EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Department of Mathematics

Module Handbook

Mathematical Physics

Master of Science

Summer Semester 2025

Version: 29th January 2025

Contents

1 Program description

1.1 Study Concept

The Master of Science Mathematical Physics is an international research-oriented two year master's program offered jointly by the departments of Mathematics and Physics within the Faculty of Science of the University of Tübingen starting every year in the winter semester. It is geared towards students with a solid background in Mathematics as well as in Physics, and it requires a bachelor's degree in physics or mathematics or an equivalent degree. The scientific discipline "Mathematical Physics" is concerned with the mathematically rigorous formulation and analysis of physical theories and models. In this master's program students will thus deepen and broaden their knowledge of Mathematics and Physics in interdisciplinary courses in Mathematical Physics as well as in disciplinary courses in Mathematics and Theoretical Physics. At the end of the program they are particularly well prepared for jobs where the typical competences of mathematicians are needed in combination with applications of physics. The program is international and cannot be pursued without a solid knowledge of the English language. Language skill on the level of B2 according to the European Framework of Reference for Languages are therefore required. All mandatory modules and a large number of facultative modules are offered only in English. Some facultative modules may sometimes be offered only in German.

1.2 Qualification Goals

Students deepen and broaden their theoretical knowledge of different areas of mathematical physics, mathematics and theoretical physics. They become proficient in general and specific methods and principles in these areas. They can connect problems and questions from physics with their counterparts in mathemtical models and are able to judge and critically question the relevance and adequacy of mathematical models and the derived consequences. They are able to report on and scrutinize the current state of research in the area of their specialisation. Graduates can apply their expanded knowledge in order to develop and successfully handle their own research projects. They are able to present, discuss, and defend the results of their research in writing and orally in front of a scientific audience. In the course of the Mathematical Physics Colloquium students practice scientific collaboration and discourse in interdisciplinary and internationally mixed groups.

Their education enables graduates in mathematical physics to successfully and professionally tackle complex mathematical modelling problems in physics and, after an appropriate familiarization with the subject, also in other areas of technology, finance or economics. They are moreover well prepared for interdisciplinary and international collaborations in mixed teams of different specialists from different cultural backgrounds, as are common nowadays in all areas of research and development.

1.3 Program Structure

The Master's Program is a two year (four terms) consecutive study program with a modular structure. Based on the foundational modules "Geometry in Physics", "Functional Analysis in Geometry", "Mathematical Quantum Theory", and "Mathematical Relativity", to be attended during the first year, students can specialise rather freely according to their personal preferences in one or more areas of Mathematical Physics, Mathematics and/or Theoretical Physics. The few restrictions are that every student must take at least one module from the Mathematics master's program and one module from the Theoretical Physics master's program, as well as a seminar. As a consequence, all graduates of the Master's Program have proven their ability to successfully conduct mathematical studies and theoretical physics studies at the master's level. A Scientific Project in the third term typically serves as a preparation for the Master Thesis (M.Sc. Thesis, 30 ECTS-points) written during the final term. During the second year students are also required to attend the Mathematical Physics Colloquium. This is a weekly colloquium where specialists lecture about recent developments in Mathematical Physics, and students have the opportunity to meet and discuss with international guest scientists and local researchers about current topics. The prescribed period of study is two years corresponding to a total of 120 ECTS points.

1.4 Mentoring

At the start of the program every student will be assigned to a mentor from the group of professors involved in the master's program for the whole duration of his/her studies. Students meet their mentor at the beginning and later at least once per term in order to plan and discuss the progress of their studies. In particular, at these meetings the study and examination plan in compliance with the examination regulations is discussed. The module selection is documented and passed on to the head of the examinations board for approval. During the first meeting possible gaps in the knowledge should be discussed in order to fill them by taking appropriate courses within the area of elective specialisation. The study and examination plan is then updated every semester during the meetings with the mentor. The mandatory mentoring program assures that students specialise in a purposeful way and select accordingly goal-oriented combinations of modules from mathematics and physics.

During the meetings with the mentor also possible time slots for a study period at a university abroad can be discussed. In principle, every semester is suitable, depending on the study progress of the student and the courses avaliable at the other institution. It is also possible to write the master's thesis during a stay abroad under the cosupervision of a scientist there.

1.5 Information for students with a bachelor's degree in Physics at the University of Tübingen

Graduates of the 4-year degree program Bachelor of Science in Physics at the University of Tübingen can already gain up to 60 credit points for the degree program Master of Science in Mathematical Physics during their bachelor studies.

In particular,

- the module BMTPKFT Klassische Feldtheorie from the bachelor's program can be credited with

9 credit points for the module MAT-40-32 Advanced Topics in Theoretical Physics in the master's program, and

- up to 21 credit points in the section Vertiefungsfach in the bachelor's program can be credited in the section Elective Studies, provided the choice is suitable.

Moreover,

- up to 27 credit points in the section Ergänzungsmodule in the bachelor's program can be gained via the modules MAT-65-11 Geometry in Physics, MAT-65-12 Mathematical Quantum Theory, MAT-65-13 Mathematical Relativity or MAT-65-14 Mathematical Statistical Physics from the master's program, and
- the bachelor's thesis can be credited with 9 credit points in the module Scientific Project.

In order to finish the Master of Science in Mathematical Physics subsequently to the bachelor's degree in Physics at the University of Tübingen it is recommended to choose in the section Vertiefungsfach in the bachelor's program courses in theoretical physics, which can be credited in the section Elective Studies in the master's program in Mathematical Physics. Moreover, it is recommended to choose in the section Ergänzungsmodule in the bachelor's program at least two of the modules MAT-65-11, MAT-65-12, MAT-65-13 or MAT-65-14 from the master's program in Mathematical Physics. Good choices would be the combinations MAT-65-11 + MAT-65-13 and MAT-65-12 + MAT-65-14. Also the combination MAT-65-11 and MAT-65-12 would be suitable.

2 Study Plans

2.1 Overview by Modules

We provide here an overview of the study plan as a table showing the modules to be taken.

Suggested Term	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Foundations							
1	MAT-65-11	Geometry in Physics	L+E	PM	EC	wr. o. or.	9
1	MAT-65-12	Mathematical Quantum Theory	L+E	PM	EC	wr. o. or.	9
2	MAT-65-13	Mathematical Relativity	L+E	PM	EC	wr. o. or.	9
Section 2: Knowledge Expansion							
1–3	MAT-40-31	Advanced Topics in Mathematics	L+E	PMW	EC	wr. o. or.	9
1–3	MAT-40-32	Advanced Topics in Theoretical Physics	L+E	PMW	EC	wr. o. or.	9
2–3	MAT-40-33	Seminar Knowledge Extension	S	PMW	s.M.	Pr	3
Section 3: Elective Specialisation							
1-3		Modules from the master's programmes of the Department of Mathematics or Physics according to Section 3.		WPM			30
Section 4: Scientific Work							
3	MAT-40-41	Scientific Project	P	PM	s.M.	-	9
3–4	MAT-40-42	Mathematical Physics Colloquium	C+C	PM	-	-	3
4	MAT-40-43	Master Thesis M.Sc. Mathematical Physics	MT	PM	s.M.	MT	30

Abbreviations:

Type of Module	: PM=compulsory module, PMW=compulsory module with choice, WPM=elective module
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Course Work	: EC=exercise certificate
Other	: h=hours, o.=or, s.M.=see module description

2.2 Overview by the Course of Studies

We first provide a general study plan showing the distribution of credit points over the different areas and the general time line. On the following pages example study plans for different types of specialisation are provided, where possible courses are assigned to the modules MAT-40-31 and MAT-40-32 as well as the modules from the area of Elective Specialisation.

<table>
<tr><th>Term</th><th>CP</th><th>Foundations of Mathematical Physics</th><th>Knowledge Expansion</th><th>Elective Specialisation</th><th>Scientific Work</th></tr>
<tr><td>1.</td><td>27</td><td rowspan="2">27 CP</td><td rowspan="3">21 CP</td><td></td><td></td></tr>
<tr><td>2.</td><td>30</td><td rowspan="2">30 CP</td><td></td></tr>
<tr><td>3.</td><td>31</td><td></td><td rowspan="2">42 CP</td></tr>
<tr><td>4.</td><td>32</td><td></td><td></td><td></td></tr>
</table>

Figure 2.1: General Study Plan

2.3 Example Study Plans

The example study plans shown below shall give an idea how the individual study in the different specialisations could look like. They are not meant as a recommendation, and it is neither guaranteed that the courses listed will be offered each year, nor that they all will be given in English.

Example Study Plan without Specialisation

<table>
<tr><th>Term</th><th>CP</th><th>Foundations of Mathematical Physics</th><th>Knowledge Expansion</th><th>Elective Specialisation</th><th colspan="2">Scientific Work</th></tr>
<tr><td rowspan="2">1.</td><td rowspan="2">27</td><td>Geometry in Physics (9 CP)</td><td rowspan="2">Linear Partial Differential Equations (9 CP)</td><td rowspan="2"></td><td rowspan="2" colspan="2"></td></tr>
<tr><td>Mathematical Quantum Theory (9 CP)</td></tr>
<tr><td rowspan="2">2.</td><td rowspan="2">30</td><td rowspan="2">Mathematical Relativity (9 CP)</td><td rowspan="2">Seminar(3 CP)</td><td>Advanced Topics in Mathematical Quantum Theory (9 CP)</td><td rowspan="2" colspan="2"></td></tr>
<tr><td>Mathematical Statistical Physics (9 CP)</td></tr>
<tr><td rowspan="2">3.</td><td rowspan="2">31</td><td rowspan="2"></td><td rowspan="2">Quantum Field Theory and Particle Physics (9CP)</td><td>Advanced Topics in Mathematical Relativity (6 CP)</td><td rowspan="3">Mathe-matical Physics Colloquium (3 CP)</td><td rowspan="2">Scientific Project (9 CP)</td></tr>
<tr><td>Advanced Topics in Mathematical Statistical Physics (6 CP)</td></tr>
<tr><td>4.</td><td>32</td><td></td><td></td><td></td><td>Master Thesis (30 CP)</td></tr>
</table>

Figure 2.2: The program Mathematical Physics can be completed to a large extent also without choosing a particular specialisation. In this case we recommend taking all four foundational modules and also all advanced courses offered. The modules from the area Knowledge Expansion should then be chosen in accordance with the planned specialisation in the Scientific Project and the Master Thesis, cf. e.g. the following study plans.

Example Study Plan Quantum Theory

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP)	Operator Theory (9 CP)			
		Mathematical Quantum Theory (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Quantum Field Theory and Particle Physics (9 CP)	Functional Analysis (9 CP)		
			Seminar(3 CP)			
3.	31			Advanced Topics in Mathematical Quantum Theory (9 CP)	Mathe-matical Physics Colloquium (3 CP)	Scientific Project (9 CP)
				Computational Methods in Physics / Astrophysics (6 CP)		
				Theoretical Condensed Matter Physics (6 CP)		
4.	32					Master Thesis (30 CP)

Figure 2.3: The mathematical foundations of quantum theory are predominantly allocated to areas of analysis. Thus we recommend that those specialising in one of the areas Mathematical Quantum Theory, Quantum Field Theory, Condensed Matter, Many-Body Quantum Systems, or Quantum Information attend mathematical courses from analysis, e.g. Operator Theory, Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Relativity

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP) Mathematical Quantum Theory (9 CP)	Astronomy and Astrophysics (9 CP)			
2.	30	Mathematical Relativity (9 CP)	Introduction to Partial Differential Equations (9 CP) Seminar(3 CP)	Riemannian Geometry (9 CP)		
3.	31			Advanced Topics in Mathematical Relativity (9 CP) Theoretical Astrophysics (6 CP) Computational methods in Physics / Astrophysics (6 CP)	Mathe-matical Physics Colloquium (3 CP)	Scientific Project (9 CP)
4.	32					Master Thesis (30 CP)

Figure 2.4: The mathematical foundations of relativity are predominantly allocated to areas of geometry and analysis. Thus we recommend that those specialising in one of the areas Mathematical Relativity, Astronomy, Cosmology, or Astro Physics attend mathematical courses from geometry, e.g. Riemannian Geometry and Lorentz Geometry, and from analysis, e.g. Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Statistical Physics

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP)	Probability Theory (9 CP)			
		Mathematical Quantum Theory (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Advanced Statistical Physics (9 CP)	Mathematical Statistical Physics (9 CP)		
				Density Functional Theory (6 CP)		
3.	31		Seminar (3CP)	Advanced Topics in Mathematical Statistical Physics (6 CP)	Mathematical Physics Colloquium (3 CP)	Scientific Project (9 CP)
				Mathematical Statistics (9 CP)		
4.	32					Master Thesis (30 CP)

Figure 2.5: The mathematical foundations of statistical physics are predominantly allocated to areas of probability. Thus we recommend that those specialising in one of the areas Mathematical Statistical Physics, Soft Matter, or Density Functional Theory attend mathematical courses from probability, e.g. Probability Theory and Mathematical Statistics.

2.4 Overview by Study Progress and Credit Requirements

Overview by Study Progress and Credit Requirements													
		Exam				Teaching				Term			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of exams / ECTS points to semesters is a recommendation only. Compulsory allocations are marked as such. The allocation of ECTS points to courses is for information only. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Foundations of Mathematical Physics:									**27**				
MAT-65-11 Geometry in Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-65-12 Mathematical Quantum Theory								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-65-13 Mathematical Relativity								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4			6		
2.	Exercises					E	o	2			3		
Knowledge Expansion:									**21**				
MAT-40-31 Advanced Topics in Mathematics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-40-32 Advanced Topics in Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4			6		
2.	Exercises					E	o	2			3		
MAT-40-33 Seminar								2	3				
1.	Seminar	Pres.	45–90	g	3	S	o	2				3	
Elective Specialisation:									**30**				
Here the modules MAT-65-15 and MAT-65-21 to MAT-65-26, as well as further suitable advanced modules from the Master's Programs in Mathematics, Physics, and Astro and Particle Physics, can be chosen. The choices need to be discussed and agreed upon with the Mentor. Modules from other areas need to be approved by the examinations board.													
MAT-65-14 Mathematical Statistical Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	4			6		
2.	Exercises					E	f	2			3		
MAT-65-21 Advanced Topics in Mathematical Quantum Theory								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	4			6		

Overview by Study Progress and Credit Requirements

		Exam				Teaching				Term			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	1. CP	2. CP	3. CP	4. CP
2.	Exercises					E	f	2			3		
MAT-65-22 Advanced Topics in Mathematical Quantum Theory (short version)								4	6				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	6	L	f	2			3		
2.	Exercises					E	f	2			3		
MAT-65-23 Advanced Topics in Mathematical Relativity								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	2				3	
2.	Exercises					E	f	2				3	
MAT-65-24 Advanced Topics in Mathematical Relativity (short version)								4	6				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	6	L	f	2				3	
2.	Exercises					E	f	2				3	
Scientific Work									**42**				
MAT-40-41 Scientific Project									9				
1.	Project	Proj.		ng	9		o					9	
MAT-40-42 Mathematical Physics Colloquium									3				
1.	Colloquium			ng			o					1	2
MAT-40-43 Master Thesis									30				
1.	Thesis	Thes.		g	30		o						30

Term note: The allocation of exams / ECTS points to semesters is a recommendation only. Compulsory allocations are marked as such. The allocation of ECTS points to courses is for information only. Credits are only awarded upon completion of the module.

Abbreviations:
Marking system : g=graded, ng=non graded
Form of examination : MA=Master Thesis, Or.=oral exam, Wr.=written exam, Pres.=presentation
Form of teaching : L=lecture, E=exercise class, S=seminar, Proj.=project work, Coll.=colloquium
Status : o=obligatory, f=fakultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points

3 Module Descriptions

Section 1: Foundations

In the case that some of the mandatory modules in this section or modules, which are essentially identical as far as the contents and competences are concerned, have been part of the Bachelor studies, which are the prerequisite for this Master's Degree Program, according to the examination regulations these modules cannot be taken in the Master's Degree Program any more. They have to be replaced by other suitable modules in the framework of the studies and examination plan.

Module Number: **MAT-65-11**	**Module Title:** Geometry in Physics		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.		
Objectives	Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometry in Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									

Literature

Possible References :

- John Lee: Introduction to smooth manifolds. Springer 2012.
- John Lee: Riemannian manifolds: An introduction. Springer 1997.
- Chris Isham: Modern differential geometry for physicists. World Scientific 1999.
- Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.

Transfer

Participation in the module is a prerequisite for participation in the module Mathematical Relativity. Successful completion of the module may be a prerequisite for participation in the module Seminar Knowledge Extension and is so for the participation in the module Scientific Project.

Prerequisites

There are no further prerequisites.

Responsible Persons

Christoph Bohle, Carla Cederbaum, Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-12	**Module Title:** Mathematical Quantum Theory		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides an introduction to mathematical quantum theory. Particular topics are the stationary and time-dependent Schrödinger equation, fundamental approximation methods as Rayleigh-Schrödinger perturbation theory and Hartree- resp. Hartree-Fock theory, the Fock space formalism, and elements of scattering theory. Optionally, other topics such as adiabatic theory and semiclassical approximations can be discussed.		
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle pysics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle pysics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Quantum Theory	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	Successful completion of module Mathematical Quantum Theory is a prerequisite for the participation in the module Advanced Topics in Mathematical Quantum Theory. Successful completion of one of the modules Mathematical Quantum Theory and Mathematical Relativity is a prerequisite for the participation in the module Scientific Project.

Prerequisites	-
Responsible Persons	Christian Hainzl, Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-13	**Module Title:** Mathematical Relativity	**Type of Module:** Compulsory Module
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h; Self-Study: 180 h
Duration	1 Semester	
Frequency	regularly in Summer Semester	
Term	2	
Language of Instruction	English	
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements	
Content	The module provides an introduction to the mathematical theory of relativity. Particular topics are Newton's theory of gravity, special theory of relativity, relativistic effects, Einstein's equations, the Schwarzschild spacetime. Optionally, other topics such as cosmological models, matter models, black holes, Cauchy problem and ADM decomposition, singularity theorems or gravitational waves can be discussed.	
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from the theory of relativity. They are able to interrelate physical problems in cosmology and astrophysics and their mathematical models through methods from differential geometry and to question the relevance and adequacy of the mathematical model and the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular in module MAT-65-11. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.	

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Relativity	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	Successful completion of module Mathematical Relativity is a prerequisite for the participation in the module Advanced Topics in Mathematical Relativity. Successful completion of one of the modules Mathematical Relativity or Mathematical Quantum Theory is a prerequisite for the participation in the module Scientific Project.
Prerequisites	Participation in the module Geometry in Physics is a prerequisite.
Responsible Persons	Carla Cederbaum, Gerhard Huisken, Frank Loose

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Knowledge Expansion

Module Number: **MAT-40-31**	**Module Title:** Advanced Topics in Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	Every semester		
Term	1–3		
Language of Instruction	English or German		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	It is required to attend one or more lectures as well as the respective exercise classes with the correspondent SWS-coverage from the Master's degree program in Mathematics. Recommended subjects are for instance Partial differential equations, Numerics of differential equations, Harmonic analysis, Lie groups, Nonlinear functional analysis, Operator theory, Stochastic processes, Calculus of variations, Symplectic geometry, Algebraic topology or Algebraic geometry. Further details can be found in the catalogue of mathematical modules starting at page 40 and in the module handbook of the degree program M.Sc. Mathematics.		
Objectives	The students aquire deepend knowledge in one selected area of mathematics independently of physical applications. They broaden the basis of their mathematical knowledge and extend the methods at hand to tackle mathematical problems. The further qualification goals, in particular the concrete content related qualifaction goals, will follow from the module description of the chosen course in the module handbook for the M.Sc. Mathematics.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Advanced Topics in Mathematics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	The module may be a prerequisite for the master thesis.
Prerequisites	See prerequisites in the Module Handbook M.Sc. Mathematics.
Responsible Persons	The dean of studies at the department of mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-32	**Module Title:** Advanced Topics in Theoretical Physics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	Every semester		
Term	1–3		
Language of Instruction	English or German		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	It is required to attend one or more advanced-level lectures from the field of theoretical physics as well as the respective exercise classes with the correspondent SWS-coverage from the Master's degree program in Physics or the Master's degree program Astro and Particls Physics. Recommended subjects are for instance Quantum field theory and Particle physics, Theoretical astrophysics, Relativistic astrophysics, Many-particle quantum systems, Advanced statistical physics, Yang-Mills theory, Condensed matter physics, Theoretical quantum optics, Quantum information theory, Cosmology, Numerical methods in physics and astrophysics, Current topics in theoretical physics. Further details can be found in the module handbook of the corresponding degree programs.		
Objectives	The students aquire deepend knowledge in one selected area of theoretical physics independently of rigorous mathematical formalism. They broaden the basis of their knowledge in theoretical physics and extend the methods at hand to tackle problems in physics. The further qualification goals, in particular the concrete content related qualifaction goals, will follow from the module description of the chosen course in the module handbook for the M.Sc. Pysics or the M.Sc. Astro and Particle Physics.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Advanced Topics in Theoretical Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	The module may be a prerequisite for the master thesis.
Prerequisites	See prerequisites in the Module Handbook M.Sc. Physics or M.Sc. Astro and Particle Physics.
Responsible Persons	Die Studiendekanin oder der Studiendekan des Fachbereichs Physik

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-33	**Module Title:** Seminar Knowledge Extension		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	Every semester		
Term	2–3		
Language of Instruction	English or German		
Forms of Teaching and Learning	Seminar: Presentation, Discussion, Teamwork, Handout		
Content	Various topics from various areas of Mathematical Physics, Mathematics or Theoretical Physics.		
Objectives	The students have learnt to develop independently or in team an acquaintance with an advanced topic in Mathematics or Physics by applying scientific methods and to present it in form of an oral presentation. They have improved their skills in the presentation of mathematical or physical results and are able to argue for these results in critical discussions.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	3	yes	Pr	45–90	g	100

Transfer	The module may be a prerequisite for the master thesis.
Prerequisites	Successful completion of one of the modules from the Section "Foundations of Mathematical Physics".
Responsible Persons	Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Elective Specialisation

Within the study area Elective Specialisation students can choose modules from the Master Programs Mathematical Physics, Mathematics, Physics, and Astro and Particle Physics according to their individual interests. In particular, courses listed in the module descriptions MAT-40-31 and MAT-40-32 but not chosen there, the module MAT-65-13 respectively MAT-65-14 not yet chosen in the area Foundations, the modules MAT-65-15 and MAT-65-21 to MAT-65-24, as well as other appropriate advanced modules from the programs Mathematical Physics, Mathematics (see **??**), Physics, and Astro and Particle Physics are available. Note that not all modules can be offered every year, but there is always a broad choice. Also note that some modules from other programs might be offered only in German, but also here a choice of English courses is ensured. The selection of modules within the area Elective Specialisation must be discussed and decided together with the mentor. Each module can be selected only once. In agreement with the mentor and upon request at the examinations board, 9 ECTS points within the area of Elective Specialisation can be allocated for modules that serve to close knowledge gaps either in mathematics or physics.

Within the area of Elective Specialisation students obtain relevant skills. They learn to independently judge which additional qualifications and competences are relevant to their studies and to select courses accordingly. They are able to acquire specific knowledge also beyond the mandatory parts of the study program. Within the area of their specialisation they can report on and scrutinize the current state of research. In the exercise classes students learn to work confidently, precisely and independently with the notions, statements and methods presented during the lectures. They also learn how to apply methods to new problems and to analyse and solve them alone or in groups.

Module Number: **MAT-65-14**	**Module Title:** Mathematical Statistical Physics		**Type of Module:** Elective Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly, in Summer Semester		
Term	2-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides an introduction to mathematical statistical physics. Particular topics are concepts of probability theory, classical statistical mechanics of gases (equivalence of ensembles, thermal equilibrium, Boltzmann equation, entropy), Brownian motion (stochastic processes, Wiener process), lattice models (Ising model, Gibbs measure, thermodynamic limit, phase transitions), statistical quantum mechanics (quantum mechanical ensembles, transition to thermal equilibrium, Bose-Einstein condensate). Optionally, other topics such as open quantum systems, transport phenomena, renormalization group theory and the fluctuation-dissipation theorem can be discussed.		

Objectives

Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from statistical physics. They are able to interrelate fundamental physical concepts, such as equilibrium, irreversibility and entropy, and their mathematical models via probabilistic methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular on probability theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Statistical Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer

Successful completion of module is a prerequisite for the participation in the module Advanced Topics in Mathematical Statistical Physics.

Prerequisites

-

Responsible Persons

Marcello Porta, Roderich Tumulka

Abbreviations:

Grading System : g=graded, ng=not graded
Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-15	**Module Title:** Foundations of Quantum Mechanics		**Type of Module:** Elective Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly every two years		
Term	2-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides an introduction to fundamental questions of quantum mechanics, including its mathematical and philosophical aspects. Various interpretations such as Copenhagen, Bohmian mechanics, many worlds and spontaneous collapse of the wave function are presented and analysed mathematically and physically. Other topics include Born's rule, Heisenberg's uncertainty principle, the quantum measurement problem, Bell's non-locality theorem, identical particles and no-hidden-variable theorems.		
Objectives	Students know and can apply the rules of quantum mechanics in different environments and understand several important theories of how the quantum world works. They acquire mathematical knowledge relevant to the application of these rules and theories and can connect the mathematical treatment with the physical meaning. They will familiarise themselves with the surprising phenomena and paradoxes of quantum mechanics. They will appreciate what is controversial about the orthodox interpretation and why, and will be able to follow the current debate on fundamental issues. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Foundations of Quantum Mechanics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

The type of examination is set by the instructor.

Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The basic modules on Analysis and Linear Algebra are required.
Responsible Persons	Roderich Tumulka

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-21	**Module Title:** Advanced Topics in Mathematical Quantum Theory		**Type of Module:** Elective Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	2-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides an introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent ferminon systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.		
Objectives	Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)		
Transfer	The module may be a prerequisite for the master thesis.		
Prerequisites	Knowledge from the module Mathematical Quantum Theory is assumed.		
Responsible Persons	Stefan Teufel		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Advanced Topics in Mathematical Quantum Theory	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-22	**Module Title:** Advanced Topics in Mathematical Quantum Theory (short version)		**Type of Module:** Elective Module
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly, in Summer Semester		
Term	2		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 2 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides a short introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent ferminon systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.		
Objectives	Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and in parts also critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		
Requirements for Obtaining Credit, Grading, Weight if applicable	see below		
Transfer	The module may be a prerequisite for the master thesis.		
Prerequisites	Knowledge from the module Mathematical Quantum Theory is assumed.		
Responsible Persons	Stefan Teufel		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Advanced Topics in Mathematical Quantum Theory	L	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-23</td><td colspan="2">Module Title:
Advanced Topics in Mathematical Relativity</td><td>Type of Module:
Elective Module</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly, in Winter Semester</td></tr>
<tr><td>Term</td><td colspan="3">3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignements</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to an advanced topic of mathematical theory of relativity. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Advanced Topics in Mathematical Relativity</td><td>L</td><td>o</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>o</td><td>2</td><td>3</td></tr>
</table>
In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module may be a prerequisite for the master thesis.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the module Mathematical Relativity is assumed.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Carla Cederbaum, Gerhard Huisken, Frank Loose</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-24	**Module Title:** Advanced Topics in Mathematical Relativity (short version)		**Type of Module:** Elective Module
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly, in Winter Semester		
Term	3		
Language of Instruction	English		
Forms of Teaching and Learning	Lectures 2 SWS + Exercise Classes 2 SWS, Homework Assignements		
Content	The module provides a short introduction to an advanced topic of mathematical theory of relativity. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.		
Objectives	Students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and in parts also critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Advanced Topics in Mathematical Relativity	L	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor. – In exceptional cases the module can be offered by the lecturer without exercises, in this case, only 3 credit points are awarded for the module instead of 6.

Transfer	The module may be a prerequisite for the master thesis.
Prerequisites	Knowledge from the module Mathematical Relativity is assumed.
Responsible Persons	Carla Cederbaum, Gerhard Huisken, Frank Loose

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 4: Scientific Work

Module Number: **MAT-40-41**	**Module Title:** Scientific Project		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 15 h	Self-Study: 255 h
Duration	1 Semester		
Frequency	Every semester		
Term	3		
Language of Instruction	English		
Forms of Teaching and Learning	Individual supervision by a mentor, study of scientific works.		
Content	• Definition of an advanced scientific project in coordination with the mentor. • Independent search and study of the relevant scientific literature. • Formulation of specific problems and methodical approach to their solution. • Written presentation of the project in conext of current state of research on 5-10 pages. This module serves generally as a preparation for the Master Thesis		
Objectives	Students • develop skills to systematically familiarize themselves with a new subject, • learn to work critically and to form a substantiated, professional and interdisciplinary judgement, • acquire qualifications in such areas as literature research, identification of relevant problems and appropriate methods, as well as in the written presentation of a research proposal.		
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)		
Transfer	Successful completion of this module is a prerequisite for participation in module Master Thesis.		
Prerequisites	Successful completion of module Geometry in Physics and of one of the modules Mathematical Quantum Theory or Mathematical Relativity.		
Responsible Persons	Stefan Teufel, Werner Vogelsang.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Scientific Project	P	o	1	9	yes	-	-	ng	-

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-42	**Module Title:** Mathematical Physics Colloquium		**Type of Module:** Compulsory Module
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 60 h	Self-Study: 30 h
Duration	2 Semester		
Frequency	Every semester		
Term	3–4		
Language of Instruction	English		
Forms of Teaching and Learning	Presentations, discussions. Specific form of study: during the final semester students present their Master thesis.		
Content	During each semester on 15 appointed dates (2 h each) there will take place presentations and discussions on current topics in mathematical physics. Speakers are the researchers of the involved departments, guest scientists and master's students, who present the results of their Master Thesis.		
Objectives	Students gain an insight into the current development of mathematical physics beyond the area of their own specialization. They develop the ability to follow scientific presentations and to discuss and challenge them within a larger group of scholars. They therefore also obtain interdisciplinary and intercultural competencies through regular cooperation and discussion in mixed groups.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Colloquium Winter Semester	C	o	2	1	no	-	-	ng	-
Colloquium Summer Semester	C	o	2	2	no	-	-	ng	-

Transfer	-
Prerequisites	-
Responsible Persons	Carla Cederbaum, Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-43	**Module Title:** Master Thesis M.Sc. Mathematical Physics		**Type of Module:** Compulsory Module
ECTS-Points	30		
Workload - Time in Class - Self-Study	Workload: 900 h	Time in Class: 0 h	Self-Study: 900 h
Duration	1 Semester		
Frequency	Every semester		
Term	4		
Language of Instruction	English or German		
Forms of Teaching and Learning	Master thesis		
Content	Students are assigned to workgroups and participate in seminars of the group. Under the supervision of the mentor students have to handle a concrete problem from mathematical physics by applying scientific methods and present it in written form in English or German. In particular this includes: • Definition of an advanced scientific task in coordination with the mentor; • Independent search and study of the relevant scientific literature; • Formulation of appropriate questions and methodical approach to their answers; • Independent execution and written presention of the project and the results in the context of the current state of research; • Presentation of the results in English in Mathematical Physics Colloquium.		
Objectives	Students are able to • develop acquaintance with a new problem within a given period of time and treat it with increasing independence by applying scientific methods; • develop acquaintance with scientific literature on a new topic; • critically interpret scientific results and integrate them into their state of knowledge; • present their results in written form based on principles of Good Scientific Practice; • present their work in an international scientific environment.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Master Thesis	MT	o	-	30	no	MT	-	g	100

Transfer	-

Prerequisites	• 27 CP from the compulsory elective Section Foundations of Mathematical Physics, • a total of 18 CP from the Sections Knowledge Expansion and Elective Specialisation, • Successful completion of module Scientific Project.
Responsible Persons	Stefan Teufel, Werner Vogelsang.

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Catalogue of Mathematics Modules

This section lists modules from the M.Sc. Mathematics programme that can be included in the area Elective Specialisation and which are not yet listed in Section 3.

Module Number: **MAT-45-01**	**Module Title:** Introduction to Commutative Algebra and Algebraic Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		
Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Commutative Algebra and Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969.
- David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
- David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995.
- Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980.
- Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
The module cannot be taken together with the module 'Commutative Algebra' due to the large overlap in content.

Prerequisites

There are no further prerequisites.

Responsible Persons

Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-02</td><td colspan="2">Module Title:
Commutative Algebra</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly in Winter Semester</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Rings and Ideals.
Localisation and local rings.
Noetherian and Artinian rings and modules.
Integral ring extensions and Cohen-Seidenberg theorems.
Krull's principal ideal theorem and dimension theory.
Primary decomposition.
Normality, regularity and discrete valuation rings.
Hilbert's Nullstellensatz and Noether normalisation.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Commutative Algebra</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Commutative Algebra and Algebraic Geometry' due to the large overlap in content.
Prerequisites	There are no further prerequisites.
Responsible Persons	Victor Batyrev, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-03</td><td colspan="2">Module Title:
Computer Algebra</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Canonical forms and standard bases for ideals and modules.
• Computation of important operations for ideals and modules.
• Syzygies, free resolutions and the proof of the Buchberger criterion.
• Calculation of the primary decomposition of ideals.
• Hilbert functions.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with important problems in the interplay of commutative algebra and algebraic geometry as well as algorithmic approaches to solving them. In particular, they are familiar with the theory of standard bases and their diverse applications. They are also familiar with important software packages in the field of symbolic computing and have implemented algorithms in these. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse. They also learnt about important software packages in the field of symbolic computing and have implemented algorithms in them themselves</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Computer Algebra</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Gert-Martin Greuel, Gerhard Pfister: A SINGULAR Introduction to Commutative Algebra. Springer 2008. • Wolfram Decker, Christoph Lossen: Computing in algebraic geometry. A quick start using SINGULAR. Springer 2006. • Wolfram Decker, Gerhard Pfister: A first Course in computational algebraic geometry. Cambridge University Press 2013. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Commutative Algebra and Algebraic Geometry are helpful however not absolutely necessary for participation in the module Computer Algebra
Responsible Persons	Hannah Markwig, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-11	**Module Title:** Algebraic Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Prevarieties and varieties. • Projektive varieties and homogeneous spectrum. • Finite and proper morphisms. • Blow-up and Grassmannians. • Rational maps. • Divisors and line bundles, class group and Picard group.		
Objectives	The students learn central terms, results and methods of modern Algebraic Geometry and they develop a deeper understanding of the interconnections between Geometry and Algebra. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for Obtaining Credit, Grading, Weight if applicable	see below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry and Toric Varieties' due to the large overlap in content.
Prerequisites	Essential knowledge from the module Commutative Algebra is assumed.
Responsible Persons	Victor Batyrev, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-12	**Module Title:** Algebraic Geometry and Toric Varieties		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Projective space. • Prevarieties, morphisms, tangent space and singularities. • Products and separation. • Projective varieties and Grassmannians. • Divisors and line bundles, class group and Picard group. • Toric varieties.		
Objectives	Students learn the central concepts, results and methods of modern Algebraic Geometry and they develop an advanced understanding of the relationships between Geometry and Algebra. Using the example of the class of toric varieties, they also learn how methods of Convex Geometry can facilitate the investigation of an important example class of Algebraic varieties, and expand the interplay of Algebra and Geometry by a further component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Geometry and Toric Varieties	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • David A. Cox, John B. Little, Henry K. Schenck: Toric varieties. American Mathematical Society 2011: • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry' due to the large overlap in content.
Prerequisites	Essential knowledge from the module Introduction to Commutative Algebra and Algebraic Geometry is assumed.
Responsible Persons	Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-13	**Module Title:** Algebraic Transformation Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Operations of algebraic groups on algebraic varieties, homogeneous spaces. • Elements of the structure theory of affine-algebraic groups and their Lie algebras. • Elements of the representation theory of affine-algebraic groups and their Lie algebras. • Quotients in algebraic geometry. • Classical invariant theory: Hilbert's finiteness theorem, calculation of invariants. • Geometrical invariant theory: Mumford's construction of quotients, variation of quotients. • Additionally certain aspects of topics from the following list are covered: – Toric varieties; – Spheric varieties.		
Objectives	The students learn basic methods for mathematical work with symmetries on geometric structures. At the same time, they experience the interaction of different algebraic concepts, for example from group and ring theory, in algebraic geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Transformation Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Armand Borel: Linear algebraic groups. Springer 1991. • Jean A. Dieudonne, James B. Carrell: Invariant theory. Academic Press 1971. • David Mumford: Geometric invariant theory. Springer 1965.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Groups' due to the large overlap in content.
Prerequisites	Knowledge of the Commutative Algebra and Algebraic Geometry modules is helpful, but not a prerequisite for participation in the Algebraic Transformation Groups module.
Responsible Persons	Victor Batyrev, Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-14	**Module Title:** Algebraic Curves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content

- Projective curves, divisors, Theorem of Riemann-Roch.
- Ramified coverings, Theorem of Hurwitz.
- Linear systems, embeddings, Castelnuovo inequality.
- Singularities of plane curves, Puiseux expansions.
- Classification and moduli spaces, Jacobi variety.

Objectives

Students have familiarised themselves with the central concepts, results and methods in a selected sub-area of algebraic geometry. familiarised themselves with it. They have developed an in-depth understanding of algebraic curves and their classification. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Curves	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Robin Hartshorne: Algebraic geometry. Springer 2006.
- Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994.
- Rick Miranda: Algebraic Curves and Riemann Surfaces. AMS 1995.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Essential knowledge from the module Commutative Algebra as well as basic knowledge from Algebraic Geometry and Complex Analysis is required.
Responsible Persons	Victor Batyrev, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-15	**Module Title:** Toric Varieties and Mori Dream Spaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content

In the lecture Mori Dream Spaces are considered as generalisations of toric varieties:

- Geometry and combinatorial theory for toric varieties and Mori Dream Spaces.
- Divisors on toric varieties and Mori Dream Spaces.
- Quotient representation and Cox ring for toric varieties and Mori Dream Spaces.
- Sheaves of divisorial algebras.
- Cox sheaves and characteristic space.
- Quotients of H-factorial affine varieties.
- Shaded rings.
- Varieties with torus operations.

Objectives

Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra. With the class of Mori dream spaces, they have become familiar with a generalisation of toric varieties and their investigation using methods of convex geometry. In doing so, they have added another important methodological component to the interplay between algebra and geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Toric Varieties and Mori Dream Spaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface. Cox rings. Cambridge University Press 2014. • Yi Hu, Sean Keel. Mori dream spaces and GIT. Michigan Math. J. 48: 331-348, 2000.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Essential knowledge from the modules Introduction to Commutative Algebra and Algebraic Geometry as well as Algebraic Geometry and Toric Varieties is assumed.
Responsible Persons	Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-16**	**Module Title:** Algebraic Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Definition and examples of algebraic groups. • Hopf algebras. • Operations of algebraic groups on varieties. • Linearisation of algebraic groups. • Group closure. • Resolvable and nilpotent groups. • The Lie algebra of an algebraic group. • Examples of Lie algebras. • Convolutions and commutators. • The adjoint representation and its differential. • The Jordan decomposition in affine algebraic groups. • Characters of an algebraic group. • Semi-invariants of a rational representation. • Existence and construction of quotients with applications. • Diagonalisable groups and tori. • Rigidity of diagonalisable groups. • Theorem of Lie-Kolchin. • Structure of affine resolvable groups. • Centralisers of semisimple elements of algebraic groups. • Borel subgroups and root systems. • Structure and classification of semisimple algebraic groups.		

Objectives	Students have learnt about a large class of important groups and algebraic varieties that play an essential role in many mathematical fields. They have learnt how methods of group theory and algebraic geometry complement each other and can lead to a deeper understanding. They have learnt about the approach to classifying mathematical objects using an important example class and have acquired knowledge of methods that also play a key role in classification in completely different mathematical areas. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- James E. Humphreys: Linear Algebraic Groups. Springer 1975. 21, Springer-Verlag 1981.
- Armand Borel: Linear algebraic groups. Springer 1991.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Transformation Groups' due to the large overlap in content.
Prerequisites	Knowledge of the modules Commutative Algebra and Algebraic Geometry are helpful, but not a prerequisite for participation in the Algebraic Groups module.
Responsible Persons	Victor Batyrev, Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-18**	**Module Title:** Cox Rings		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Divisorial algebras. • Cox rings. • Charakteristic spaces. • Good quotients. • Geometric invariant theory. • Gale-duality. • Connections to toric geometry. • Defining data for varieties with finitely generated Cox ring. • Singularities. • Picard group. • Basis locus. • Ampleness. • Kanonical class. • Intrinsic quadrics. • k*-surfaces. • Varieties with torus action.		
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra and combinatorics. They have familiarised themselves with the Cox ring as an algebraic object for investigating special classes of geometric spaces. In doing so, they expand the interplay between algebra and geometry with another important methodological component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Cox Rings	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, Antonio Laface: Cox Rings. CUP 2014.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of commutative algebra and algebraic geometry to the extent of the module Introduction to commutative algebra and algebraic geometry is assumed.
Responsible Persons	Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-19	**Module Title:** Real Algebraic Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 3 SWS		
Content	This course aims to dive into different aspects of the study of the topology of real algebraic varieties. This involves questions related to the 16th Hilbert problem: we look at obstructions of topological types for real algebraic varieties and at the realisation of topological types via different construction techniques, with special emphasis to low dimensional cases.		
Objectives	Students learn about some fundamental differences in algebraic geometry over the complex and real numbers. They are familiar with the application of topological and algebraic methods to the study of real algebraic varieties. They have learnt how modern methods can be used to investigate and answer unsolved scientific questions of the late 19th century. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Real Algebraic Geometry	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Frederice Mangolte: Real Algebraic Varieties. Springer 2020.
- Robert Silhol: Real Algebraic Surfaces. Springer 1989.
- Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Herrmann 1990.
- Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge in algebraic geometry or algebraic topology is helpful, but not mandatory.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-20**	**Module Title:** Introduction to Berkovich Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 3 SWS		
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.		
Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Berkovich Geometry	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of topological concepts is assumed.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-21</td><td colspan="2">Module Title:
Algebraic Number Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Rings of integers.
• Class numbers.
• Dirichlet’s unit theorem.
• Extension of Dedekind rings.
• Valuation theory.
• Local fields.
• Adeles and ideles.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learned the central terms, results and methods of algebraic number theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Algebraic Number Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007.
• Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007.
• Andre Weil: Basic number theory. Springer 1995.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Victor Batyrev, Anton Deitmar, Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-22</td><td colspan="2">Module Title:
Number Theory and Cryptography</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• RSA cryptosystem, primality tests, AKS algorithm.
• Factorisation methods, number field sieve.
• Quadratic reciprocity in cryptography.
• Evaluation of the discrete logarithm.
• Dynamical systems and Pollard's rho algorithm.
• Elliptic curve cryptography.
• Lattices and post-quantum cryptography.
• Zero-knowledge proofs, digital signatures and hash functions.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Number Theory and Cryptography</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Elliptic Curves and Cryptography' due to the large overlap in content.
Prerequisites	The contents of the module Algebra from the study program Bachelor of Science are presumed.
Responsible Persons	Elena Klimenko, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-24	**Module Title:** Elliptic Functions and Elliptic Curves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content

- Elliptic functions, Weierstrass-P-function, Riemann surfaces, complex tori.
- Plane projective curves, Theorem of Bezout, elliptic curves.
- Curves over finite fields, rational points.
- Applications in cryptography.
- Additionally a selection of the following:
  - Modular forms;
  - Classification of elliptic curves;
  - Moduli spaces.

Objectives

The students have expanded their competence in mathematical-interdisciplinary studies. They have learnt about elliptic curves as a class of mathematical objects, which has comprehensive relevance in a wide spectrum of mathematical areas. The students have studied the notions, methods and results from the disciplines Complex Calculus, Algebraic Geometry, Number Theory, Topology and Cryptography, which are relevant in the given context, and they understand their mutual interrelations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Functions and Elliptic Curves	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Wolfgang Fischer, Ingo Lieb: Funktionentheorie. Vieweg 2005. • Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the lecutre Introduction to Complex Analysis is needed.
Responsible Persons	Jörg Zintl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-25	**Module Title:** Elementary Number Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical Theorems. • Applications.		
Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elementary Number Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, only basic knowledge of groups and rings from linear algebra is required.
Responsible Persons	Victor Batyrev, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-26	Module Title: Introduction to Analytic Number Theory		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Arithmetic functions and Dirichlet series, • Prime number theorem and Dirichlet's prime number theorem, • Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula.		
Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.		
Requirements for Obtaining Credit, Grading, Weight if applicable	see table below		
Literature	**Possible References :** • Komaravolu Chandrasekharan: Introduction to Analytic Number Theory. Springer 1968.		
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.		
Prerequisites	There are no further prerequisites		
Responsible Persons	Anton Deitmar		

Requirements for Obtaining Credit, Grading, Weight if applicable:

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Analytic Number Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-27	**Module Title:** Elliptic Curves and Cryptography		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Basic concepts of cryptography. • Symmetric cryptosystems, public key systems, discrete logarithm, RSA. • Factorisation into primes, attacks on cryptosystems. • Basic concepts of plane projective geometry. • Elliptic curves as Abelian groups. • Curves over finite fields, Frobenius morphism, endomorphism ring. • Counting points, Hasse bound, Weil conjectures, Schoof's algorithm. • Cryptosystems on elliptic curves, algorithms and attacks.		
Objectives	Students are familiar with the basic tasks and concepts of cryptography. They understand the cryptographically motivated questions relating to elliptic curves and have an insight into the advanced algebraic and geometric techniques for answering them. They understand the challenges of algorithmic implementation and are familiar with standard algorithms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Curves and Cryptography	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the modules 'Number Theory and Cryptography' due to the large overlap in content
Prerequisites	There are no further requirements.
Responsible Persons	Jörg Zintl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-28	**Module Title:** Elliptic Curves and Taniyama-Shimura		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content

A selection of the following topics will be covered:

- Group-law, arithmetic of elliptic curves.
- Modular curves and forms.
- Riemann surfaces, abelian differentials, Jacobian.
- Geometric version of Taniyama-Shimura-conjecture (i.e. Wiles' modularity theorem) explained.
- Connection to Fermat's last theorem.
- L-series.

Objectives

The students have learnt and understood the interplay of methods of algebra, number theory and geometry to answer profound mathematical questions using the example of the conjecture of Taniyama-Shimura and its application to the proof of Fermat's theorem. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Curves and Taniyama-Shimura	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Joseph H. Silverman: The Arithmetic of Elliptic Curves. Springer 2009.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Introduction to Riemann surfaces and Algebraic number theory are assumed.
Responsible Persons	Ivo Radloff

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-29	**Module Title:** Introduction to Modular Forms		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The study of modular forms traces back its roots to the late 19th and early 20th century with Gauss, Eisenstein and Ramanujan, and is a fascinating blend of analysis and algebra. They have many surprising applications to number theory, including a beautiful proof of Lagrange's four square theorem and the ground-breaking proof of Fermat's last theorem in 1995. This course aims to give an introductory understanding of this broad topic. • Modular forms for the Modular group and congruence subgroups. • Examples: Eisenstein series, the Ramanujan Delta function, Theta series. • Arithmetic applications and conjectures. • Hecke operators and eigenforms.		
Objectives	Students have learnt the basic concepts, results and methods of the classical theory of modular forms. They are familiar with analytical, algebraic and geometric aspects of modular forms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Modular Forms	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Henri Cohen, Fredrik Stromberg: Modular forms. A classical approach. AMS Graduate Studies of Mathematics 2017. • Fred Diamond, Jerry Shurman: A first course in modular forms. Springer 2005. • Max Koecher, Aloys Krieg: Elliptische Funktionen und Modulformen. Springer 2007. • Toshitsune Miyake: Modular forms. Springer 1989. • Lloyd James Peter Kilford: Modular forms: A classical and computational introduction. Imperial College Press 2015. • Deitmar Anton: Automorphic forms. Springer 2013.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Modular Forms' due to the large overlap in content.
Prerequisites	There are no further prerequisites, but basic knowledge of algebra and function theory is helpful.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-30	**Module Title:** Primes of the form $x^2 + ny^2 andClassFieldTheory$		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		

Content

Just as Gauss once described number theory as the queen of mathematics, Class Field Theory can be described as her Crown Jewel. Class field theory brings together the Galois theoretic structure of of a number field, and links it to the arithmetic structure of its the ring of integers, through a deep understanding of reciprocity laws. This course aims to introduce the central notions and concepts of Class Field theory naturally through solving the elementary motivating problem of what primes can be expressed as the form $x^2 + ny^2$, and bridge elementary number theory to modern number theory.

- The theory of Quadratic forms and Genus theory.
- Generalisations of the law of quadratic reciprocity. Arithmetic in number fields and the Hilbert Class field.
- A statement of Artin Reciprocity, and a solution to the problem of describing primes of the form $x^2 + ny^2$.

Objectives

Students have learned the foundational concepts and results of algebraic number theory and of class field theory in the case of quadratic field extensions of Q, and have a very hands on understanding of where these concepts come from through an elementary example. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Primes of the form $x^2 + ny^2 andClassFieldTheory$	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- David Cox: Primes of the form $x^2 + ny^2.Wiley2013, JamesMilne : Classfieldtheory.OnlineNotes2020.$
- Jürgen Neukirch: Algebraic number theory. Springer 1999.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	An understanding of Galois theory and other basic algebraic structures (groups and rings) is essential. Deep results taught in any first course in algebraic number theory will be clearly stated and introduced, but not proven, and will be treated as a black box.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-45-31	Representation Theory of Finite Groups		Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Groups and group actions. • Representations, irreducibility, Schursch's lemma. • Semisimplicity, Maschke's theorem. • Characters, orthogonality relations. • Isotypical decomposition, character tables. • Representations of the symmetric group. • Semi-simple Artinian algebras.		
Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Representation Theory of Finite Groups	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. It can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Group Representations in Physics' due to the large overlap in content.
Prerequisites	In terms of content, only basic knowledge of linear algebra is required.
Responsible Persons	Victor Batyrev, Jürgen Hausen, Milena Wrobel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-40**	**Module Title:** Introduction to Combinatorial Birational Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Smooth projective toric surfaces P_Δ associated with lattice polygons Δ. The moment map for surfaces X over the complex numbers. The rational equivalence on boundary divisors. The divisor class group $Cl(X)$ and the bilinear intersection pairing on $Cl(X)$. • Cartier divisors, line bundles, invertible sheaves. The canonical divisor of a normal variety. Ample and very ample divisors. • Nondegenerate algebraic curves in toric surfaces. Blow ups of toric surfaces. Birational modification of surfaces via blow ups and blow downs. • The cone of curves of a surface. The Zariski decomposition. Birational Cremona transformations. • Desingularization of nondegenerate curves D on smooth toric surfaces X via blow ups. Combinatorial constructing minimal models of pairs (X, D) for normal toric surfaces X. • Cyclic quotient surface singularities and their combinatorial minimal desingularization. • Finite subgroups of $SU(2)$ and surface Du Val singularities and their minimal desingularization. • Birational classification of nondegenerate surfaces in 3-dimensional toric varieties via the Fine interior $F(\Delta)$ of their Newton polytopes Δ. • The Kodaira dimension of algebraic varieties. Combinatorial constructing minimal models of nondegenerate surfaces. • Combinatorial formulas for the Hodge numbers of minimal models.		
Objectives	In the lecture, students learn how to apply concepts, results and methods of convex geometry in order to analyse important classes of algebraic surfaces. They learn to recognise and calculate complex algebro-geometric constructions. They are familiarised with an interesting and deep classification problem, the minimal models for algebraic surfaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Combinatorial Birational Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Laurent Buse, Fabrizio Catanese, Elisa Postinghel: Algebraic Curves and Surfaces: A History of Shapes. Springer 2023.
- Klaus Hulek: Elementare Algebraische Geometrie. Springer 2012.
- Tadao Oda: Convex Bodies and Algebraic Geometry: An Introduction to the Theory of Toric Varieties. Springer 1988.
- Robin Hartshorne: Algebraic Geometry. Springer 1977.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge of commutative algebra and algebraic geometry are assumed: some of the essential concepts from these areas are briefly repeated at the beginning of the course.

Responsible Persons

Victor Batyrev

Abbreviations:

Grading System : g=graded, ng=not graded
Examination Type : MT=master’s thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-41	**Module Title:** Introduction to Combinatorial Mirror Symmetry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Quintic 3-folds in projective 4-space and their mirrors. • Toric varieties associated with fans of rational polyhedral cones. Toric varieties associated with lattice polyhedra. Smoothness. • Resolution of singularities. Cohomology rings of smooth projective toric varieties. • Construction of Calabi-Yau varieties as hypersurfaces in toric varieties associated with reflexive polyhedra. • A combinatorial formula for Hodge numbers of Calabi-Yau 3-folds. Monomial-divisor correspondence. • Combinatorial mirror construction for Calabi-Yau complete intersections. Mirrors of rigid Calabi-Yau varieties. • Computation of periods of Calabi-Yau hypersurfaces using generalized hypergeometric functions. • Stringy Hodge numbers of singular Calabi-Yau varieties. • Moduli spaces. Boundary points in moduli spaces of Calabi-Yau hypersurfaces and secondary polytopes. • Computation of Gromov-Witten invariants of Calabi-Yau complete intersections. • A combinatorial approach to Berglund-Hübsch mirror symmetry. Combinatorial mirror symmetry proposes a purely combinatorial approach to mirror symmetry based on polar duality in the class of reflexive lattice polytopes. The Platonic solids provide the most famous examples of polar dual pairs of polyhedra: Cube-octahedron, icosahedron-dodecahedron. In combinatorial mirror symmetry, an essential fact is that the vertices of the considered reflexive polyhedra Δ are elements of the lattice M and the vertices of the dual reflexive polyhedra Δ^* belong to the dual lattice N. The lattice M can be identified with the lattice of characters of an algebraic torus T, and the dual lattice N becomes the lattice of one-parameter subgroups in T. For this reason, the main tool of the combinatorial approach is the theory of toric varieties. Combinatorial mirror symmetry allows us to interpret the mirror symmetry discovered by physicists for 3-dimensional Calabi-Yau manifolds by going from M to N and from Δ to Δ^*. The aim of the module is to explain the connection between reflexive polyhedra and Calabi-Yau manifolds in the most understandable way possible and to inform students about further results obtained using this combinatorial duality.		

Objectives

Students are familiar with the complex issues of mirror symmetry, which establishes a duality between manifolds of symplectic and algebraic geometry and was first postulated by physicists. They have learnt how methods of toric geometry and discrete mathematics can be used for very important classes of Calabi-Yau varieties in order to calculate the mirrors of the manifolds and their invariants in concrete terms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Combinatorial Mirror Symmetry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Victor Batyrev: Dual Polyhedra and Mirror Symmetry for Calabi-Yau Hypersurfaces in Toric Varieties. J. Alg. Geom. 3 (1994), no. 3, 493–535.
- Victor Batyrev, Duco van Straten: Generalized hypergeometric functions and rational curves on Calabi-Yau complete intersections in toric varieties. Comm. Math. Phys., 168:3 (1995), 493–533.
- Victor Batyrev and Lev A. Borisov: Dual cones and mirror symmetry for generalized Calabi-Yau manifolds. Mirror Symmetry II, AMS/IP Stud. Adv. Math. 1, Amer. Math. Soc., Providence, RI (1997), 71–86.
- David Cox, Sheldon Katz: Mirror Symmetry and Algebraic Geometry. Mathematical Surveys and Monographs, Vol. 68, AMS, 1999.
- Israil Gelfand, Mikhail Kapranov, Andrei Zelevinsky: Discriminants, Resultants and Multidimensional Determinants. Springer-Birkhäuser 1994.
- Masao Jinzenji: Classical Mirror Symmetry. SpringerBriefs in Mathematical Physics, Band 29, 2018.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge of the modules Commutative Algebra and Algebraic Geometry are assumed.

Responsible Persons

Victor Batyrev

Abbreviations:

Grading System : g=graded, ng=not graded
Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-02**	**Module Title:** Convex Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.		
Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Besides they enhance their geometric view and their spatial sense. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Convex Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-03	**Module Title:** Tropical Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Tropical numbers and polynomials. • Tropical hypersurfaces and varieties. • Tropical toric varieties. • Matroid fans and tropical abstract varieties. • Tropical modifications, stable intersections and rational equivalence. • Tropical curves and linear systems. • Tropical (p, q)-homology. • Correspondence theorems.		
Objectives	The students know and understand the subjects studied from tropical geometry and the fundamental techniques for working them. They have reached a deepend understanding of convex geometry and they have learned, how concepts from combinatorics can be applied successfully in algebraic geometry. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018. • Diane Maclagan, Bernd Sturmfels: Introduction to tropical geometry. AMS 2015.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites. Knowledge from the modules Algebraic Geometry and Differential Geometry is helpful, however.
Responsible Persons	Hannah Markwig, Johannes Rau

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-04**	**Module Title:** Tropical Enumerative Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Tropical and classic Gromov-Witten theory in genus 0. • Real counts, Welschinger invariants and polynomial invariants. • Hurwitz numbers. • Tropical correspondences for Hurwitz numbers. • Real Hurwitz numbers and Zigzag numbers.		
Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards modular spaces and Gromov-Witten theory. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Enumerative Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Familiarity with the module Tropical Geometry is assumed.
Responsible Persons	Hannah Markwig, Johannes Rau

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-05	**Module Title:** Introduction to Tropical Enumerative Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Real counts, Welschinger invariants and polynomial invariants.		
Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards moduli spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Tropical Enumerative Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be counted alongside it.
Prerequisites	Familiarity with the module Tropical Geometry is helpful, but not necessary.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-06**	**Module Title:** Tropical Enumerative Geometry - Part 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Plane curves of higher genus. • Multiplicities. • Welschinger invariants. • Lattice paths. • Floor diagrams. • Hurwitz numbers. • Tropical moduli spaces.		
Objectives	The students deepen their knowledge on the combinatorics of plane tropical curves. They get acquainted with various methods to enumerate tropical curves, as well as with various enumerative problems which can be solved with the aid of tropical geometry. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Enumerative Geometry - Part 2	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Diane Maclagan, Bernd Sturmfels: Introduction to tropical geometry. AMS 2015.
- Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be included alongside it.
Prerequisites	Familiarity with the module Tropical Enumerative Geometry is expected.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-10</td><td colspan="2">Module Title:
Geometry of Manifolds 1</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Manifolds and submanifolds.
• Vector fields and flows.
• Metrics, foundations of Riemannian geometry.
• Complex structures.
• Theorem of Gauß-Bonnet on surfaces.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. They have deepened their understanding especially of differential and integral calculus and have exemplarily experienced how mathematical concepts are naturally used in geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Geometry of Manifolds 1</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. Due to a significant overlap in contents the module cannot be taken together with either of the modules 'Geometry in Physics' and 'Introduction to Differential Geometry'.
Prerequisites	There are no further prerequisites.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-11	**Module Title:** Geometry of Manifolds 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Global aspects of Riemannian geometry • Cohomology of manifolds. • Analysis of differential operators on manifolds. • Applications on Riemann surfaces (and complex manifolds).		
Objectives	The students are familiar with the fundamental concepts of global real and complex differential geometry. They have deepened their understanding of methods in differential geometry, and they have exemplarily experienced how local and global aspects in geometry interact. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometry of Manifolds 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011. • John Milnor: Morse Theory. PUP 1963. • Donu Arapura: Algebraic Geometry over the Complex Numbers. Springer 2012. • Sundararaman Ramanan: Global Calculus. AMS 2005.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules 'Geometry on Manifolds' or 'Geometry in Physics' is assumed.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-12	**Module Title:** Information Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). • Application to neural data processing (in particular supervised learning in artificial neural networks).		
Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Information Geometry	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001.
- Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005.
- Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998.
- Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.

Transfer

The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Basic knowledge from differential geometry (Riemannian metrics, connections and curvature, geodesics) and from stochastics is assumed.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-13	**Module Title:** Information Geometry and Neural Data Processing 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Further basics of information geometry (e.g. dual flat structures for exponential families, Pythagoras' theorem and information projections, em algorithm). • Application to neural data processing (in particular *Unsupervised Learning* in artificial neural networks, e.g. Boltzmann and Helmholtz machines).		
Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Information Geometry and Neural Data Processing 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001.
- Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005.
- Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998.
- Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The module Information Geometry and Neural Data Processing 1 is a prerequisite.

Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-14	**Module Title:** Mathematical Aspects of Neuronal Information Processing 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Artificial neural networks and their training using backpropagation/stochastic gradient methods. • Dynamic interpretation as the flow of data/activations through the network (fast dynamics) and the change of weights during training (slow dynamics). • Simple neuroscientific models for the dynamics of neural networks. • Recent work on the theoretical foundations of deep learning and biologically plausible machine learning. • In a continuation of the lecture, the focus will shift to the processing of data with a dynamic origin, such as time series.		
Objectives	The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with dynamical systems as a possible framework for theoretical and mathematical investigations. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Aspects of Neuronal Information Processing 1	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Anthony C. C. Coolen, Reimer Kühn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. • Dmitry Krotov, John J. Hopfield: Unsupervised learning by competing hidden units. PNAS 2019. • Guan-Horng Liu, Evangelos A. Theodorou: Deep Learning Theory Review - An Optimal Control and Dynamical Systems Perspective. arXiv:1908.10920.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-15	**Module Title:** Introduction to Riemann Surfaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		

Content

- Coverings and fundamental groups.
- Topological classification of the surfaces.
- Theorem of Riemann-Hurwitz.
- Differential forms and integration.
- Sheaves and cohomology.
- Theorem of Riemann-Roch.
- Serre duality.
- Kobayashi metric.
- Theorem of Picard.

Objectives

Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. Using the sheaf concept, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Riemann Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature	**Possible References :** • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Riemann surfaces' due to the large overlap in content
Prerequisites	Knowledge from the lecture Introduction to Complex Analysis is required.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-16	Module Title: Riemannian Geometry		Type of Module: Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Riemannian manifolds. • Geodesics. • Curvature. • Geometry of submanifolds.		
Objectives	The students have learned and understood definitions and main examples of Riemannian manifolds from a classical point of view. In addition, topics related to geodesics were discussed. The students were exposed to important geometric results involving geodesics which are sufficient to study their role in different areas of differential geometry. Intuition for various notions of curvature was developed by the students and familiarity with computations in differential geometry was achieved in the exercise sessions. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Riemannian Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- John M. Lee: Riemannian manifolds: An introduction to curvature. Springer 1997.
- Barret O'Neill: Semi-Riemannian geometry. With applications to relativity. Academic Press 1983.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Geometry in Physics is assumed.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-17**	**Module Title:** Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Integrable systems are differential or difference equations with extraordinarily large symmetry group. The course will focus on equations related to the Korteweg de Vries (KdV) equation and discrete counterparts. Originally a mathematical model for the soliton phenomenon discovered during a famous horse ride along a canal, equations of KdV type have now many applications and the underlying theory involves various mathematical disciplines. A fundamental idea for understanding and solving KdV type equations is their interpretation as spectrum preserving deformations of underlying auxiliary linear operators - in the simplest case symmetric matrices. We study an important class of explicit solutions that includes solitons and finite gap (or algebro-geometric) solutions. This class of solutions can be described using a combination of Riemann surface theory and classical mechanics. The relevant parts of classical mechanics, Riemann surface theory, and spectral theory will be explained in the lecture. We will also briefly touch upon an integrable systems interpretation of the QR-algorithm of numerical linear algebra. The KdV equation is related to geometry in several different ways: for example, it can be interpreted as a dynamical system on the space of parametrized curves in the plane; it is deeply related to the geometry of Lie algebras and Lie groups; the special solutions discussed in the lecture are related to the geometry of Riemann surfaces... In a sequel to the lecture, it is planed to explain how infinite dimensional projective geometry allows to interpret generalizations of the KdV equation as quadratic equations and to finally linearize their dynamics.		
Objectives	The students have seen and understood relations between classical topics like Riemann surfaces, mechanics, and spectral theory – as well as other branches of mathematics – that were discovered mainly in the second half of the twentieth century during the emergence of a branch of mathematics sometimes called *soliton theory* or *integrable mathematics* The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Integrable Systems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature

Possible References :

- Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004.
- Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003.
- Alan C. Newell: Solitons in mathematics and physics. SIAM 1985.
- Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).

Transfer

The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

The module Introduction to Complex Analysis and Ordinary Differential Equations is required. Basic knowledge of differential geometry (manifolds, differential shapes) is helpful, but not necessary.

Responsible Persons

Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-18	**Module Title:** Integrable Systems (and Infinite Dimensional Lie Algebras)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Integrable systems are differential or difference equations with extraordinarily large symmetry group. The course will focus on equations related to the Korteweg de Vries (KdV) equation and discrete counterparts. Originally a mathematical model for the soliton phenomenon discovered during a famous horse ride along a canal, equations of KdV type have now many applications and the underlying theory involves various mathematical disciplines. A fundamental idea for understanding and solving KdV type equations is their interpretation as spectrum preserving deformations of underlying auxiliary linear operators - in the simplest case symmetric matrices. This lecture is the continuation of the lecture called Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory). This continuation will investigate integrable equations using sl(2,C)–loop algebras. In particular, we will study explicit solutions that can be described using the theory of hyperelliptic Riemann surfaces.		
Objectives	The students have aquired a uniform point of view on integrable equations related to the loop algebra of sl(2,C). The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Integrable Systems (and Infinite Dimensional Lie Algebras)	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004. • Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003. • Alan C. Newell: Solitons in mathematics and physics. SIAM 1985. • Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge from the module Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory) is assumed.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-19</td><td colspan="2">Module Title:
Mathematical Aspects of Neuronal Information Processing 2</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Advanced applications of artificial neural networks, e.g. processing of time-dependent data.
• Dynamic interpretation of neural data processing methods as flow of data/activations through the network (fast dynamics) and change of weights during training (slow dynamics).
• Simple neuroscientific models of neural network dynamics. Current work on biologically plausible machine learning.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have learnt the basics of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with dynamic systems as a possible framework for theoretical and mathematical investigations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td>Mathematical Aspects of Neuronal Information Processing 2</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016.
• Anthony C. C. Coolen, Reimer Kühn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005.
• Simon Haykin: Neural Networks: A Comprehensive Foundation. Pearson 1998.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisations Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In terms of content, the module Dynamic Systems and information processing 1 module is a prerequisite.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-50-20	Topology	Compulsory Module with Choice

ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		

Content

- Review of metric spaces: Closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces.
- Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms.
- Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions.
- Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems.
- Outlook on algebraic topology.

Objectives

Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topology	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-21	**Module Title:** Algebraic Topology 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-22	**Module Title:** Algebraic Topology 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Further development of singular homology theory. • Simplicial complexes and their simplicial homology. • CW spaces and their cellular homology. • Axiomatic homology. • Homological algebra. • Cohomology. • Homology and Cohomology with coefficients. • Product structures in homology and cohomology. • The Poincaré duality theorem for topological manifolds.		
Objectives	The students extend their ability to transfer concrete topological problems into algebraic constructions. They deepen their knowledge in abstract mathematical disciplines to accomplish even technically very challenging tasks. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Content-wise, the module Algebraic Topology 1 is a prerequisite for participating in this module.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-23	**Module Title:** Algebraic Topology 3		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	A selection of the following topics will be covered: • Basic concepts of homotopy theory; • Homotopy group of spheres; • Spectral sequences; • K-theory; • Characteristic classes.		
Objectives	With the in-depth knowledge of algebraic topology that they have acquired, students are now introduced to current areas of research and they tackle a small research project themselves, which can lead to a Master's thesis, for example. They will also lay the foundations for a possible doctorate in algebraic topology. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology 3	L	f	2	3	no	P		g	100

Specifics on the portfolio will be explained by the examiner at the beginning of the course.

Literature

Possible References :

- Allen Hatcher: Algebraic topology. Cambridge University Press 2009.
- Allen Hatcher: Vector bundles and K-theory. Manuskript 2009.
- John W. Milnor, James D. Stasheff: Characteristic classes. Princeton University Press 1974.
- John W. Milnor: Lectures on the h-cobordism theorem. Princeton University Press 1965.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the modules Algebraic Topology 1 and 2 are prerequisite for participation in this module.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-24**	**Module Title:** Introduction to K-theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Vector bundles. • Topological K-theory. • Künneth formula and Bott periodicity. • Characteristic classes. • Chern character. • Algebraic K-theory • Plus construction.		
Objectives	The students have learnt an important mathematical field that combines analysis, geometry, algebra and number theory. They have learnt to recognise and use the connections between different areas. They can understand and use terms such as vector or fibre bundles or categorical K-groups and apply them. They have learnt to think in large contexts. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to K-theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Literature

Possible References :

- Michael Atiyah: K-theory. Addison-Wesley 1989.
- Max Karoubi: K-theory. Springer 2008.
- Emilio Lluis-Puebla, Jean-Louis Loday, Henri Gillet, Christophe Soule, Victor Snaith: Higher algebraic K-theory: an overview. Springer 1992.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis und Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-25	**Module Title:** Applied topology 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Simplicial complexes and their homology. • Persistent homology. • Basic notions from topological data analysis.		
Objectives	The students are familiar with basic concepts of algebraic topology and their application in the context of topological data analysis. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Applied topology 1	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Herbert Edelsbrunner, John L. Harer: Computational Topology. AMS 2010. • Robert Ghrist: Elementary Applied Topology. Create Space 2014. • Sergey V. Matveev: Lectures on Algebraic Topology. EMS 2006.
Transfer	The module belongs to the *specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Stochastics*. Taking the personal specialisation and the restrictions of the sections into account, the module can be assigned to the Section *Specialisation*, *Specialisation Knowledge Mathematics* or *Elective Specialisation*.
Prerequisites	There are no further prerequisites.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-26	**Module Title:** Applied Topology 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Advanced aspects of persistent homology (e.g. stability). • Applied Morse theory. • Applied sheaf theory.		
Objectives	The students are familiar with advanced concepts of applied topology and topological data analysis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Applied Topology 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Herbert Edelsbrunner, John L. Harer: Computational Topology. AMS 2010. • Robert Ghrist: Elementary Applied Topology. Create Space 2014. • Sergey V. Matveev: Lectures on Algebraic Topology. EMS 2006.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The contents of the module ’Applied Topology 1’ are assumed. Moreover, basic knowledge from differential geometry is needed.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-27	**Module Title:** Topological Vector Spaces and Distributions		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	A selection of the following topics will be covered: • Locally convex topological vector spaces, Frechet spaces, LF spaces and LB spaces. • Duality: Hahn-Banach theorem, dual space, topologies on the dual space. • Generalised functions, Radon measures and distributions. • Properties of distributions and operations on the space of distributions. • Applications and examples.		
Objectives	Students master the basic principles and techniques of the theory of topological vector spaces and understand how to apply this to the theory of generalised functions according to L. Schwartz. Students are also able to name the main applications of the theory and show which classical questions of mathematical physics can be treated with it. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topological Vector Spaces and Distributions	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Gerald Folland: Real Analysis. Wiley 1999. • Helmut H. Schäfer: Topological Vector Spaces. Springer 1999. • Laurant Schwartz: Theorie des Distributions. Hermann 1998. • Laurant Schwartz: Mathematics for the Physical Sciences. Dover 2008. • Francois Trèves: Topological Vector Spaces, Distributions and Kernel. Dover 1967.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Functional Analysis and basic knowledge of set-theoretical topology is assumed.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-28</td><td colspan="2">Module Title:
Uniformisation of Riemann Surfaces</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Uniformisation of Riemann surfaces</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learnt how to determine the simply connected Riemann surfaces by successively solving suitable differential equations. They are then able to classify Riemann surfaces under suitable conditions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Uniformisation of Riemann Surfaces</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>2</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Algebra and Geometry and Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">The module Introduction to Riemann Surfaces as well as the Compulsory Modules of the Bachelor of Science Mathematics programme for Analysis are required.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Reiner Schätzle</td></tr>
</table>

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-29	**Module Title:** Algebraic Curves and Riemann Surfaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Compact Riemann surfaces. • Normalisation of plane curves. • Topological genus. • Coverings. • Forms and integration. • Sheaves and cohomology. • Hodge theory. • Arithmetic and geometric genus. • Abel's theorem. • Riemann-Roch theorem. • Serre duality. • Jacobian and Abelian varieties. • Riemann bilinear relations. • Jacobi inverse problem. • Elliptic curves and functions. • j-Invariant. • Uniformisation. • Topology of non-compact Riemann surfaces.		
Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Curves and Riemann Surfaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.

Literature

Possible References :

- Frederice Mangolte: Real Algebraic Varieties. Springer 2020.
- Robert Silhol: Real Algebraic Surfaces. Springer 1989.
- Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Herrmann 1990.
- Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
The module cannot be taken together with the module *Introduction to Riemann Surfaces* due to the large overlap in content.

Prerequisites

In terms of content, the courses on integration and measure theory as well as the introduction to function theory of ordinary differential equations are assumed.

Responsible Persons

Ivo Radloff

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-30	**Module Title:** Geometric Group Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Group actions on graphs, free groups. • Quasi isometries. • Growth types. • Hyperbolic groups. • Ends.		
Objectives	Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Group Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Clara Löh: Geometric Group Theory - an Introduction. Springer 2017.
- Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-40	Module Title: Gromov-Witten Theory		Type of Module: Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		

Content

- Enumerative geometry,
- Moduli spaces of stable curves,
- Moduli spaces of stable maps,
- Universal families,
- Forgetful maps,
- Gluing maps,
- Gromov-Witten invariants,
- Computation of Gromov-Witten invariants,
- Divisor equations,
- Kontsevich's formula.

Objectives

Students are based on their knowledge in Algebraic geometry introduced into the current research field of Gromov-Witten theory and enumerative geometry. The students know and understand important example classes of enumerative invariants and know how to present them as cut products on moduli spaces. Students master the basic algorithms for calculating Gromov-Witten invariants. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Gromov-Witten Theory	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Joachim Kock, Israel Vainsencher: An invitation to quantum cohomology: Kontsevich's formula for rational plane curves. Birkhäuser 2007. • Ravi Vakil: The moduli space of curves and Gromov-Witten theory. Enumerative invariants in algebraic geometry and string theory. Lecture Notes in Mathematics, 1947. Springer 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Algebraic Geometry is required.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-50**	**Module Title:** Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge of the interweaving of geometry and algebra. The students are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959.
- Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000.
- Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Geometry is helpful but not required.
Responsible Persons	Hermann Hähl, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-01**	**Module Title:** Functional Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Normed spaces, Banach spaces, Dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact Operators, normal operators, spectral theorems.		
Objectives	The students are aquainted with the basic principles and techniques of the theory of infinte dimensional spaces and can apply them to problems in analysis and geometry. They understand the complex of problems of spectral theory and can use its results for the solution of analytical problems. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Functional Analysis	L	f	4	6	yes	K o. mP o. H	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus *Numerical Mathematics and Optimisation*.
Prerequisites	There are no prerequisites.
Responsible Persons	Carla Cederbaum, Anton Deitmar, Gerhard Huisken, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-02	**Module Title:** Non-Linear Functional Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Differentiation and integration in Banach spaces. • Compact, coercive, proper mappings and gradient mappings. • Fredholm mappings. • Continuity method. • Degree of mapping. • Fixed point theorems. • Variational inequalities. • Monotone operators.		
Objectives	Students master the differentiation and integration of non-linear functions and various functional analytical methods for solving non-linear equations in infinite-dimensional spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes, students have acquired confidence in the technical handling of the methods they have learnt and can apply them independently to other problems. They are able to present their problem solutions and participate in discourses on problems in this field of research.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Linear Functional Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Melvyn Berger: Nonlinearity in Functional Analysis. Elsevier 1977. • Klaus Deimling: Nonlinear Functional Analysis. Springer 1985. • Eberhard Zeidler: Nonlinear Functional Analysis and its Applications I. Fixed-Point Theorems. Springer 1986.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The Integration and Measurement Theory module and the Functional Analysis module must have been successfully completed.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-03	**Module Title:** Operator Theory	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 4 SWS	
Content	• Operator semigroups and abstract Cauchy problems. • Theorem of Hille-Yosida. • Applications of concrete evolution equations. • Spectral theory of semigroups and their generators. • Asymptotic of semigroups. • Applications: – Semigroups of ordinary and partial differential equations; – Semigroups of transport problems; – Semigroups of control theory.	
Objectives	Students have understood the concept of the operator-valued exponential function. They are able to deal with concrete evolution equations in this abstract form. They are able to prove well-posedness using the Hille-Yosida theorem and discuss the qualitative behaviour of solutions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Operator Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Bruce Blackadar: Operator algebras. Springer 2006. • Klaus Jochen Engel, Rainer Nagel: One-parameter semigroups for linear evolution equations. Springer 2000. • Klaus Jochen Engel, Rainer Nagel: A short course on operator semigroups. Springer 2006. • Gert Pedersen: Analysis now. Springer 1995.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Anton Deitmar, Rainer Nagel, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master’s thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-04**	**Module Title:** Operator Algebras		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Geometry of Hilbert spaces. • Operators on Hilbert spaces and their spectral properties. • Spectral theory in Banach algebras. • Commutative Banach algebras and the representation theorem of Gelfand and Gelfand-Naimark. • The spectral theorem for normal operators of a Hilbert space. • Operator topologies and von Neumann's bicommutant theorem. • Kaplansky's density theorem. • Von Neumann algebras and their classification according to Murray-von Neumann, construction of examples. • The axiomatics of C^*- and W^*-algebras, the theorem of Gelfand-Naimark-Segal theorem for C^*-algebras and the representation theorem of Sakai for W^*-algebras. • Applications and outlook.		
Objectives	The students have familiarised themselves with the central concepts, results and methods of the theory of operator algebras. They have learnt the interplay between algebra and topology using the example of von Neumann algebras and their classification. The students also recognise how taking a higher point of view, i.e. the axiomatic nature of the problem, allows different questions to be dealt with and solved simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Algebras	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		ü	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									

Literature

Possible References :

- Bruce Blackadar: Operator algebras. Springer 2006.
- Ola Bratelli, Derek Robinson: Operator Algebras and Quantum Physics. Springer 1997.
- Richard Kadison, John Ringrose: Fundamentals of the Theory of Operator Algebras I - IV. AMS 1997.
- Gert Pedersen: Analysis now. Springer 1995.
- Shoichiro Sakai: C^*- and W^*-Algebras. Springer 1998.
- Masamichi Takesaki: Theory of Operator Algebras I - II. Springer 2002.

Transfer

The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

The content of the Functional Analysis module is prerequisite for participation in this module.

Responsible Persons

Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-05	**Module Title:** Ergodic Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Topological and measure-theoretical dynamical systems. • Recurrence and mixing properties. • Ergodic theorems of von Neumann and Birkhoff. • Spectral theory of the Koopman operator. • Operators with discrete spectrum (Halmos-von Neumann) • Applications in stochastics and number theory.		
Objectives	The students have familiarised themselves with the central concepts, results and methods of ergodic theory. They have experienced the profound interplay between measure theory and topology using the example of dynamic systems and their classification. The functional-analytical perspective makes it possible to deal with and solve various problems simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Ergodic Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Manfred Einsiedler, Thomas Ward: Ergodic Theory with a View Towards Number Theory. Springer 2011. • Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015. • Paul Halmos: Lectures on Ergodic Theory. Martino Fine Books 2013. • Marcelo Viana, Krerley Oliveira: Foundations of Ergodic Theory. CUP 2016.
Transfer	The module belongs to the *Study specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is Prerequisite for participation in this module.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-06</td><td colspan="2">Module Title:
Control Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Introduction to finite-dimensional linear control systems with examples from mechanics.
– Controllability, observability, stabilisability.
– Kalman criterion. Feedback systems.
– Stabilisability through feedback.
– Examples.
• Introduction to infinite-dimensional control theory.
– Mathematical framework and examples.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students learn important foundations of finite- and infinite-dimensional control theory. They are able to use the theory in areas of application such as mechanics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Control Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Hans W. Knobloch: Lineare Kontrolltheorie. Springer 1985. • Hans W. Knobloch, Alberto Isidori, Dietrich Flockerzi: Topics in control theory. Birkhäuser 1993. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Rurth F. Curtain, Hans Zwart: An Introduction to Infinite Dimensional Systems Theory. Springer 1995.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-07	**Module Title:** Linear Control Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Linear Control Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985.
- Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992.
- Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the modules Analysis and Linear Algbra is sufficient.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-08	**Module Title:** Spectral Theory of Positive Operators	**Type of Module:** Compulsory Module with Choice
ECTS-Points	6	
Workload - Time in Class - Self-Study	Workload: 180 h; Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS	
Content	Starting from the classical theorems of Perron and Frobenius on the spectrum of positive matrices, positive linear mappings to C^*- and W^*-algebras and their spectral and algebraic properties are analysed. The ergodic properties of these operators, i.e. the convergence of powers and means, can then be derived from these. We then discuss the generalisation to operator semigroups. Applications of the theory can be found in mathematical physics, among others.	
Objectives	Students learn the basic spectral properties of positive operators on C^*- and W^*-algebras and the connections with non-commutative ergodic theory. In the seminar following the lecture, students can work on topics that lead to a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.	

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Spectral Theory of Positive Operators	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Tanja Eisner, Markus Haase, Rainer Nagel : Operator Theoretic Aspects of Ergodic Theory. Springer 2015.
- Ulrich Groh: Spectral Theory of Completely Positive Maps on C^*- and W^*-Algebras. Preprint.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from functional analysis and operator algebras is assumed.

Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master’s thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-09	**Module Title:** Non-Commutative Ergodic Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	Firstly, the essential basic concepts and properties of C* and W* algebras are introduced and discussed. Then, based on commutative theory, non-commutative dynamical systems are defined. With the help of the so-called cross products it is then shown how such non-commutative dynamical systems can be characterised with the help of the group representation. The significance in mathematical physics is always emphasised.		
Objectives	The students have learnt the central concepts, results and methods of non-commutative ergodic theory, i.e. of dynamical systems on operator algebras. They have experienced the fascinating interplay between the structure of von Neumann algebras and the (asymptotic and spectral-theoretical) behaviour of operators on these algebras. The students realised how an axiomatic and structural point of view makes it possible to treat and solve different problems simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Commutative Ergodic Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015.
- Bruce Blackadar: Operator Algebras. Springer 2006.
- Alai Guichardet: Systèmes dynamiques non commutatifs. Astérisque 13-14 1974.
- Dirk Werner: Funktionalanalysis. Springer 1995.
- Volker Runde: A Taste of Topology. Springer 2005.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Good knowledge of functional analysis and basic knowledge of topology. Interest in mathematical quantum mechanics.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-10</td><td colspan="2">Module Title:
Pseudo Differential Operators</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Fourier transform and Sobolev spaces.
• Pseudodifferential operators on manifolds.
• Finite propagation velocity.
• Fredholm operators and elliptic complexes .
• The heat conduction kernel and the local index theorem.
• The Atiyah-Bott-Patodi theorem.
• Von Neumann algebras and representations.
• The L2 index theorem.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn basic techniques in the theory of elliptic differential operators and spectral geometry. They will understand the connection between differential and integral operators and how both merge into the more general calculus of pseudo-differential operators and how the transition from one to the other results in solution techniques for differential equations. They will be able to use theoretical approaches to solve specific problems in concrete cases. They will learn to use modern approaches of L2 theory to prove deep group theoretical statements. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td>Pseudo Differential Operators</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Peter B. Gilkey: Invariance theory, the heat equation, and the Atiyah-Singer index theorem. Publish or Perish 1984. • Wolfgang Lück: L2-invariants: theory and applications to geometry and K-theory. Springer 2002. • Michael Taylor: Pseudo differential operators. Springer 1974. • Man-Wah Wong: An introduction to pseudo-differential operators. World Scientific Publishing 2014.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Functional Analysis is assumed.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-11	**Module Title:** Introduction to Harmonic Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Fourier series and Fourier transformation. • Plancherel theorem and inverse theorems. • Poisson summation formula. • tempered distributions. • Additionaly a selection of the following topics will be covered: – LCA groups; – general Fourier transformation; – non-abelian groups and representations; – Sobolev-spaces; – Singular integrals; – Poisson integrals.		
Objectives	Students can combine algebraic and analytical methods and apply them to solve problems. They recognise the interplay between the properties of functions and their Fourier transforms and can apply the knowledge gained from this to questions in physics, analysis and number theory. They understand the interaction of group theory and analysis and gain deep insights into various function spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for Obtaining Credit, Grading, Weight if applicable	see table below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Harmonic Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Anton Deitmar: A first course in harmonic analysis. Springer 2005. • Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press, 1970. • Elias M. Stein, Guido Weiss: Introduction to fourier analysis on euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-12	**Module Title:** Harmonic Analysis in Euclidean Space		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Fourier transformation. • Covering-, decomposition- and interpolation theorems. • Singular integrals, Poisson integrals. • Hardy- and BMO-spaces, multiplier theorems, Littlewood-Paley theory.		
Objectives	The students got to know the central terms, results and methods of harmonic analysis in euclidean space. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Harmonic Analysis in Euclidean Space	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Charles L. Feffermann, Elias M. Stein: H^p spaces of several variables. Acta Mathematica 129, pp. 137-193, 1972. • Christopher D. Sogge: Fourier integrals in classical analysis. Cambridge University Press 2017. • Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press 1970. • Elias M. Stein: Harmonic analysis. Princeton University Press 1993. • Elias M. Stein, Guido Weiss: Introduction to Fourier analysis on Euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the modules functional analysis and Introduction to Harmonic Analysis are a prerequisite for participation in this module.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-13</td><td colspan="2">Module Title:
Harmonic Analysis on Abelian Groups</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Locally compact groups, existence and uniqueness of Haar measures.
• Convolution algebras, Banach algebras, the Gelfand-Neumark theorem.
• LCA groups, Pontryagin duality, Plancherel theorem, structure theory of LCA groups.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have become familiar with the central concepts and methods of abstract harmonic analysis and know how to use them. They have understood the connection between topological/analytical/geometric concepts such as LCA groups and their expression in algebraic structures such as C^*-algebras and are able to apply this way of thinking to other theories. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Harmonic Analysis on Abelian Groups</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Anton Deitmar: A first course in Harmonic Analysis. Springer 2005.
• Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008.
• Edwin Hewitt, Kenneth Ross: Abstract harmonic analysis. Vol. I. Springer 1979.
• Walter Rudin: Fourier analysis on groups. John Wiley 1990.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Functional Analysis is a prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-14</td><td colspan="2">Module Title:
Harmonic Analysis on General Groups</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Representation theory of compact groups, Peter-Weyl theorem.
• Representation theory of general groups.
• trace formula and applications to the Heisenberg group and $SL_2(\mathbb{R})$.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have familiarised themselves with the deeper concepts and methods of abstract harmonic analysis and know how to use them. They have mastered the trace formula and understand its far-reaching implications, also for other areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Harmonic Analysis on General Groups</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008.
• Gerald B. Folland: A course in abstract harmonic analysis. Studies in Advanced Mathematics. Boca Raton 1995.
• Michael E. Taylor: Noncommutative Harmonic Analysis. AMS 1986.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In terms of content, the module Harmonic Analysis on abelian groups is a prerequisite for participation in this module.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-15	**Module Title:** Selected Chapters from Operator Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Spectral theory of restricted and unrestricted linear operators, especially spectral calculus. • Spectral theory of positive operators – Perron-Frobenius theory. • Spectral theory for operators of ergodic theory.		
Objectives	Students master the concepts of spectral theory and in particular the abstract functional calculus. They can then apply this to concrete operators and discuss properties such as asymptotic behaviour. They are also able to recognise cross-connections to other mathematical fields such as stochastics, ergodic theory or number theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Selected Chapters from Operator Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Klaus Jochen Engel, Rainer Nagel: One-parameter semigroups for linear evolution equations. Springer 2000. • Markus Haase: The Functional Calculus for Sectorial Operators. Birkhäuser 2006.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Solid knowledge of operator theory, in particular Hille-Yosida theory for operator semigroups is a prerequisite.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-21	**Module Title:** Introduction to Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		
Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Partial Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus *Numerical Mathematics and Optimisation*.
Prerequisites	There are no further prerequisits.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-22**	**Module Title:** Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Schauder estimates. • Calderon-Zygmund estimates. • Harnack's inequality. • Hölder regularity. • Viscosity solutions. • Existence of solutions according to Perron. • Evans-Krylov theorem.		
Objectives	After the students have learnt the basic concepts and methods in Introduction to Partial Differential Equations, this knowledge is deepened. Students are prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Partial Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Luis Angel Caffarelli, Xavier Cabre: Fully nonlinear elliptic equations. American Mathematical Society 1995. • Michael G. Crandall, Hitoshi Ishii, Pierre-Louis Lions: User's Guide to Viscosity Solutions of second Order Partial Differential Equations. Bulletin of the American Mathematical Society 27, No. 1, pp. 1-67, 1992. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear elliptic equations.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Instationary Differential Equations, it can be included in the specialisation *Numerical Mathematics and Optimisation*.
Prerequisites	The content of the module Introduction to Partial Differential Equations is a prerequisite for the participation in this module.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-24**	**Module Title:** Nonlinear Elliptic Partial Differential Equations in Minimal Surface Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	The course will consider PDE aspects of minimal surfaces, beginning with the existence theory for minimal graphs with prescribed boundary data. Emphasis will be placed on the De Giorgi-Nash estimate, which is one of the key achievements of 20th Century mathematics, and is foundational in the study of quasilinear elliptic and parabolic equations. We will also explore connections between minimal surfaces and the Allen-Cahn equation, which is a semilinear equation arising in the theory of phase transitions. Here the focus will be on rigidity results for entire solutions (namely the Bernstein problem and closely related De Giorgi conjecture) and their use in proving regularity via rescaling.		
Objectives	The students obtain an advanced working knowledge of nonlinear elliptic PDE theory, and an understanding of connections between this theory and profound problems in geometry. They will acquire an array of new techniques for establishing quantitative and qualitative control on objects governed by nonlinear differential equations. These techniques include advanced applications of Sobolev theory, rescaling and compactness arguments, Stampacchia iteration, Moser iteration, and the use of monotonicity formulae. Students will be able to assess if and when these techniques are applicable to a given problem. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Nonlinear Elliptic Partial Differential Equations in Minimal Surface Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Lawrence C. Evans: Partial Differential Equations. AMS 2010. • David Gilbarg, Neil Trudinger: Elliptic partial differential equations of second order. Springer 1083. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications. Siam 2000.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basics of linear elliptic PDE (Schauder theory, existence for Dirichlet problem) are desirable but not completely necessary.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-25	**Module Title:** Introduction to Partial Differential Equations – Part 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Partial Differential Equations – Part 1	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010.
- David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001.
- Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules 'Numerics of Stationary Differential Equations' or 'Numerics of Nonstationary Differential Equations', it can be included in the Specialisation *Numerical Mathematics and Optimisation*. The module is part of the module Introduction to Partial Differential Equations and cannot be taken together with this module.
Prerequisites	There are no further prerequisites.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-26**	**Module Title:** Partial Differential Equations in Conformal Geometry: the Yamabe Problem		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The uniformisation theorem states that a closed two-dimensional Riemannian manifold is conformally equivalent to one with constant sectional curvature. The appropriate higher-dimensional analogue problem is finding a conformal metric with constant scalar curvature. This is the so-called Yamabe problem, introduced by Yamabe in 1960. The flaws in his proof were fixed only in 1984, by combining the works of Trudinger, Aubin, Schoen, and others. This course provides an overview of the complete proof of the Yamabe problem, which is extremely rich in techniques of calculus of variation, geometric analysis, and elliptic partial differential equations. It also employs crucial results of mathematical relativity, such as the Positive Mass Theorem, which is motivated by physics but possesses powerful analytical consequences. Moreover, basic results for non-compact manifolds or manifolds with boundary and the parabolic analogue, the Yamabe flow, will be discussed.		
Objectives	The students can state the Yamabe problem and explain the structure of the proof. The students are familiar with basic methods of the calculus of variation, particularly concerning the (sub-critical) Yamabe energy functionals and the associated elliptic semi-linear PDEs. They can study the conformal Green's function and relate it to the solvability of the Yamabe problem. The students are also familiar with the notion of mass in mathematical relativity and the renowned Positive Mass Theorem, still linking it to the solution of the Yamabe problem; they can provide the main ideas of the proof of the Positive Mass Theorem and, in particular, master elements of the minimal surface theory. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Partial Differential Equations in Conformal Geometry: the Yamabe Problem	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • John M. Lee, Thomas H. Parker: The Yamabe problem. Bull. Am. Math. Soc., New Ser. 17, 37–91, 1987. • Richard Schoen, Shing-Tung Yau: Lectures on differential geometry. International Press 1994. • Thierry Aubin: Some nonlinear problems in Riemannian geometry. Springer 1998. • Michael Struwe: Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems. Springer 2008.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Geometry in Physics and Introduction to Partial Differential Equations is assumed.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-27	**Module Title:** Fully Non-Linear Elliptic Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Solution of general fully non-linear equations with elliptic equations. • Solution of the Monge-Ampere equation.		
Objectives	The students learn the techniques to successively estimate the supremum, the gradient and the second derivatives of a given solution of a fully non-linear elliptic equation. The students learn how the modulus of continuity of the second derivatives is then estimated using the Evans-Krylov theorem and learn the continuity method, which leads to the existence of a solution. In particular, they are able to apply the methods to general uniformly elliptic equations and to the special, non-uniformly elliptic Monge-Ampere equation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Fully Non-Linear Elliptic Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Luis A. Caffarelli, Joseph Kohn, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. I. Monge-Ampere equation. In: Communications on Pure and Applied Mathematics 37,3 pp. 369-402.
- Luis A. Caffarelli, Joseph Kohn, Luis Nirenberg, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. II. Complex Monge-Ampere, and uniformly elliptic, equations. In: Communications on Pure and Applied Mathematics 38,2 pp. 209-252.
- David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order. Springer 1998.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module MAT-60-36 'Fully nonlinear elliptic and parabolic partial differential equations' due to the large overlap in content.
Prerequisites	For participation, the modules Introduction to Partial Differential Equations and Partial Differential Equations are required.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-28**	**Module Title:** Morse Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Topology of differentiable manifolds. • Riemannian metrics on differentiable manifolds. • Dynamic systems on differentiable manifolds. • Homotopy type of differentiable manifolds. • Main approaches of Morse theory. • Outlook on Morse homology.		
Objectives	Students learn how to analyse problems in the differential topology of differentiable manifolds using the tools of analysis, in particular the theory of dynamical systems. In particular, they learn how the level surfaces of non-degenerate functions, so-called Morse functions, can be used to obtain statements about the homotopy type of manifolds. They also build a bridge to algebraic topology, which analyses the topology (of manifolds) using algebraic methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Morse Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- John Milnor: Morse Theory. Annals of Math. Studies, Number 51. Princeton University Press 1961.
- Morris W. Hirsch: Differential Topology. Graduate Texts in Mathematics 33. Springer 1988.

Transfer	The module belongs to the *Study Specialisation Studienschwerpunkt Analysis and Differential Geometry* and *Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of differentiable manifolds and of dynamic systems is helpful.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-32	**Module Title:** Selected Chapters from Dynamical Systems Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	A selection of the following topics will be covered: • dynamical systems as solution flows of ordinary and partial differential equations; • isomorphic invariants of dynamical systems, especially the discrete spectrum; • linear skew-product flows; • applications in number theory, combinatorics and stochastics.		
Objectives	Students are familiar with qualitative questions in the theory of ordinary and partial differential equations and the methods used to analyse them. On the basis of solid knowledge of functional analysis, operator theory and ergodic theory, they have experienced the diverse applicability of abstract mathematical concepts. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Selected Chapters from Dynamical Systems Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator theoretic aspects of ergodic theory. Springer 2015.
- Manfred Einsiedler, Thomas Ward: Ergodic theory: with a view towards Number Theory. Springer 2011.
- David Kerr, Hanfeng Li: Ergodic theory: independence and dichotomies. Springer 2016.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. It can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Dynamic Systems Prerequisite for participation in this module.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-33	**Module Title:** Abstract Dynamical Systems		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	The central properties of topological dynamic systems such as minimality, recurrence and (topological) ergodicity are repeated. Subsequently, the statements proven there are applied to category theoretical foundations. Important structural results such as the Jacobs-deLeeuw-Glicksberg decomposition, the theorem of Halmos-von Neumann's theorem and the Furstenberg-Zimmer structure theory are discussed and generalised. In this context, current research topics are addressed and a category-theoretical perspective is developed. Among other things, the application of ergodic theory to number theory and combinatorics is presented.		
Objectives	The students have learnt how abstract theories (here dynamic systems, Koopman systems) can be developed and further abstracted from concrete questions (in number theory). They can apply the techniques developed in these areas, to deal with concrete (e.g. number-theoretical or ergodic-theoretical) problems. The students have thus learnt important examples of the usefulness of abstract mathematical theories. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Abstract Dynamical Systems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Tanja Eisner et al.: Operator theoretic aspects of ergodic theory. Springer 2015.
- Jan de Vries: Topological dynamical systems. An introduction to the dynamics of continuous mappings. De Gruyter 2014.
- Saunders Mac Lane: Categories for the working mathematician. Springer 1998.
- Helmut H. Schaefer: Banach lattices and positive operators. Springer 1978.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Dynamical Systems' due to the large overlap in content.
Prerequisites	Solid knowledge of Topology, Functional Analysis and Operator Theory, in particular spectral theory of positive operators is required. operators are assumed. Fundamentals of Ergodic Theory and Category Theory are also very useful, but not strictly necessary. necessary.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-34	**Module Title:** Introduction to Dynamical Systems		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do exist mathematical solutions? Are there equilibrium states or periodic orbits? They are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introducation to Dynamical Systems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Literature

Possible References :

- Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974.
- Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010.
- Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Dynamical Systems' due to the large overlap in content.
Prerequisites	There are no further prerequisites.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-41**	**Module Title:** Introduction to Geometric Measure Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and Cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Geometric Measure Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-42</td><td colspan="2">Module Title:
Geometric Measure Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• First and second variation for varifolds.
• Monotonicity formula.
• Allard’s integral compactness theorem.
• Lipschitz approximation.
• tilt-excess descent.
• Allard’s regularity theorem.
• General and rectifible flows.
• Deformation theorem.
• Surface minimizing flows.</td></tr>
<tr><td>Objectives</td><td colspan="3">After having learned the basic terms and methods in introduction to geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Geometric Measure Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Regarding to content the module Introduction to Geometric Measure Theory is a prerequisite for participation in the module Geometric Measure Theory.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-43	Module Title: Area Minimising Flows	Type of Module: Compulsory Module with Choice
ECTS-Points	5	
Workload - Time in Class - Self-Study	Workload: 150 h; Time in Class: 45 h; Self-Study: 105 h	
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS	
Content	• Compactness theorem for integral flows. • Regularity of area minimising flows.	
Objectives	After having learned the essential notions and methods of geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Area Minimising Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Introduction to Geometric Measure Theory and Geometric Measure Theory is expected.

Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-44</td><td colspan="2">Module Title:
Introduction to Geometric Measure Theory – Measure Theoretic Methods</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities.
• Isodiametric inequality.
• Rademacher’s theorem and Whitney’s embedding theorem.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Introduction to Geometric Measure Theory – Measure Theoretic Methods</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>2</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992.
• Herbert Federer: Geometric measure theory. Springer 1969.
• Leon Simon: Lectures on geometric measure theory. Australian National University 1984.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	There are no further prerequisites.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-45	**Module Title:** Introduction to Geometric Measure Theory – Varifolds		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Surface- and Cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Geometric Measure Theory – Varifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992.
- Herbert Federer: Geometric measure theory. Springer 1969.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	The module Integration and Measure Theory from the B.Sc. Mathematics or an equivalent module must have been successfully completed during the course of studies.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-46	**Module Title:** Elastic Curves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Classification of elastic curves according to Langer and Singer. • Order reduction of the Euler-Lagrange equation of the elastic energy of a curve. • Qualitative behaviour of an elastic curve. • Solving the Willmore equation under axial symmetry with variational methods.		
Objectives	Students learn how to deal with a geometrically relevant functional and its critical points using the example of the elastic energy of a curve. This gives them an insight into the theory of fourth-order elliptic differential equations where familiar techniques, such as the maximum principle, can no longer be used. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elastic Curves	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Filippo Gazzola, Hans-Christoph Grunau, Guido Sweers: Polyharmonic Boundary Value Problems, Springer 2010.
- David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 1998.
- Joel Langer, David A. Singer: The total squared curvature of closed curves, J. Differential Geom. Band 20, Nummer 1, Seiten 1-22, 1984.
- John M. Lee: Introduction to smooth manifolds. Springer 2013.
- Michael Struwe: Variational Methods. Springer 2008.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the module Introduction to Partial Differential equations and basic knowledge of differential geometry are assumed.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-47</td><td colspan="2">Module Title:
Geometric Measure Theory – Varifolds</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• First and second variation for varifolds.
• Monotonicity formula.
• Allard’s integral compactness theorem.
• Lipschitz approximation.
• tilt-excess descent.
• Allard’s regularity theorem.</td></tr>
<tr><td>Objectives</td><td colspan="3">After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Geometric Measure Theory – Varifolds</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>2</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Herbert Federer: Geometric measure theory. Springer 1969.
• Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984.
• Leon Simon: Lectures on geometric measure theory. Australian National University 1984.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	In terms of content, the module Introduction to Geometric Dimension Theory is a is a prerequisite for participation.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-48	**Module Title:** Geometric Measure Theory – Flows		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• General and rectifiable flows. • Deformation theorem. • Surface minimizing flows.		
Objectives	After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Measure Theory – Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Herbert Federer: Geometric measure theory. Springer 1969.
- Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	In terms of content, the module Introduction to Geometric Dimension Theory is a is a prerequisite for participation.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-49	**Module Title:** Calculus of Variations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Calculus of Variations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the modules Introduction to Partial Differential Equations and Functional Analysis is an advantage, but not essential.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-51	**Module Title:** Lie Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		
Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can disciplines of mathematics can work together extremely successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Lie Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-52	**Module Title:** SL2(R)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	Deutsch und Englisch		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Structure theory of the Lie group $SL_2(\mathbb{R})$. • Introduction to the representation theory of $SL_2(\mathbb{R})$. • Computation of the unitary dual. • Proof of the explicit Plancherel formula.		
Objectives	With the $SL_2(\mathbb{R})$ the students have studied an important Lie group in detail. That way they have become familiar with the basics of the representation theory of Lie groups as well as with the basics of hyperbolic geometry. The students have learned to construct, to split and to classify representations. Moreover, they are able to transfer their knowledge to analyse other Lie groups, and they have gathered a deeper understanding of the theory of Lie groups. They understand the analysis lying underneath the Theorem of Plancherel and apply it successfully. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
$SL_2(\mathbb{R})$	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Literature	**Possible References :** • Anthony Knapp: Representation theory of semisimple groups. PUP 2001. • Serge Lang: SL2(R). Springer 1985.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-53	**Module Title:** Automorphic Forms		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.		
Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Automorphic Forms	L	f	2	3	yes	K o. mP o. H	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Deitmar, Anton: Automorphic Forms. Springer 2012.
- Goldfeld, Dorian: Automorphic forms and L-functions for the group GL(n,R). Cambridge University Press 2015.
- Serre, Jean-Pierre: A course in arithmetic. Springer 1973.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Introduction to Geometric Measurement Theory and cannot be taken together with this module.
Prerequisites	Basic knowledge of function theory is assumed.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-60	**Module Title:** Introduction to Mathematical Logic		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to the Mathematical Logic	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Literature

Possible References :

- Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008.
- Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.

Transfer	The module is not assigned to a specialisation. It can be included in the Sections *Advanced Knowledge in Mathematics* or *Elective Specialisation* in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-61	**Module Title:** Cohomology and Sheaves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content

It is shown how different cohomology theories (singular, de Rham, Cech) can all be understood as derivatives of the section functor from sheaf theory and thus their equality (after coefficient expansion) can be shown very easily:

- Introduction to category theory.
- Presentation of the current cohomology theories.
- Sheaves, derived functors, sheaf cohomology.
- Comparison of cohomology theories.

Objectives

The students see and understand the connections between seemingly widely divergent theories. They understand mechanisms that combine algebraic, geometric and analytic methods. They have learned to abstract arbitrary mathematical theories using category theory, to appreciate cohomology theory as a general obstacle theory in applications and to use sheaves as generalisations of function spaces for topological questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Cohomology and Sheaves	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 6 credit points will be awarded for the module instead of 9.

Literature

Possible References :

- Saunders Mac Lane: Categories for the working mathematician. Springer-Verlag 1971.
- Allen Hatcher: Algebraic topology. Cambridge University Press 2002.
- Glen Bredon: Sheaf theory. Springer-Verlag 1997.
- Joseph Rotman: An introduction to homological algebra. Springer-Verlag 2008.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, only basic knowledge from the analysis and linear algebra is required.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-62</td><td colspan="2">Module Title:
Consistency Proofs</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes).
• Philosophical foundational programs (logicism; formalism; intuitionism).
• The Hilbert program and Gödel’s theorems.
• Gentzen’s transfinite consistency proof for number theory.
• Alternative approaches to consistency (including Gödel’s T).
• Current situation of consistency proofs.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn about the historical context from which the question of the non-contradiction of formal mathematical theories arose, as well as the relevant modern techniques for investigating this question mathematically. They are able to categorise the problem of non-contradiction in mathematics both historically and philosophically. In addition, they have acquired the mathematical tools to be able to comprehend the corresponding proofs of non-contradiction and, to a certain extent, to carry them out themselves. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Consistency Proofs	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic mathematical knowledge to the extent of the basic lectures is assumed. Previous knowledge of mathematical logic is helpful, but not necessary.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-63	**Module Title:** Introduction to set theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	**Content:** •		
Objectives	- The students are able to view mathematical theorems and theories in the context of mathematical logic. They understand the boundaries of the mathematically knowable, the difference between truth and provability, and are able to apply model-theoretic considerations to mathematical problems. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to set theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Literature	**Possible References :** •
Transfer	
Prerequisites	There are no further prerequisites.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-64	**Module Title:** Theory of Mathematical Proofs		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.		
Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Theory of Mathematical Proofs	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Wolfram Pohlers. Proof Theory. Springer 2009.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Basic mathematical knowledge to the extent of the basic lectures is assumed. Previous knowledge of mathematical logic is helpful, but not necessary.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-65	**Module Title:** Explicit Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.		
Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Explicit Mathematics	L	f	2	3	yes	or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 6.

Literature	**Possible References :** • Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium ?78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. Textos de Matematica, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3),141-162, 2001.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of mathematical logic.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-70**	**Module Title:** Selected Chapters from Functional Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	A selection of the following topics will be covered: • Topological vector spaces and duality theory. • (LB) and (LF) spaces and distributions. • Compactness concepts (Eberlein's theorem, Banach-Alaoglu, Krein-Milman, Smulian). • Theorems from topology (Tietze, Urysohn, Stone-Cech) and their applications in functional analysis. • Uniform spaces.		
Objectives	Students are familiar with the basic concepts of topological vector spaces and have learned to apply their methods and results to concrete examples from the field of functional analysis, such as the theory of distributions. They have recognised and understood the many cross-connections to other parts of mathematics, such as measure theory or topology. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Selected Chapters from Functional Analysis	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Gerald Folland: Real Analysis. Wiley 1999. • Helmut H. Schäfer: Topological Vector Spaces. Springer 1999. • Volker Runde: A Taste of Topology. Springer 2005. • Gert K. Pedersen: Analysis Now. Springer 1989. • Paul R. Halmos: Measure Theory. Springer 1950.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the Functional Analysis module is assumed.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-55-71	Operator Algebras and their Applications to Statistical Mechanics		Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		
Content	• Basics on operator algebras (C*-algebras, algebraic states, inductive limits); • Introduction to algebraic deformation quantization (general set-up, coherent states, examples); • Applications to the classical limit of quantum mechanics and statistical mechanics including asymptotic emergence (phase transitions, large deviations (entropy), spontaneous symmetry breaking).		
Objectives	The students have obtained deepend knowledge in selections questions in algebraic quantum theory with an emphasis on algebraic deformation quantisation and their applications to the classical limit of quantum mechanics and statistical mechanics. They have learned algebraic techniques in order to develop abstract structures encoding the features of a physical theory. They are familiar with techniques to prove existence results of limits of sequences/nets encoded by algebraic states, examine these, and put them into a general perspective. Moreover, they understand the physical relevance of the results and are able to relate them to features of equilibrium thermodynamics, such as phase transitions and spontaneous symmetry breaking. They are able to describe the current state of research in the specific area. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Operator Algebras and their Applications to Statistical Mechanics	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Klaas Landsman: Foundations of Quantum Theory, From Classical Concepts to Operator Algebras. Springer 2017.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry*, *Mathematical Physics* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of C*-algebras and functional analysis as well as in thermodynamics are assumed.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-01	**Module Title:** Geometric Evolution Equations	**Type of Module:** Compulsory Module with Choice
ECTS-Points	3	
Workload - Time in Class - Self-Study	Workload: 90 h; Time in Class: 30 h; Self-Study: 60 h	
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 2 SWS	
Content	• Examples of geometric evolution equations such as Mean curvature flow, Ricci flow, Inverse mean curvature flow. • Parabolic maximum principles. • Regularity theory for parabolic evolution equations. • Rescaling techniques and description of singularities. • Asymptotic behavior of solutions.	
Objectives	Students learn to combine their knowledge of differential geometry and partial differential equations and apply it to specific problems in selected geometric evolution equations. They learn techniques for checking solutions of non-linear evolution equations, which enables them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.	

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Evolution Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Nonstationary Differential Equations or Numerics of Differential Equations on Surfaces, it can be included in the subject specialisation *Numerical Mathematics and Optimisation*.
Prerequisites	Knowledge from the module Introduction to Partial Differential Equations and basic knowledge of differential geometry are required.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-60-02</td><td colspan="2">Module Title:
Geometric Variation Problems</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
• Examples of geometric variation problems such as minimal surfaces, capillary surfaces, harmonic mappings and associated boundary value problems.
• Direct methods of calculating variations.
• Regularity theory for solutions of variational problems.
• Relationship between variational problems and partial differential equations.
• Stability properties of solutions.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn to combine their knowledge of differential geometry and analysis and apply it to specific problems in selected geometric variational problems. They learn techniques for proving solutions to various variational problems and for analysing the properties of solutions, which provide a basis for independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td>Geometric Variation Problems</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the module Introduction to Partial Differential Equations and basic knowledge of differential geometry are required.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Carla Cederbaum, Gerhard Huisken</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-60-03**	**Module Title:** Topics in Mathematical Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• A selection of concrete models of Mathematical Relativity, such as black holes, static metrics, physical invariants of isolated systems, positivity estimates for energy and mass. • Geometric and analytical structure of the models, existence and properties of concrete models as solutions to Einstein's equations.		
Objectives	Students acquire in-depth knowledge of selected issues in the mathematical theory of relativity. They learn analytical and geometric techniques for proving and investigating solutions to Einstein's equations and are able to categorise the physical relevance of the mathematical results. The lecture introduces students to their first independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)		
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.		
Prerequisites	In terms of content, the modules Mathematical Relativity and Introduction to Partial Differential Equations are assumed.		
Responsible Persons	Carla Cederbaum, Gerhard Huisken		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topics in Mathematical Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-04	**Module Title:** Space-Like Hypersurfaces in Lorentzian Manifolds		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	Space-like hypersurfaces of a Lorentzian manifold play a crucial role in the study of solutions to Einstein's equations modelling some phenomenon in General Relativity. The course explores how geometric choices of space-like hypersurfaces such as maximal surfaces, constant mean curvature surfaces or solutions of mean curvature flow and inverse mean curvature flow can be used to achieve a splitting of space and time that is suitable for the study of both isolated gravitating systems and cosmological spacetimes.		
Objectives	The students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Space-Like Hypersurfaces in Lorentzian Manifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Barrett O'Neill: Semi-Riemannian Geometry - With applications to Relativity. Academic Press 1983. • Andrejs E. Treibergs: Entire space-like hypersurfaces of constant mean curvature in Minkowski space. Inventiones Math. 66, (1982) 39–56. • Klaus Ecker, Gerhard Huisken: Parabolic methods for the construction of spacelike slices of prescribed mean curvature in cosmological spacetimes. Comm. Math. Phys. 135 (1991), 595–613. • Helmut Friedrich, Alan Rendall: The Cauchy Problem for the Einstein Equations. In: Schmidt B.G. (eds) Einstein's Field Equations and Their Physical Implications. Lecture Notes in Physics, vol 540. Springer 1999. • Hans Ringström: The Cauchy Problem in General Relativity. European Math. Society 2009.
Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Mathematical Relativity is expected.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-05	**Module Title:** Limits of Spaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		
Content	• Basic concepts of metric geometry, e.g. geodesics, doubling property and Hausdorff measure. • Generalized curvature conditions in the sense of Alexandrov and Busemann. • Gromov-Hausdorff and ultra convergence. • Gromov's Precompactness Theorem and stability theorems.		
Objectives	Students generalise their knowledge in analysis and know how to apply the methods to particular problems in metric geometry. They get to know different convergence notions and learn which properties are stable in the limiting process. In addition, the students are familiar with synthetic and concrete curvature notions, which help to better understand curvature notions in differential geometry and general relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Limits of Spaces	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Jeff Cheeger, David Ebin: Comparison Theorems in Riemannian Geometry. AMS 1975.
- Dimitri Burago, Yuri Burago, Sergei Ivano: A Course in Metric Geometry. AMS 2001.
- Mikhail Gromov: Metric Structures for Riemannian and Non-Riemannian Spaces. Springer 2007.

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge in analysis and measure theory is assumed.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-06	**Module Title:** The Ricci Flow of Riemannian Metrics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The lecture introduces the basic properties of the Ricci flow and develops the necessary techniques, e.g. tensor maximum principles and regularity estimation. The long-term existence of solutions and resulting classifications for metrics of positive curvature are presented. Finally, the monotonicity of functionals according to Perelman is derived and used for the classification of possible singularities, with an outlook on the surgery methods of Hamilton and Perelman, which have led to the proof of the Poincaré and geometrisation conjectures.		
Objectives	The students have learnt basic methods for the treatment of geometric evolution equations in Riemannian geometry. At the same time, they have experienced the interplay of local geometric assumptions on the curvature properties of a metric with analytic techniques for the study of parabolic equations and have learnt and understood how the local assumptions have global consequences for the geometry and topology of the underlying manifolds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
The Ricci Flow of Riemannian Metrics	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	ü	o	2	3					

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.

Literature

Possible References :

- Simon Brendle: Ricci-flow and the sphere theorem. AMS 2010.
- Peter Topping: Lectures on the Ricci-Flow. Lecture Notes 2006.
- Richard Hamilton: Riemannian 3-manifolds with positive Ricci curvature. J. Diff. Geom. 17, 1982.

Transfer

The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the module Introduction to Partial Differential Equations as well as fundamental knowledge in differential geometry is required.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-07	**Module Title:** Special Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Derivation of the Minkowski metric from basic physical assumptions. • Physical consequences of relativity such as length contraction, time dilation and some popular paradoxes.		
Objectives	Students have learnt and understood the derivation of the special theory of relativity and important concepts such as length contraction and time dilation. They are familiar with important paradoxes that arise. Students have developed an intuition for various aspects of the theory of relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Albert Einstein: Relativity: the special and general theory. Public domain 1920.
- Thomas A. Moore: Six ideas that shaped physics: unit R. McGraw-Hill 2003.
- Robert Resnick: Introduction to Special Relativity. Wiley 2007.
- Bernard Schutz: A First Course in General Relativity. Cambridge University Press 2009.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Introduction to Partial Differential Equations and basic knowledge of differential geometry are required.

Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-08	**Module Title:** Null Geometry in General Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	This module provides an introduction to null geometry. Topics include the properties of light-like vector fields and curves, as well as the geometry of light-like hypersurfaces that carry a degenerate induced metric. Another major topic is the extrinsic curvature of space-like surfaces in higher codimension, which are considered in particular along light-like hypersurfaces. Optionally, geometric flows along light-like hypersurfaces can also be treated.		
Objectives	Students know and understand the concepts and methods mentioned and can use them to analyse known and new questions from null geometry. Furthermore, they link physical problems in cosmology and astrophysics and their mathematical modelling using differential geometric methods and are able to question the relevance and adequacy of mathematical modelling and the mathematical results derived from it. In particular, they expand on the methods learnt in the MAT-65-11 module and connect their methodological and specialist knowledge. They are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Null Geometry in General Relativity	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Barrett O'Neill: Semi-Riemannian Geometry. Academic Press 1983.
- Johannes Sauter: Foliations of Null hypersurfaces and the Penrose Inequality. Dissertation (ETH Zürich), url: https://www.research-collection.ethz.ch/handle/20.500.11850/150826.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Geometry in Physics module is a prerequisite.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><th>Module Number:
MAT-60-09</th><th colspan="2">Module Title:
The Einstein Constraint Equations</th><th>Type of Module:
Compulsory Module with Choice</th></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Introduction to GR and the Einstein equations:
The Einstein equations, special solutions and geometric constructions;
The Cauchy problem for the Einstein equations.
The constraint equations and the conformal method:
The conformal method;
Overview of the elliptic theory on closed manifolds;
Classification of constant mean curvature on closed manifolds.
Asymptotically Euclidean (AE) initial data:
AE manifolds and elliptic operators;
Constructions of AE initial data sets.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students can use conformal methods to transform Einstein’s constraints into a system of elliptic partial differential equations and thus describe parts of the solution spaces of Einstein’s equations and analyse properties of the associated solutions. They have learnt about connections between the theory and questions of geometric analysis such as the scalar curvature problem and the Yambe problem and are familiar with the interplay of methods of Riemannian geometry, geometric analysis and physics for answering questions of general relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
</table>

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	The Einstein Constraint Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.									
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.									
Prerequisites	Basic knowledge of differential geometry and Riemannian geometry is assumed. Previous knowledge of partial differential equations is an advantage, but not essential. Previous knowledge of general relativity is also useful, but the necessary concepts are also covered in the lecture.									
Responsible Persons	Carla Cederbaum									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-60-10	Special Topics in Evolution Equations for Submanifolds (with Exercise Class)	Compulsory Module with Choice

ECTS-Points	6
Workload - Time in Class - Self-Study	Workload: 180 h; Time in Class: 60 h; Self-Study: 120 h
Duration	1 Semester
Frequency	not regularly
Term	1-3
Language of Instruction	English
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Topics in Evolution Equations for Submanifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* und *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Introduction to Partial Differential Equations and basic knowledge of differential geometry is required.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-11	**Module Title:** Special Topics in Evolution Equations for Submanifolds (without Exercise Classes)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.		
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Topics in Evolution Equations for Submanifolds	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Introduction to Partial Differential Equations and basic knowledge of differential geometry is required.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-60-30**	**Module Title:** Gravitational Collapse and Singularities in General Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		

Content

The course is divided into three parts: First, we will study the causal structure of spacetime in general relativity, the causal hierarchy and various theorems related to causality. Then we will study singularities and the celebrated singularity theorems by Penrose and Hawking. And finally we will study Penrose's cosmic censorship conjecture, some properties of black holes, the phenomenon of gravitational collapse, which is the reason for the formation of singularities, and some examples of gravitational collapse that apparently does not obey the cosmic censorship conjecture. The content is as follows:

- Causality theory:
  - Time orientation, causal hierarchy, global hyperbolicity.
- Singularities:
  - Raychoudhuri's equations, conjugate points, singularity theorems.
- Black holes:
  - Cosmic censorship, properties of black holes, naked singularities.

Objectives

Students have acquired in-depth knowledge of causality theory and singularity theorems in general relativity. They will learn to apply topological methods in causality theory and in proving singularity theorems. They will also get an overview of cosmic censorship conjecture and naked singularities. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Gravitational Collapse and Singularities in General Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Robert M. Wald: General Relativity. The University of Chicago Press 1984. • Stephen W. Hawking and George F. R. Ellis: The large scale structure of spacetime. Cambridge Monographs on Mathematical Physics 1973. • Pankaj S. Joshi: Gravitational collapse and spacetime singularities. Cambridge University Press 2007. • Barret O'Neill: Semi-Riemannian Geometry with applications to relativity. Academic Press 1983.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of relativity is required to follow the course.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-60-35**	**Module Title:** Non-Linear Elliptic and Parabolic Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Semilinear and quasilinear elliptic and parabolic partial differential equations; • Minimum surface operator and surfaces of prescribed mean curvature; • Parabolic geometric equations, e.g. flow along the mean curvature; • Hölder continuity according to De Giorgi and Nash; • Inner regularity and boundary regularity of solutions.		
Objectives	Students have learnt analytical methods that are central to the treatment of non-linear partial differential equations of second order of the elliptic and parabolic type. Using concrete examples of partial differential equations from mathematical physics and differential geometry, techniques were learnt to prove the existence and regularity of solutions to such equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes, students have acquired confidence in the technical handling of the methods they have learnt and can apply them independently to other problems. They are able to present their problem solutions and participate in discourses on problems in this field of research.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptic PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Fritz John: Introduction to Partial Differential Equations. Springer 1982. • Jürgen Jost: Partielle Differentialgleichungen. Springer 1998. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications, Pure and Applied Mathematics, Vol. 88. Academic Press 1980.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, the module Introduction to Partial Differential Equations is a prerequisite.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-60-36**	**Module Title:** Fully Non-Linear Elliptic and Parabolic Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The lecture investigates fully non-linear elliptic and parabolic partial differential equations of second order. Classical examples are the Monge-Ampère equation, the equation of prescribed Gaussian curvature or, more general, equations of prescribed other scalar invariants of curvature together with their parabolic analogues. They also arise in problems of stochastic control and optimal transport. The course establishes basic techniques for solving Dirichlet- and Neumann boundary value problems for such equations, in particular techniques for deriving the necessary a priori estimates for solutions.		
Objectives	The students have learnt analytical methods that are central for the treatment of strongly non-linear partial differential equations of the elliptic and parabolic type. Using concrete examples of such differential equations, techniques were learnt to prove the existence and regularity of solutions of such equations and the associated boundary value problems. Students can apply the methods they have learnt to other problems and related equations independently. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Fully Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptc PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Ilya J. Bakelman: Convex functions and nonlinear geometric elliptic equations. Springer 1994. • Luis Caffarelli, Xavier Cabrè: Fully nonlinear elliptic equations. AMS 1995.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'MAT-55-27 Fully Nonlinear Elliptic Equations' due to the large overlap in content.
Prerequisites	At least one course on partial differential equations, basic concepts of differential geometry.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-65-05**	**Module Title:** Groups and Representations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. • Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. • Applications: symmetries and degeneracies in quantum mechanics, selection rules. • Representations of finite groups: group algebra, regular representation, ideals, idempotents. • Symmetric groups: Young tableaux, Young operators, dimensions and characters. • Applications: identical particles in quantum theories. • Lie groups: Haar measure, representations, Lie algebras. • Tensor representations of classical groups: symmetry classes, Young tableaux. • Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour) • Moreover a selection of the following: – Irreducible representations of the Lorentz and Poincaré groups. – Applications: notion of particles in quantum theories. – Roots and weights; Killing-Cartan classification of semi-simple Lie algebras		
Objectives	The studens know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deepend understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Groups and Representations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.									
Literature	**Possible References :** • Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. • Barry Simon: Representations of Finite and Compact Groups. AMS 1996. • Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.									
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Representation Theory of Finite Groups' due to the large overlap in content.									
Prerequisites	There are no further prerequisites.									
Responsible Persons	Stefan Keppeler									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-31	**Module Title:** Mathematical Methods for Condensed Matter Physics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	The course provides an introduction, with an analytic perspective, to the basic mathematical tools necessary to have a deeper understanding of the mathematical theories of topological insulators. In particular, the course will cover the following topics: • Direct integrals on Hilbert spaces. • Stability theorems for relatively bounded perturbations. • Bloch-Floquet transformations and their application to the periodic Schrödinger operator. • Introduction to the theory of vector bundles and Chern classes. • Definition of the Bloch bundle.		
Objectives	The students know, understand and are familiar with the concepts of the lecture. In particular, they have developed a deeper understanding of how mathematical concepts are applied in a natural way in solid state physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Methods for Condensed Matter Physics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, only knowledge from the basic courses of the first two years in the B.Sc. Mathematik are required.

Responsible Persons	Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-32</td><td colspan="2">Module Title:
Mathematical Aspects of the Quantum Hall Effect</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Exercise Class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The course is focused on the description of mathematical models for the quantum Hall effect. In particular, the course will cover the following topics:
• Review of the classical Hall effect and historical introduction on the quantum Hall effect.
• Analysis of the Landau Hamiltonian and of the geometry of the Landau levels.
• Linear response theory and derivation of the Kubo formula.
• Wannier functions and their relations to the Hall conductivity.
• Magnetic perturbations and Streda formula.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learned, understood, and become familiar with the concepts explained in the lectures. In particular, they have developed a deep understanding of the mathematical aspects of the quantum Hall effect. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Mathematical Aspects of the Quantum Hall Effect</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the specialisations Analysis and Differential Geometry and Mathematical Physics. Taking the personal specialisation and the restrictions of the sections into account, the module can be assigned to the Section Specialisation, Specialisation Knowledge Mathematics or Elective Specialisation.</td></tr>
</table>

Prerequisites	It is strongly recommended that the students have attended the course mathematical methods for condensed matter physics.
Responsible Persons	Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-33	**Module Title:** Wave Equations of Relativistic Quantum Mechanics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Klein-Gordon equation. • Dirac equation. • Representation Theory of the Lorentz Group. • Relativistic Many-Particle Systems (Multi-Time Formalism).		
Objectives	The students obtain knowledge and understanding of wave equations in relativistic quantum mechanics. They learn analytical techniques for proving the existence and uniqueness of solutions of the Klein-Gordon equation and the Dirac equation, as well as for investigating their properties. The students are able to assess the physical relevance of the mathematical results. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Wave Equations of Relativistic Quantum Mechanics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Bernd Thaller: The Dirac equation. Springer 1992. • Silvan S. Schweber: An introduction to relativistic quantum field theory, Chap. 2-4. Dover Books 2005. • Paul R. Garabedian: Partial differential equations. AMS 1998. • Erich Zauderer: Partial differential equations of applied mathematics. Wiley 2006.
Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge in Quantum Mechanics and Special Relativity Theory is expected. Moreover, basic knowledge of Functional Analysis and Partial Differential Equations would be helpful but is not mandatory.
Responsible Persons	Roderich Tumulka

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-36</td><td colspan="2">Module Title:
Quantum Information Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
• Basic notions on the universal quantum computer: Quantum gates, quantum circuits, universality and measurements.
• Quantum algorithms: Deutsch-Jozsa, Shor and Grover.
• Quantum communication: No-cloning theorem, quantum teleportation and superdense coding. Quantum key distribution.
• Physical realizations: DiVincenzo criteria, Cirac Zoller quantum computer, Circuit QED.
• Decoherence and open quantum systems.
• Quantum error correction. Fault tolerant quantum computing.
• Alternative quantum computing models: Adiabatic quantum computation.
• Introduction to the theory of entanglement: Definition, criteria and measurement of entanglement, multipartite entanglement.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with the basic concepts and theoretical tools of quantum information processing. They understand the concept of quantum algorithms and quantum circuits and have learnt how to program a quantum computer. They understand how important quantum algorithms work and can describe quantum channels. They know the principles of quantum error correction and entanglement theory and also understand the most advanced concepts of physical realisations of quantum computers. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Quantum Information Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>ü</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Michael A. Nielsen, Isaac L. Chuang: Quantum Computation and Quantum Information. http://mmrc.amss.cas.cn/tlb/201702/W020170224608149940643.pdf • Ronald de Wolf: Quantum Computing: Lecture Notes. https://homepages.cwi.nl/~rdewolf/qcnotes.pdf • John Preskill: Quantum Computation. Lecture Notes. http://theory.caltech.edu/~preskill/ph219/index.html
Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no prerequisites.
Responsible Persons	Angela Capel Cuevas

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-37	**Module Title:** Matrix Analysis and Applications		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		
Content	• Foundations of operators and matrices: Square matrices and tensor products. • Mappings and algebras. • Positive matrices. • Functional calculus and derivations. • Matrix monotone functions and convexity. • Matrix means and inequalities. • Applications in quantum information theory.		
Objectives	Students have acquired in-depth knowledge of matrix analysis from the perspective of functional analysis. They have become familiar with some aspects of analysis in the context of matrices, including topics such as monotone matrix functions, matrix averages, majorisation, entropies, quantum Markov triplets, etc. They are also familiar with several typical applications of matrix analysis in quantum information theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Marix Analysis and Applications	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Fumio Hiai, Denes Petz: Introduction to Matrix Analysis and Applications. https://math.bme.hu/~petz/matrixPD.pdf • Denes Petz: Matrix Analysis with some Applications. https://math.bme.hu/~petz/matbme.pdf • Rajendra Bhatia: Matrix Analysis. Springer 1997. • Rajendra Bhatia, Positive Definite Matrices. Princeton University Press 2007.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of functional analysis is desirable.
Responsible Persons	Angela Capel Cuevas

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-38	**Module Title:** Hamiltonian Systems		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	The module provides an introduction to the theory of Hamiltonian systems as used in classical mechanics. This builds a bridge between the fields of differential geometry, symplectic geometry and dynamical systems as well as theoretical physics. The main points of the lecture are: • Symplectic manifolds and the canonical 1-form of the cotangent bundle. • Darboux-Moser theorem. • Lagrangian and Hamiltonian systems. • Integrable systems and Arnold-Liouville theorem. • Moment mappings. • Symplectic reduction. • Symplectic manifolds and toric effects.		
Objectives	The students are familiar with the theory of Hamiltonian systems and their investigation using methods of symplectic geometry. They are familiar with the interplay of methods and questions of different areas of mathematics (differential geometry, geometry, dynamical systems) and theoretical physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Hamiltonian Systems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 1989. • Ana Cannas da Silva: Lectures on symplectic geometry. Springer 2001.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the Geometry in Physics module is assumed.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-39	**Module Title:** Propagation of Chaos		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.		
Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Propagation of Chaos	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	?	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. • Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.
Transfer	The module belongs to the *Study Specialisations Mathematical Physics* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to the basics of analysis and linear algebra, the content of the Stochastics module is a prerequisite.
Responsible Persons	Peter Pickl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-70-01**	**Module Title:** Algorithms of Numerical Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calulation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.		
Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algorithms of Numerical Mathematics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008.
- Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	There are no further prerequisites.
Responsible Persons	Christian Lubich, Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-70-02**	**Module Title:** Numerics of Stationary Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.		
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Stationary Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Dietrich Braess: Finite Elemente. Springer Spektrum 2013. • Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the numerical algorithms module is helpful, but not mandatory.
Responsible Persons	Christian Lubich

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-03	**Module Title:** Numerics of Instationary Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	Numerical treatment of transient (i.e. time-dependent) differential equations, such as: stiff ordinary differential equations, stochastic differential equations, parabolic and hyperbolic partial differential equations.		
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of instationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Instationary Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Ernst Hairer, Gerhard Wanner: Solving Ordinary Differential Equations II. Stiff Problems. Springer 1996. • Vidar Thomee: Galerkin Finite Element Methods for Parabolic Problems. Springer 1997.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Numerical Mathematics of Stationary Differential Equations is helpful, but not absolutely necessary.
Responsible Persons	Christian Lubich

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-04	**Module Title:** Ordinary Differential Equations - Analysis and Numerics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. • Numerical approximation: linear multi-step processes, adaptive processes, geometric integration.		
Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Ordinary Differential Equations - Analysis and Numerics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Basic knowledge of the theory of the ordinary differential equations are required, such as those taught in the module Algorithms of Numerical Mathematics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-05</td><td colspan="2">Module Title:
Optimal Control Theory with Ordinary Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Brief overview of existence and uniqueness theory for ODEs.
• Numerical solutions to ODEs.
• Introduction to optimal control problems with ODEs.
• Existence and uniqueness theory for linear quadratic optimal control problems (LQ problems).
• Pontryagin’s maximum principle.
• Numerical approximation of LQ problems.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with the problems of optimal control using ordinary differential equations and various approaches to solving the problem. They are also familiar with qualitative statements on unambiguous solvability. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Optimal Control Theory with Ordinary Differential Equations</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>ü</td><td>f</td><td>1</td><td>2</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Matthias Gerdts: Optimal Control of ODEs and DAEs. De Gruyter 2012.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Analysis and the sub-module Introduction to Ordinary Differential Equations is assumed.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-70-06	Numerics of Differential Equations of Surfaces		Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		

Content

- Numerical treatment of differential equations on moving (or stationary) surfaces.
- Semi- and fully discretization of elliptic and parabolic equations on surfaces using surface finite elements and efficient time integrators.
- Implementation of the algorithms.

Objectives

Students have learned the basic methods and techniques of numerics for problems on (moving) surfaces. In particular, they are familiar with the discussed energy techniques, which are very strong, general and rich in application, even in surface-independent areas of numerics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Differential Equations of Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Gerhard Dziuk: Finite elements for the Beltrami operator on arbitrary surfaces. 1988.
- Gerhard Dziuk, Charles M. Elliott: Finite elements on evolving surfaces. 2007.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge of the numerical algorithms module is helpful, but not mandatory.

Responsible Persons	Christian Lubich

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-11</td><td colspan="2">Module Title:
Stochastic Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Stochastic processes, filtrations, martingales.
• Wiener process, random walk, Donsker's theorem.
• Diffusion semigroup, Ito's integral.
• Solution of a stochastic differential equation.
• Markov property, Malliavin calculus, rough path theory.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Stochastic Differential Equations</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>ü</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Bernt Oksendal: Stochastic differtial equations. Springer 2000.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge of the modules Stochastics and Introduction to Integration and Measurement Theory from the Bachelor of Science programme is assumed.</td></tr>
</table>

Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-12	**Module Title:** Introduction to Stochastic Differential Equations - Part 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Introduction to Brownian motion and stochastic integration. • Solution concepts for stochastic differential equations. • Stability of stochastic differential equations. • Numerical approximation of stochastic differential equations.		
Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Stochastic Differential Equations - Part 1	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Bernt Oksendal: Stochastic differential equations. Springer 2000.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the modules Stochastics and Introduction to Integration and Measure Theory from the Bachelor of Science programme are required.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-15	**Module Title:** Numerics of Stochastic Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.		
Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Stochastic Differential Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the Stochastics module in the Bachelor of Science is required.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-16	**Module Title:** Stochastic Optimal Control in Infinite Dimensions		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The course covers aspects of stochastic optimal control, an interdisciplinary subject at the overlap of analysis, optimisation, partial differential equations and stochastics, which lead the participants to topics in current research. The choice of contents takes the knowledge of the participants into consideration.		
Objectives	The students aquire deepend knowledge in stochastic optimal control that introduce them to a current area of research and that allow them to start a small research project. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Stochastic Optimal Control in Infinite Dimensions	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The contents of the module Numerical Mathematics are assumed.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-20	**Module Title:** Introduction to Optimisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		
Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Optimisation	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019.
- Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Only basic knowledge of linear algebra and analysis is required.
Responsible Persons	Christian Lubich

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-21	**Module Title:** Non-Linear Optimisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method. • Restricted optimisation, Farkas' lemma, tangent cone. • Abadie CQ, KKT conditions, Slater conditions. • Linear programme, duality, simplex method. • Penalty and barrier methods, interior point method. • Nonlinear programs, SQP methods, non-smooth optimisation.		
Objectives	Students master the basic principles and techniques of analysis and numerics of constrained optimisation problems. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Linear Optimisation	L	f	4	6	yes	or.	20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	There are no further prerequisites.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-22	**Module Title:** Optimisation with Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Direct method in the calculus of variations, Euler-Lagrange equation. • Brouwer-Minty theorem, non-linear evolution equations. • Gateaux and Frechet differentiability. • Proof of existence of optimal controls, necessary optimality conditions. • Adjoint, convergent optimisation methods in Banach spaces. • Variational discretisation concepts.		
Objectives	Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Optimisation with Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The content of the Functional Analysis module is prerequisite for participation in this module.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-25	**Module Title:** Numerical Optimisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	An introduction to numerical methods for solving optimisation problems in science and technology with a focus on continuous optimisation and non-linear programming. • Basic concepts of optimization. • Unconstrained optimization and Newton-type algorithms. • Optimization with equations as constraints. • Optimization with inequalities as constraints. • Applications: – Economy: resource allocation in logistics, investments, etc. – Science: model estimation and adaptation to measurement data, experimental design. – Engineering: design and operation of technical systems such as bridges, cars, airplanes, digital devices, etc.		
Objectives	Students are familiar with the problems and numerical methods of optimization. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerical Optimisation	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	1	2					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Jorge Nocedal, Stephen J. Wright: Numerical Optimization. Springer 2006. • Stephen Boyd, Lieven Vandenberghe: Convex Optimization. Cambridge University Press 2004.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further requirements.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-30	**Module Title:** Theoretical Aspects of Machine Learning		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		

Content

The lecture covers some recent aspects of theoretical machine learning such as:

- The theory of Reproducing Kernel Hilbert Spaces (RKHS).
- Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings.
- Approximation capabilities of neural networks.
- Dynamics of neural networks and the neural tangent kernel.
- Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.

Objectives

The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Theoretical Aspects of Machine Learning	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.

Literature	**Possible References :** • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge in linear algebra, analysis and probability theory is needed as well as some knowledge in elementary Hilbert space theory.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-70-31	Statistical Learning Theory for Nonparametric Regression 1		Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.		
Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Statistical Learning Theory for Nonparametric Regression 1	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.

Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the Stochastics and Probability Theory modules is assumed.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-70-32	Statistical Learning Theory for Nonparametric Regression 2	Compulsory Module with Choice

ECTS-Points	9
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h; Self-Study: 180 h
Duration	1 Semester
Frequency	not regularly
Term	1-3
Language of Instruction	German
Forms of Teaching and Learning	Lecture 4 SWS

Content

- The uniform law of large numbers on function classes (Vapnik-Chervonenkis theory).
- Abstract (strong) consistency theory for *least-squares* regression estimators on (approximating) function classes.
- Examples, in particular the *data dependent partitioning* estimator and the *least squares neural networks* estimator.
- Rate convergence for *least-squares* estimators.

Objectives

Students are familiar with in-depth methods of stochastic learning and their analysis, as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Statistical Learning Theory for Nonparametric Regression 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge from the module Statistical Learