G	ruyter 2012.						

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Analysis and the sub-module Introduction to Ordinary Differential Equations is assumed.
Responsible Persons	Andreas Prohl

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-06	Module Title: Numerics of Differential Equa	ations	of Sı	urfac	es			of Module: ulsory Modu	le with	Choice
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h		lass	:		Self-St 120 h	tudy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS	re 2 SWS								
Content	Numerical treatment of	of diffe	renti	al ec	luatic	ns on	moving (or	stationary) s	urface	S.
		Semi- and fully discretization of elliptic and parabolic equations on surfaces using surface finite elements and efficient time integrators.								
	Implementation of the	• Implementation of the algorithms. Students have learned the basic methods and techniques of numerics for problems on (moving) surfaces. In particular, they are familiar with the discussed energy techniques, which are ery strong, general and rich in application, even in surface-independent areas of numerics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods in new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.								
Objectives	ing) surfaces. In particular, to very strong, general and rick The students are capable of assessing and explaining the In the exercise classes they the terms, statements and mon new problems, to analyse									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Numerics of Differential Equations of Surfaces	L ü	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise or examination the coursework oral is decided by the instruc	must	nave	bee	n acc	uired.	Whether th	e examinati		
Literature		· · · · · ·								
Transfer	The module belongs to the Taking into account the chos tions Study Focus, Advanced dance with the restrictive rec	en pe d <i>Kno</i> i	rson vledg	al St ge in	udy s <i>Mati</i>	Special <i>hemati</i> d	isation, it cans s or <i>Electi</i> v	an be includ	led in t	the Sec-
Prerequisites	Knowledge of the numerical	algorit	hms	mod	dule i	s helpfı	ul, but not m	nandatory.		_

Responsible Persons	Christian Lubich
Abbreviations:	
Grading System : g	=graded, ng=not graded
Examination Type : M	IT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	electure, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, etutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-11	Module Title: Stochastic Differential Equation	ons						of Module: ulsory Modul	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-S 180 h	tudy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS										
Content	Stochastic processes,	 Stochastic processes, filtrations, martingales. Wiener process, random walk, Donsker's theorem. 									
	Wiener process, rander										
	Diffusion semigroup, I	to's in	tegra	ıl.							
	Solution of a stochast	ic diffe	renti	al ed	quatio	on.					
	Markov property, Mall	avin c	alcul	us, r	ough	path t	heory.				
Objectives	Students master the basic p differential equations. The s of the lecture as well as asse In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present their	tudent essing have a ethode them a	s are and acqui s of t and t	e cap explired he le o wo	able aining a corecture ork or	of nar g the p nfident, e. They n solution	ning and processoring and processoring are precise are have learred at the strategies.	roving the estance tions. In the discount of the estance of the es	ssentia ent har er the r vn or in	I results adling of methods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Stochastic Differential	L	f	4	6	yes	wr. o.	90-180	g	100	
	Equations	ü	f	2	3	,,,,	or.	o. 20-30	9		
	In this module an exercise context examination the coursework oral is decided by the instruction	must l	nave	bee	n acc	uired.	Whether th	ne examination			
Literature	Possible References :			_	_						
	Bernt Oksendal: Stoc	Bernt Oksendal: Stochastic differtial equations. Springer 2000.									
Transfer	Stochastics. Taking into account in the Sections Study Focus,	e module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> and ochastics. Taking into account the chosen personal Study Specialisation, it can be included the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge of the modules S ory from the Bachelor of Scientific Sci							ion and Mea	surem	ent The-	

Responsible Persons	Andreas Prohl
Abbreviations:	
Grading System : g	=graded, ng=not graded
Examination Type : M	T=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, =tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-12	Module Title: Introduction to Stochastic Dif	ferent	ial E	quati	ons -	- Part 1		of Module: ulsory Modul	e with	Choice	
ECTS-Points	5										
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	•		Self-St 180 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German	rman									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	cture 2 SWS + Exercise Class 1 SWS									
Content	Solution concepts for a stability of stochastic stability.	 Introduction to Brownian motion and stochastic integration. Solution concepts for stochastic differential equations. Stability of stochastic differential equations. Numerical approximation of stochastic differential equations. 									
Objectives	differential equations. The st of the lecture as well as asse In the exercise classes they the terms, statements and m on new problems, to analyse	Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Stochastic Differential Equations - Part 1	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	bee	n acc	quired.	Whether th	e examination			
Literature	Possible References :										
	Bernt Oksendal: Stock	nastic	diffe	renti	al eq	uations	s. Springer	2000.			
Transfer	and Stochastics. Taking into included in the Sections Stud	The module belongs to the Study Specialisations Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be accluded in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the modules from the Bachelor of Science						on to Integr	ation and M	easure	Theory	
Responsible Persons	Andreas Prohl										

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-15	Module Title: Numerics of Stochastic Diffe	rential	Equ	atior	ıs			f Module: Ilsory Modul	le with	Choice
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	_	lass	:		Self-St 60 h	udy:		
Duration	1 Semester									
Frequency	not regularly									
Term	1-3	3								
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	Random number gene	Random number generator, Ito-Taylor expansion.								
	Strong and weak appropriate the strong appro	oxima	ition,	con	siste	ncy.				
	Euler-Maruyama meth	nod, M	lilstei	n me	ethoc	d, stoch	astic Runge	e-Kutta meth	nod.	
	Approximation of stop	ped d	iffusi	on p	roces	sses.				
Objectives	Students master the basic pritions of stochastic differential essential results of the lecture	l equa	tions	. Th	e stu	dents a	are capable	of naming a	nd pro	ving the
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Numerics of Stochastic Dif- ferential Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instru	uctor with a	pprova	al by the
Literature	Possible References :									
	Peter E. Kloeden, Ecl tions. Springer 1999.	khard	Plate	en: N	Nume	erical s	olution of st	ochastic dif	ferenti	al equa-
Transfer	The module belongs to the S Stochastics. Taking into account the Sections Study Focus, in accordance with the restrict	ount th <i>Adva</i>	e ch nced	oser <i>Kno</i>	pers wlea	sonal S <i>lge in N</i>	tudy Specia <i>lathematics</i>	llisation, it ca or <i>Elective</i>	an be i	ncluded
Prerequisites	Knowledge from the Stochas	tics m	odul	e in t	the B	achelo	r of Science	is required	-	
Responsible Persons	Andreas Prohl									

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-16	Module Title: Stochastic Optimal Control in	Infini	te Di	men	sions	3		of Module: ulsory Modu	le with	Choice	
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	•		Self-St 60 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	rman or English									
Forms of Teaching and Learning	Lecture 2 SWS										
Content	overlap of analysis, optimisat	ne course covers aspects of stochastic optimal control, an interdisciplinary subject at the verlap of analysis, optimisation, partial differential equations and stochastics, which lead the articipants to topics in current research. The choice of contents takes the knowledge of the articipants into consideration.									
Objectives	to a current area of researc	The students aquire deepend knowledge in stochastic optimal control that introduce them to a current area of research and that allow them to start a small research project. The students can name and prove the central results of the lecture and they can explain their intrinsic connections									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Stochastic Optimal Control in Infinite Dimensions	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instr	uctor with a	pprova	l by the	
Transfer	The module belongs to the sand Stochastics. Taking into included in the Sections Stuccialisation, in accordance with	acco	ount cus, 7	the d Adva	chos ncea	en per Know	sonal Study <i>ledge in Ma</i>	Specialisa thematics o	tion, it <i>Electi</i>	can be	
Prerequisites	The contents of the module Numerical Mathematics are assumed.										
Responsible Persons	Andreas Prohl	Andreas Prohl									
Examination Type : N	=graded, ng=not graded //T=master's thesis, or.=oral exa =lecture, LE=lecture with inte							-	•		

Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-20	Module Title: Introduction to Optimisation		of Module: ulsory Modul	le with	Choice						
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass			Self-St 120 h	udy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German	rman									
Forms of Teaching and Learning	Lecture 3 SWS + Exercise CI	ass 1	SWS	3							
Content	problems with constraintFoundations of the theDuality theory for convergence	 Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. Foundations of the theory of convex sets and functions. Duality theory for convex and linear optimisation problems. Solution methods for linear optimisation problems. 									
Objectives	Students know and understatimisation problems. They heconomics, technology or phimitations of using the methotial results of the lecture as with the exercise classes they have terms, statements and meaning on new problems, to analyse they are able to present their	ave le ysics. Ids. The ell as nave a ethods them a	earnt The ne st asse acqui s of t	to a will uder essinated a he le o wo	pply If be its ar g and a cor cture rk on	the mable to e capa dexplain explain they are the map of the map o	ethods to so critically as ble of nami ining the proprecise an have learn on strategies	imple problessess the poing and provesented conditional independent to transfession their own	ems re ossibiliting the nnection ent har er the r	lated to ties and e essen- ns. ndling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	J Status	SWS 3	ECTS 4,5	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Introduction to Optimisation E f 1 1,5 yes or. 90-180 g										
	examination the coursework i	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									
Literature	Possible References :										
	Methoden. Springer 20	 Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006. 									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.	
Prerequisites	Only basic knowledge of linear algebra and analysis is required.	
Responsible Persons	Christian Lubich	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-70-21	Module Title: Non-Linear Optimisation							of Module: ulsory Modu	le with	Choice					
ECTS-Points	9														
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-Si 180 h	Self-Study: 180 h							
Duration	1 Semester														
Frequency	regularly														
Term	1-3														
Language of Instruction	German														
Forms of Teaching and Learning	Lecture 4 SWS														
Content	method. Restricted optimisation Abadie CQ, KKT cond Linear programme, du	 Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method. Restricted optimisation, Farkas' lemma, tangent cone. Abadie CQ, KKT conditions, Slater conditions. Linear programme, duality, simplex method. Penalty and barrier methods, interior point method. 													
Objectives	• Nonlinear programs, S Students master the basic proptimisation problems. The sof the lecture as well as asselin the exercise classes they the terms, statements and monnew problems, to analyse They are able to present their	inciple tuden ssing have a ethods	es ar ts ar and acqu s of t	d ted e cap explaired a he le o wo	chniq pable aining a cor cture rk on	lues of e of nar g the partident, e. They n solution	analysis ar ming and presented co precise ar have learn on strategie	nd numerics roving the est onnections. Id independent and to transfers on their ow	ssentia ent har er the r vn or in	I results adling of methods a team.					
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade					
	Non-Linear Optimisation	L ü	f	4	6	yes	or.	20-30	g	100					
	In this module an exercise ce examination the coursework oral is decided by the instruction	must l	nave	beer	acc	uired.	Whether th	e examinati							
Literature		Possible References: Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.													
Transfer	The module belongs to the Taking into account the chos tions <i>Study Focus</i> , <i>Advanced</i> dance with the restrictive req	en pe <i>l Kno</i> ı	rson: vledg	al Sti ge in	udy S <i>Matl</i>	Special hematic	isation, it c s or <i>Electi</i>	an be includ	led in t	he Sec-					

Prerequisites	There are no further prerequisites.
Responsible Persons	Andreas Prohl
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, P=project, S=seminar, IC=inverted classroom} \\ \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-70-22	Module Title:Type of Module:Optimisation with Differential EquationsCompulsory Module with Choice											
ECTS-Points	9											
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h											
Duration	1 Semester											
Frequency	not regularly	not regularly										
Term	1-3											
Language of Instruction	German or English											
Forms of Teaching and Learning	ecture 4 SWS											
Content	 Brouwer-Minty theorem Gateaux and Frechet Proof of existence of control Adjoint, convergent op 	 Direct method in the calculus of variations, Euler-Lagrange equation. Brouwer-Minty theorem, non-linear evolution equations. Gateaux and Frechet differentiability. Proof of existence of optimal controls, necessary optimality conditions. Adjoint, convergent optimisation methods in Banach spaces. Variational discretisation concepts. 										
Objectives	Students master the basic pr totypical control problems we students are capable of nan assessing and explaining the In the exercise classes they the terms, statements and me on new problems, to analyse They are able to present thei	th corning a presentate the presenta	nstra nd p ented acqui s of t	ints rovir con red he le o wo	in the nection control to the contro	e form e esse ions. nfident, e. They n solution	of partial on the precise and have learn on strategies	differential east of the lection of	quationure as ent hare the r	well as adding of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Optimisation with Differential Equations	L ü	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise ce examination the coursework oral is decided by the instruc	must ł	nave	bee	n acc	uired.	Whether th	e examinatio				
Literature	Possible References : • Michael Hinze, Rene constraints. Springer 2		u, M	icha	el Ull	brich, §	Stefan Ullric	ch: Optimiza	ition w	ith PDE		

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Andreas Prohl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \quad : L=\mbox{lecture, } \mbox{ L=lecture with integrated exercises, } \mbox{ SL=seminar or lecture, } \mbox{ E=exercise class, } \mbox{ T=tutorial, } \mbox{ P=project, } \mbox{ S=seminar, } \mbox{ IC=inverted classroom.}$

Status : o=obligatory, f=facultative

Module Number: MAT-70-25	Module Title: Numerical Optimisation							of Module: ulsory Modu	le with	Choice			
ECTS-Points	5												
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	Class			Self-St 180 h	udy:					
Duration	1 Semester												
Frequency	not regularly	not regularly											
Term	1-3												
Language of Instruction	German												
Forms of Teaching and Learning	Lecture 2 SWS												
Content	An introduction to numerical methods for solving optimisation problems in science and technology with a focus on continuous optimisation and non-linear programming.												
	Basic concepts of opt	Basic concepts of optimization.											
	Unconstrained optimi	zation	and	New	ton-ty	ype alg	orithms.						
	Optimization with equ	ations	as c	onst	raints	S.							
	Optimization with inection	qualitie	s as	cons	strain	its.							
	Applications:												
	Economy: resoScience: mode				•				. expe	rimental			
	design.												
	 Engineering: digital airplanes, digital 				ation	or teci	inicai syste	ms sucn as	briage	es, cars,			
Objectives	Students are familiar with the are capable of naming and and explaining the presented in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	proving d conne have a lethods them a	g the ection acqu s of t and t	e ess ins. ired the le	entia a cor cture rk on	I resul nfident, e. They n solution	ts of the led precise and have learn on strategies	cture as wel d independe ed to transfe s on their ow	I as as ent har er the r n or in	sessing adling of nethods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Numerical Optimisation	L ü	f	2	3 2	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise context examination the coursework oral is decided by the instruction	must h	nave	bee	n acc	uired.	Whether the	e examination					

Literature	Possible References :
	Jorge Nocedal, Stephen J. Wright: Numerical Optimization. Springer 2006.
	Stephen Boyd, Lieven Vandenberghe: Convex Optimization. Cambridge University Press 2004.
Transfer	The module belongs to the Study Specialisation Numerical Mathematics and Optimisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further requirements.
Responsible Persons	Andreas Prohl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-30	Module Title: Type of Module: Theoretical Aspects of Machine Learning Compulsory Module with Choice										
ECTS-Points	6										
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h		Class			Self-Si 120 h	tudy:			
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	English	English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise C	Lecture 2 SWS + Exercise Class 2 SWS									
Content	The lecture covers some rec	The lecture covers some recent aspects of theoretical machine learning such as:									
	The theory of Reprod	ucing	Kern	el Hil	bert	Space	s (RKHS).				
	 Applications of RKHS mean embeddings. 	theory	y suc	h as	SVM	ls, kerr	nel regressi	on, kernel P	CA, an	d kernel	
	Approximation capabi	lities c	of ne	ural r	etwo	orks.					
	Dynamics of neural	etwork	s an	d the	neu	ral tanç	gent kernel.				
	 Recent advances in h generalisation. 	nigh di	men	siona	ıl sta	tistics,	in particula	ar overparan	netrisa	tion and	
Objectives	The students learn the math works, support vector mach modern topics in machine leand conceptual tools as need able to name and prove the explain the context develope able to describe and critically. Through homework assignment and independent acquaintar lectures. They learn how to develop solution strategies solutions and to stand for the	ines a earning eded for essent ed in the challe ents a nce with transi	and hand hand he leed the leed	kerned with a discontinuous di	el me h the cussi nents e and curre se cl tions meth und w	ethods. eir theo ion and s and c I to put ent stat asses s , staten nods to vithin a	They are retical basid justification oncepts from it into a late of research students dements, and new probligroup. The	familiar with is, mathematic of algorithm the lectural reger framework in the spectage of the control of the co	h fund atical anms. The as work. The cific and fident, kplaine the cifice the control of the con	amental pproach hey are ell as to hey are rea. precise, d in the em and	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Theoretical Aspects of	L	f	2	3	yes	wr. o.	90-180	g	100	
	Machine Learning	E	f	2	3	,	or.	o. 20-30			
	In this module an exercise context examination the coursework oral is decided by the instruction of the exceptional cases, be offer only 3 credit points are award	must I tor wit ed wit	have h ap thout	beer prova exer	n acc al by cises	quired. the Bo	Whether th ard of Exar	ie examination miners. – Th	on is w e mod	ritten or ule may,	

Literature	Possible References :
	 Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012.
	 Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014.
	 Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021.
	 Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge in linear algebra, analysis and probability theory is needed as well as some knowledge in elementary Hilbert space theory.
Responsible Persons	Andreas Prohl
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-31	Module Title: Statistical Learning Theory for Nonparametric Regression 1 Type of Module: Compulsory Module with Choice										
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 180 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	1-3									
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS	_ecture 4 SWS									
Content	Non-parametric regre	Non-parametric regression, regression estimator.									
	(Universal) consisten	cy.									
	Rate convergence.										
	Stone's theorem.										
	Kernel estimator, k-N	N estir	nator								
	Slow rate convergence	e, min	imax	con	verge	ence ra	tes.				
Objectives	Students are familiar with ba universal consistency and ra methods of stochastic learn are capable of naming and and explaining the presented In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	ate coring as proving deconnique the the the the the the the the the th	requirection of the control of the c	ence ired ess ns. red he le o wo	e. The for mention of the control of	ney are naching al result nfident, e. They n solution	familiar with learning at soft he learning at soft he learning precise an have learn on strategies	h the basic applications. Sture as well dindependent to transfers on their ow	princip The solass as ent harer the round or in	eles and students sessing adling of nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Statistical Learning Theory for Nonparametric	L	f	4	6	yes	wr. o.	90-180	g	100	
	Regression 1	ü	f	2	3	,55	or.	o. 20-30	9		
	In this module an exercise c examination the coursework oral is decided by the instruc	must	nave	bee	n acc	uired.	Whether th	e examination			
Literature	Possible References :										
	 Laslo Györfi, Michael nonparametric regres 						rro Walk: A	distribution	-free th	neory of	

Stochastics. Taking into account the chosen personal Study in the Sections Study Focus, Advanced Knowledge in Mathe in accordance with the restrictive requirements of the respec	The module belongs to the Study Specialisation Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.		
	Prerequisites	Knowledge of the Stochastics and Probability Theory modules is assumed.	
	•	Andreas Prohl	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-32	Module Title: Statistical Learning Theory sion 2	for N	onpa	rame	etric	Regres		of Module: ulsory Modul	le with	Choice	
ECTS-Points	9						·				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass	:		Self-Si 180 h	tudy:			
Duration	1 Semester										
Frequency	not regularly	not regularly									
Term	1-3	1-3									
Language of Instruction	German	German									
Forms of Teaching and Learning	Lecture 4 SWS	ecture 4 SWS									
Content	 The uniform law of large numbers on function classes (Vapnik-Chervonenkis theory). Abstract (strong) consistency theory for <i>least-squares</i> regression estimators on (approximating) function classes. Examples, in particular the <i>data dependent partitioning</i> estimator and the <i>least squares neural networks</i> estimator. Rate convergence for <i>least-squares</i> estimators. 										
Objectives	Students are familiar with in quired for machine learning a essential results of the lecture. In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present their	pplicate as whate as when the as when the as when the as well	ations ell as acqu s of t and t	s. The assired and the leads of	e stu essir a cor cture rk on	dents and affident, e. They solution	re capable explaining to precise and have learn on strategie	of naming a the presentend independent ned to transfers on their ow	nd pro d conn ent har er the r n or in	ving the ections. Indling of the nethods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Statistical Learning Theory	Type of Course	+ Status	SWS	n ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	for Nonparametric Regression 2 L f 4 6 ü f 2 3 wr. o. 90-180 o. 20-30 g 100 In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										
Literature	Possible References : • Laslo Györfi, Michael nonparametric regres:	Kohle	r, Ad	am I	Krzyz	ak, Ha			-free tl	neory of	
Transfer	The module belongs to the S Stochastics. Taking into accoin the Sections Study Focus, in accordance with the restrict	unt th <i>Advai</i>	e ch nced	osen <i>Kno</i>	pers wled	sonal S ge in M	tudy Specia <i>athematics</i>	alisation, it can	an be i	ncluded	
Prerequisites	Knowledge from the module	Statis	tical	Lear	ning	1 is as	sumed.				

Responsible Persons	Andreas Prohl
Abbreviations:	
Grading System : g	=graded, ng=not graded
Examination Type : M	IT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
	electure, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, etutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-33	Module Title: Theory and Numerics for Constrained Optimisation Problems							of Module: ulsory Modu	le with	Choice	
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h										
Duration	1 Semester	Semester									
Frequency	not regularly										
Term	1-3										
Language of Instruction	English	nglish									
Forms of Teaching and Learning	Lecture 4 SWS	ecture 4 SWS									
Content	method with step size control imum, as well as its variants Central to this is the convex of a minimum with (necessa Abadie condition, Karush-Kı	We start with the unconstrained convex minimisation problem (on spaces), and the gradient nethod with step size control according to Armeijo for the approximate calculation of a minnum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, labadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods assed on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.									
Objectives	The participants have become constrained optimisation prosimplex method, interior point be able to analyse the algornaming and proving the essenth presented connections. In the exercise classes they the terms, statements and monnew problems, to analyse They are able to present the	blems: at methithms ential have a ethods them	the nods and resu acqu s of t	se incomplets of the letter would be incompleted and the letter work in the letter would be incompleted.	clude nalisa pare the the a con ecture rk or	e gradie ation me their co lecture nfident, e. They n solutio	ent methods ethods and omplexity. It as well as precise an have learn on strategie	s with step s the SQP me the students assessing d independe ed to transfe s on their ow	ize corethod. are calend exent hare retherenterenterenterenterenterenterente	atrol, the You will pable of plaining adling of methods a team.	
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Theory and Numerics for Constrained Optimisation Problems	L ü	f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	examination the coursework oral is decided by the instruct exceptionally be offered by	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.									
Literature	Possible References :										
	Carl Geiger, Christiar gaben. Springer 2002		ow:	The	orie	und Nu	merik restr	ingierter Op	timieru	ingsauf-	

Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.	
Prerequisites	There are no further requirements.	1
Responsible Persons	Andreas Prohl	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-70-40	Module Title: Game Theory		Type of Module: Compulsory Module with Choice								
ECTS-Points	3										
Workload - Time in Class - Self-Study	Workload: 90 h							Self-Study: 60 h			
Duration	1 Semester						•				
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English	German or English									
Forms of Teaching and Learning	Lecture 2 SWS										
Content	The focus is on Nash and ger	neralis	ed N	ash	equil	librium	problems ar	nd their num	erical	solution.	
Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Game Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100	
	Whether the examination is Board of Examiners.	writte	n or	oral	is de	ecided	by the instr	uctor with a	pprova	ıl by the	
Literature	Possible References :	exandr	a Scl	nwar	tz: S	Spielthe	orie. Birkha	euser 2018.			
Transfer	The module belongs to the Study Specialisation Numerical Mathematics and Optimisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.										
Prerequisites	Basic knowledge of analysis dents of related fields with ba							ule is also s	uitable	for stu-	
Responsible Persons	Andreas Prohl										
Abbreviations:											

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, } \mbox{LE=lecture with integrated exercises, } \mbox{SL=seminar or lecture, } \mbox{E=exercise class, } \mbox{T=tutorial, P=project, S=seminar, IC=inverted classroom}$

Status : o=obligatory, f=facultative

Module Number: MAT-70-51	Module Title: Financial Mathematics and N		Type of Module: Compulsory Module with Choice							
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 120 h									
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English	English								
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Cl	-								
Content	els and numerical techniques that are essential for understanding and solving problems in note of the problem of the problems of the problems in the problems									
Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Financial Mathematics and Numerics	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For examination the coursework must have been acquired. Whether the exami oral is decided by the instructor with approval by the Board of Examiners. – in exceptional cases, be offered without exercises at the discretion of the lection only 3 credit points are awarded instead of 6.						e examination niners. – Th	on is w e modu	ritten or ıle may,		
Literature	Possible References : • Steven Shreve: Stoche	astic (Calcu	ılus f	or Fi	nance.	Springer 20	005.		

Transfer	The module belongs to the Study Specialisations Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of calculus, linear algebra, basic programming, ordinary differential equations theory and introductory probability is recommended.
Responsible Persons	Andreas Prohl

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-01	Module Title: Probability Theory	Type of Module: Compulsory Module with Choice								
ECTS-Points	9	9								
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h									
Duration	1 Semester						'			
Frequency	regularly in Winter Semester									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS								
Content	 Characteristic functions and additions to the central limit theorem. Conditional expectations and further measure-theoretic foundations. Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. Introduction to processes in contiuous time like Poisson processes and Brownian motion. 									
Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Probability Theory	L E	f	2	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written oral is decided by the instructor with approval by the Board of Examiners.									

Literature	Possible References :
	 Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010.
	Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2010.
	Hans-Otto Georgii: Stochastik. De Gruyter 2009.
	Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004.
	 Olav Kallenberg. Foundations of Modern Probability. Springer 2002.
	Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013.
	David Meintrup, Stefan Schäffler: Stochastik. Springer 2005.
	Albert N. Shiryaev: Probability-1. Springer 2016.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Martin Möhle, Martin Zerner

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\begin{array}{lll} \text{Teaching Format} & : \text{L=lecture, } & \text{LE=lecture with integrated exercises, } & \text{SL=seminar or lecture, } & \text{E=exercise class, } \\ & & \text{T=tutorial, P=project, S=seminar, IC=inverted classroom} \\ \end{array}$

Status : o=obligatory, f=facultative

Module Number: MAT-75-02	Module Title: Combinatorics		of Module: ulsory Modul	le with	Choice						
ECTS-Points	9						'				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass			Self-St 180 h	udy:			
Duration	1 Semester	1 Semester									
Frequency	not regularly										
Term	1-3										
Language of Instruction	German	German									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS									
Content	Basic combinatorial o	bjects.					·				
	Generating functions.										
	Partial orders, Möbius	invers	sion.								
	Method of Polya and	Redfie	ld.								
	Symbolic combinatori	cs.									
	Transfer matrix methor	od.									
	Euler-Maclaurin sumr	mation	form	ula.							
	Asymptotic methods.										
Objectives	The students have learned the application of basic combinatoric methods. They can analyse discrete structures and counting problems. Furthermore they are familiar with applying common identities and handling counting coefficients. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Combinatorics	L	f	4	6	VAS	wr. o.	90-180		100	
	Combinatorics	Е	f	2	3	yes	or.	o. 20-30	g	100	
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature	Possible References :
	Martin Aigner: Combinatorial theory. Springer 1997.
	Martin Aigner: A Course in Enumeration. Springer 2007.
	Richard P. Stanley: Enumerative combinatorics. Volume 1. Cambridge University Press 2011.
	 Francois Bergeron, Gilbert Labelle, Pierre Leroux. Combinatorial species and tree-like structures. Cambridge University Press 1998.
	Philippe Flajolet, Robert Sedgewick. Analytic Combinatorics. Cambridge University Press 2009.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of algebra (group actions), function theory (Cauchy's integral formula) and the foundations of discrete mathematics are expected.
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-03	Module Title: Mathematical Statistics		Type of Module: Compulsory Module with Choice								
ECTS-Points	9	9									
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 180 h										
Duration	1 Semester	1 Semester									
Frequency	regularly										
Term	1-3										
Language of Instruction	German	German									
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	Lecture 4 SWS + Exercise Class 2 SWS									
Content	 Statistical models, exponential families, sufficient statistics. Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. Hypothesis testing, confidence interval, Neyman-Pearson lemma. Testing methods, UMPU tests, 1- and 2-sample tests. Models with growing density quotients, non parametric models. Introduction in regression and variance analysis. 										
Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	- Status	SMS 4	о ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Mathematical Statistics	E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										

Literature	Possible References :				
	 Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. 				
	Hans-Otto Georgii: Stochastik. De Gruyter 2009.				
	Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005.				
	Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998.				
	Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009				
	Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000.				
	Mark J. Schervish: Theory of Statistics. Springer 1995.				
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.				
Prerequisites	Knowledge of the probability theory module is helpful, but is not mandatory.				
Responsible Persons	Martin Möhle, Martin Zerner				
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio					

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-04	Module Title: Stochastic Processes							of Module: ulsory Modul	le with	Choice		
ECTS-Points	9						'					
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h		lass	:		Self-St 180 h	udy:				
Duration	1 Semester											
Frequency	regularly											
Term	1-3	1-3										
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 4 SWS											
Content	Stochastic processes in continuous time, such as											
	Markov processes;	Markov processes;										
	Martingale;											
	Brownian motion, Poi	sson p	roce	sses	and	genera	al Levy proc	esses;				
	Gaussian processes.											
	Among other things, existe these processes are analyse		nd co	nver	genc	e state	ements as v	well as path	prope	erties of		
Objectives	The students have learnt the stochastic processes in con are capable of naming and and explaining the presented in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	tinuous provin d conn have s nethod them	s tim g the ection acqu s of tand t	e and e ess ns. ired the le	d car entia a cor ecture rk on	n hand Il resul nfident e. They n solution	e them mat ts of the lec precise and have learne on strategies	thematically. Sture as well d independe ed to transfe s on their ow	The solution as	students sessing adling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Stochastic Processes	L	f	4	6	yes	wr. o.	90-180	g	100		
		ü	f	2	3	, 55	or.	o. 20-30	9			
	In this module an exercise c examination the coursework oral is decided by the instruc	must	have	bee	n acc	uired.	Whether the	e examinatio				

Literature	Possible References :
	 Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010.
	Joseph L. Doob: Stochastic Processes. Wiley 1990.
	 Samuel Karlin, Howard Taylor: A First Course in Stochastic Processes. Academic Press 1975.
	 Samuel Karlin, Howard Taylor: A Second Course in Stochastic Processes. Academic Press 1981.
	Götz Kersting, Anton Wakolbinger: Stochastische Prozesse. Birkhäuser 2014.
	Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013.
	James R. Norris: Markov Chains. Cambridge University Press 1997.
Transfer	The module belongs to the <i>Study Specialisation</i> Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of the Probability Theory module is assumed.
Responsible Persons	Martin Möhle, Martin Zerner

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-05	Module Title: Percolation Theory							f Module: Ilsory Modul	le with	Choice			
ECTS-Points	3												
Workload - Time in Class - Self-Study	Workload: 90 h	Time 30 h	in C	lass	:		Self-St 60 h	udy:					
Duration	1 Semester												
Frequency	not regularly	not regularly											
Term	1-3												
Language of Instruction	German												
Forms of Teaching and Learning	Lecture 2 SWS												
Content	 Edge percolation on graphs, especially on multidimensional grids. Phase transitions. Number of clusters and cluster sizes. Special features in two dimensions. Alternative percolation models. 												
Objectives	Students can interpret special spatially indexed families of random variables as random geometric structures and apply probability theory methods to analyse them. Using simple models, they learn how microscopic changes can result in macroscopic phase transitions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.												
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Percolation Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100			
Literature	Possible References : • Béla Bollobás, Oliver • Geoffrey Grimmett: Po						bridge Unive	ersity Press	2006.	1			
Transfer	The module belongs to the Physics and Stochastics. To can be included in the Section tive Specialisation, in accordance	aking ons <i>St</i>	into i udy	acco <i>Focu</i>	unt t is, A	the cho dvance	sen person d Knowledg	al Study Sp e in Mather	ecialis natics	sation, it or <i>Elec-</i>			
Prerequisites	Knowledge of the module Pro	obabil	ity Tł	neor	y is h	elpful,	but not esse	ential.					
Responsible Persons	Elmar Teufl, Martin Zerner												

 $Grading \ System \quad : g=graded, \ ng=not \ graded$

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-06	Module Title: Stochastic Analysis							of Module: ulsory Modul	le with	Choice			
ECTS-Points	9						'						
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	_	lass			Self-St 180 h	udy:					
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German or English												
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS												
Content	Martingales and stopping times in continuous time.												
	Doléans measure, compensator, Doob-Meyer decomposition.												
	 Stochastic integral for square integrable martingales (in particular for non-continuous martingales). 												
	Semimartingales, tra	Semimartingales, transformation of stochastic integrals.											
	Itô formula (in particula)	lar for	proc	esse	s with	h jump	s).						
	Stochastic differentia	l equat	ions										
Objectives	The students know the main and they know how to hand the lecture and they can exp In the exercise classes they the terms, statements and n to new problems, to analysteam. They are capable of p discourse.	le then plain the have a nethod e then	n. Their in acques of to acques of the acques of the anconstructure of the acques of t	ne stu trinsi ired a he le d to	udent c cor a cor cture work	ts can nnection fident, e. They on so	name and pons. precise and have learn lution strate	orove the ce d independe ed to transfe egies on the	ntral re ent har er the r eir own	esults of adling of nethods or in a			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Stochastic Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100			
	In this module an exercise of examination the coursework oral is decided by the instruc	must	have	beei	n acc	uired.	Whether th	e examination					

Literature	Possible References :								
	Fabrice Baudoin: Diffusion Processes and Stochastic Calculus. EMS 2014.								
	 Kai Lai Chung and Ruth J. Williams: Introduction to Stochastic Integration. Birkhäuser 1990. 								
	Richard Durrett: Stochastic Calculus. CRC Press 2006.								
	Albrecht Irle: Finanzmathematik. Teubner 2003.								
	 Ioannis Karatzas, Steven Shreve: Brownian Motion and Stochastic Calculus. Springer 1991. 								
	Michel Métivier: Semimartingales. De Gruyter 1982.								
	Bernt Oksendal: Stochastic Differential Equations. Springer 2007.								
	 Nicolas Privault: Stochastic Analysis in Discrete and Continuous Settings. Springer 2009. 								
	Daniel Revuz, Marc Yor: Continuous Martingales and Brownian Motion. Springer 1999.								
	Heinrich von Weizsäcker, Gerhard Winkler: Stochastic Integrals, Vieweg 1990.								
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.								
Prerequisites	A solid knowledge on probability theory is a prerequisite.								
Responsible Persons	Martin Möhle								
Persons									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio $\label{eq:lecture} \mbox{Teaching Format} \ : \mbox{L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom \\$

: o=obligatory, f=facultative Status

Module Number: MAT-75-07	Module Title: Information Theory							of Module:	le with	Choice		
ECTS-Points	9							•				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass:			Self-St 180 h	udy:				
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS											
Content	Entropy and entropy r	Entropy and entropy rates in the discrete case.										
	Theorem of Shannon-McMillan-Breiman.											
	Entropy rates of Mark	Entropy rates of Markov chains.										
	Kolmogorov complexi	ty.										
	Data compression.											
	Chanel capacity.											
	Differential entropy.											
Objectives	Students learn to describe in the basic theory to concrete also apply the theoretical concapable of naming and provexplaining the presented correctly in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	e rando oncepts ing the nnectio have a them a	om es s to s e ess ns. acqui s of th	xper speci entia red a he le o wo	imen ific p al res a cor cture rk on	its and roblem sults of nfident, e. They i solution	stochastic s in coding the lecture precise an have learn on strategie	processes. theory. The as well as d independe ed to transfe s on their ow	Stude e stud assess ent har er the r	ents can ents are sing and adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Information Theory	L	f	4	6	yes	wr. o.	90-180	g	100		
		E	f	2	3	, 50	or.	o. 20-30	9			
	In this module an exercise or examination the coursework oral is decided by the instruc	must h	nave	beer	n acc	uired.	Whether th	e examination				

Literature	Possible References :
	Robert B. Ash: Information Theory. Wiley. 1965.
	Thomas M. Cover, Joy A. Thomas: Elements of Information Theory. Wiley 2006.
	 David J.C. MacKay: Information Theory, Inference and Learning Algorithms. Cambridge 2003.
	 Claude Shannon, Warren Weaver: The Mathematical Theory of Communication. University of Illinois Press 1949.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the modules Stochastics and Probability Theory is assumed.
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl
Abbreviations:	

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-08	Module Title: Mathematical Population Ge	netics						f Module:	le with	Choice			
ECTS-Points	6												
Workload - Time in Class - Self-Study	Workload: 180 h	Time 60 h	in C	lass:			Self-Str 120 h	udy:					
Duration	1 Semester												
Frequency	not regularly												
Term	1-3												
Language of Instruction	German												
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS												
Content	Exchangeable population models.												
	Probability of extinction.												
	Descendants and ancestors.												
	Duality of Markoff pro	cesses	S.										
	Coalescent processes	s and a	assoc	ciate	d cor	nverge	nce rates.						
	Simple mutation mod	els, Ew	ens	sam	pling	formu	a.						
	Statistical application:	s, e.g.	estin	natin	g the	mutat	ion rate.						
Objectives	In the lecture, students lear an understanding for the int capable of naming and provexplaining the presented corcurrent state of research in the exercise classes they the terms, statements and mon new problems, to analyse They are able to present the	eractio ing the inection he sub have a lethods them a	n of e ess ns. S ject a acqui s of the	geor sentia Stude area. ired a he le o wo	metrion al resents von a cor cture rk on	c and sults of vill be a fident, e. They solution	algebraic m the lecture able to reflect precise and have learned on strategies	ethods. The as well as a well as a well as a continuation of the and critical discounting the as a continuation of the as	e stude assess ally ana ent har er the r on or in	ents are sing and lyse the adling of nethods a team.			
Requirements for Obtaining Credit, Grading, Weight if applicable	Title Mathematical Population	Type of Course	J Status	SWS 2	ω ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade			
	Genetics	E	f	2	3	yes	or.	o. 20-30	g	100			
	In this module an exercise or examination the coursework oral is decided by the instruc	must h	nave	beer	n acq	uired.	Whether the	e examinatio					

Literature	Possible References :
	Jean Bertoin: Random Fragmentation and Coagulation Processes. Cambridge 2006.
	Stewart N. Ethier, Thomas G. Kurtz: Markov Processes. Wiley 1986.
	Warren J. Ewens: Mathematical Population Genetics. Springer 2004.
	Jim Pitman: Combinatorial Stochastic Processes. LNM 1875. Springer 2006.
	John Wakeley: Coalescent Theory. Roberts & Company Publishers 2008.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of probability theory is assumed.
Responsible Persons	Martin Möhle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title:							f Module:		O		
MAT-75-09	Point Processes						Compi	Ilsory Modu	le with	Choice		
ECTS-Points	6											
Workload - Time in Class	Workload:	Time	in C	lass			Self-St	udy:	ıdy:			
- Self-Study	180 h	60 h					120 h					
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 2 SWS											
Content	Random measures, point processes, Poisson processes.											
	Factorial measure, Mecke equation.											
	Transformation, labelling, thinning.											
	Characterisation of po	int pro	ces	ses.								
	Stationary Poisson pr	ocesse	es.									
	Poisson integrals.											
	Cox processes.											
Objectives	The students have familiaris examples of the theory of po are capable of naming and p explaining the presented con current state of research in t In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	int pro- roving nectione sub have a ethoda them a	cess the e ns. S ject acqu s of t and t	es aresser Stude area ired a he le o wo	nd cantial rents was core controls with the core core core core core core core cor	in hand esults vill be a nfident, e. They n solution	dle them ma of the lectur able to reflect precise and have learn on strategies	thematically e as well as et and critica d independe ed to transfe s on their ow	The sassessally ana ent har er the ron or in	students sing and alyse the adling of methods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Point Processes	L	f	2	3	yes	wr. o.	90-180	g	100		
		ü	f	2	3		or.	o. 20-30				
	In this module an exercise context examination the coursework oral is decided by the instruction	must I	nave	beei	n acc	uired.	Whether the	e examination				

Litavatova	Descible Deferences :
Literature	Possible References :
	 Daryl John Daley, David Vere-Jones: An Introduction to the Theory of Point Processes. Springer 2008.
	Martin Jacobsen: Point Process Theory and Applications. Birkhäuser 2006.
	Olav Kallenberg: Foundations of Modern Probability. Springer 2002.
	John F. C. Kingman: Poisson Processes. Clarendon Press 1993.
	Günter Last, Mathew D. Penrose: Lectures on the Poisson Process. Cambridge 2016.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of probability theory is assumed.
Responsible Persons	Martin Möhle

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title: Graph Theory							of Module:	le with	Choice		
ECTS-Points	9						Compe	aloony ivioual				
Workload - Time in Class - Self-Study	Workload: 270 h	Time 90 h	in C	lass:			Self-St	udy:				
Duration	1 Semester						'					
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS											
Content	Basic concepts in gra	Basic concepts in graph theory,										
	Basic graph theory algorithms,											
	Flows, cuts, connectedness, matchings,											
	Cycle and cut space (cohomology theory),											
	Spectral graph theory	, matrix	tree	e the	orem	١,						
	 Planar graphs, theore 	m of Ku	urato	owsk	i and	Wagn	er,					
	Planar embeddings,											
	Graph colorings,											
	Theory of minors.											
Objectives	Students know the basic co use graph theory methods in algebra and be able to benef essential results of the lectur In the exercise classes they the terms, statements and m on new problems, to analyse They are able to present the	n praction to the practic from the	ce. Themell as cqui of the control o	They a. The ass red a he le o wo	will a e stu essir a cor cture rk on	also re dents and and and afident, e. They solution	cognise cor are capable explaining t precise an have learn on strategie	nnections to of naming a he presented independed ed to transfe s on their ow	geome and pro d conn ent har er the n	etry and ving the ections. adling of nethods a team.		
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	, ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Graph Theory	E	f f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100		
	In this module an exercise or examination the coursework oral is decided by the instruc	must h	ave	beer	n acc	juired.	Whether th	e examination				

Literature	Possible References :
	Bela Bollobas: Modern graph theory, Springer, 1998.
	John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008.
	Reinhard Diestel: Graph theory, Springer, 2018.
	 Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Transfer	The module belongs to the Study Specialisations Algebra and Geometry and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Elmar Teufl
Abbreviations:	-graded na-not graded

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-11	Module Title: Markov Chains and Applications							of Module: ulsory Modul	e with	Choice	
ECTS-Points	9						·				
Workload - Time in Class - Self-Study	Workload: Time in Class: Self-Study: 270 h 90 h 180 h										
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Exercise C	lass 2	SWS	3							
Content	Fundamentals and advanced topics on Markov chains and related stochastic models are discussed. In particular, the long-term behaviour of Markov chains is examined. Furthermore, applications of Markov chains, such as Markov chain Monte Carlo simulation, randomised search algorithms, graphical models, entropy rates of Markov chains, are discussed.										
Objectives	The students have learnt the basic concepts of the theory of Markov chains and related models. They are also familiar with applications of the theory and have experienced the interaction of probability theory and algorithms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
	Markov Chains and Applications	L E	f	4 2	6	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise ce examination the coursework oral is decided by the instruc	must l	nave	beer	acc	uired.	Whether th	e examinatio			
Literature	Possible References: • Pierre Bremaud: Discrete Probability Models and Methods. Springer 2017. • Pierre Bremaud: Markov Chains. Springer 1999. • Olle Häggström: Finite Markov Chains and Algorithmic Applications. Cambridge University Press 2002. • Kevin Murphy: Machine Learning: A Probabilistic Perspective. MIT Press 2012. • James Spall: Introduction to Stochastic Search and Optimization. Wiley 2003.										
Transfer	The module belongs to the sen personal Study Specialis Knowledge in Mathematics quirements of the respective	ation, or <i>Ele</i>	it ca <i>ctive</i>	n be	inclu	uded in	the Section	ns <i>Study Foo</i>	cus, Ad	dvanced	

Prerequisites	Good knowledge of linear algebra and stochastics is required. Knowledge from the probability theory module is helpful, but is not required.
Responsible Persons	Elmar Teufl

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number:	Module Title: Type of Module:											
MAT-75-12	Foundations of Discrete Mat	hemat	ics				Compu	ulsory Modu	le with	Choice		
ECTS-Points	9	1										
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h 180 h										
Duration	1 Semester											
Frequency	not regularly											
Term	1-3											
Language of Instruction	German											
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS											
Content	• Logic.											
	Sets, relations, functions.											
	Partial orders.											
	Combinatorics.											
	Number theory.											
	Graph theory.											
	Algorithms and forma	l langu	ages	S.								
	Discrete optimization											
Objectives	Students have learned how to use basic methods of discrete mathematics. They can analyze discrete structures and identify discrete structures in different contexts. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.											
Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade		
	Foundations of Discrete	L	f	4	6	yes	wr. o.	90-180	g	100		
	Mathematics	E	f	2	3		or.	o. 20-30				
	examination the coursework	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										

Literature	Possible References :								
	 Ronald Graham, Donald Knuth, Oren Patashnik: Concrete Mathematics. Addison-Wesley 1994. 								
	Kenneth H. Rosen: Discrete Mathematics and Its Application. McGraw-Hill 2019.								
	Ralph P. Grimaldi: Discrete and Combinatorial Mathematics. Addison-Wesley 2004.								
	Norman L. Biggs: Discrete Mathematics. Oxford University Press 2002.								
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.								
Prerequisites	There are no further prerequisites.								
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl								

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-20	Module Title: Probability Distances for Data Science							of Module: ulsory Modul	e with	Choice	
ECTS-Points	6	6									
Workload - Time in Class - Self-Study	Workload: 180 h						Self-Study: 120 h				
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	English	English									
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS										
Content	We study different concepts of distances between probability measures aimed at applications in data science. The classes of distances which are studied include optimal transport distances, f-divergences and integral probability metrics. The focus is on fundamental mathematical properties of these distances, like duality, famous inequalities, geometric aspects, and quantisation. Several applications in the area of data science and machine learning are illustrated throughout, for instance related to clustering, autoencoders, GANs, image processing, and compression.										
Objectives	Students are familiar with commonly used distances on the space of probability measures, particularly optimal transport distances, divergences, and integral probability metrics. They understand key mathematical results in this area, for instance related to duality, geometric aspects, and quantisation, as well as the interplay between different distances. They have further obtained an understanding of computational aspects and applicability in selected areas of data science. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed. In the exercises, they have developed a confident, precise and independent approach to the concepts, statements and methods from the lecture. They have learned to transfer the methods to new problems, to analyse them and to develop solution strategies alone or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for Obtaining Credit, Grading, Weight if applicable	Type of Course Status SWS ECTS Coursework Type of Exam Type of Exam Grading								Weight for Grade		
	Probability Distances for Data Science	L E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.										

Literature	Possible References :
	 Gabriel Peyre, Marco Cuturi: Computational optimal transport: with ap- plications to data science. Foundations and Trends in Machine Learning 11.5-6 (2019): 355-607.
	 Alison L. Gibbs, Francis Edward Su: On choosing and bounding probability metrics. International Statistical Review 70.3 (2002): 419-435.
	Cedric Villani: Topics in optimal transportation. American Mathematical Society, 2003.
	 Imre Csiszar, Paul C. Shields: Information theory and statistics: a tutorial. Foundations and Trends in Communications and Information Theory 1.4 (2004). 417-528.
	 Ily Tolstikhin et al.: Wasserstein auto-encoders. 6th International Conference on Learning Representations (ICLR 2018)
	 Siegfried Graf, Harald Luschgy: Foundations of quantization for probability distributions. Springer, 2007.
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	The course is mostly self-contained, but students benefit from basic knowledge in analysis, probability theory, optimisation, and Python.
Responsible Persons	Stephan Eckstein

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Module Number: MAT-75-21	Module Title: Bayesian Networks and Causality Type of Module: Compulsory Module with Cho										
ECTS-Points	5										
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h								
Duration	1 Semester										
Frequency	not regularly										
Term	1-3	-3									
Language of Instruction	English	inglish									
Forms of Teaching and Learning		ecture 2 SWS + Exercise Class 1 SWS									
Content	to deal with it. Therefore, the corporate probabilities was human understanding goes ever the sprinkler is on, to postul the plants will be wet, but if In this course, Bayesian ne probability distributions. In pudiscussed. Moreover, it will be distributions, but are also ab of Bayesian networks will be data. • Part I: Bayesian Networks will be data. • Part II: Bayesian Networks will be data.	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able of deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, numan understanding goes further than that: We go beyond observing events, such as Whenever the sprinkler is on in my greenhouse, the plants are wet or Whenever the plants are wet, the sprinkler is on, to postulating a relationship of cause and effect: If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler. In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inferenence and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: — Computing probabilities using Bayesian networks. — d-Separation: A graphical criterion for probabilistic independence. — Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation of Causal Knowledge: — Functional causal models: A representation of causal knowledge. — Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. — Causal Bayesian networks. — Causal Bayesian networks. — Causal structure discovery: Learning causal relationships from data. — Counterfactual identifiability: Answering counterfactual. questions using causal									
Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.										

Requirements for Obtaining Credit, Grading, Weight if applicable	Title Bayesian Networks and Causality Whether the examination is was Board of Examiners.	Type of Course	f f or o	SMS 2 1 oral i	SLO3 3 2 s de	yes ecided I	wr. o. or.	Onr. of Exam (min) 90-180 o. 20-30	g Grading	Meight for Grade
Literature	Possible References: • Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.									
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	The module Stochastics is ass	sume	d.							
Responsible Persons	Stephan Eckstein									

Grading System : g=graded, ng=not graded

Examination Type: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio Teaching Format: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,

T=tutorial, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative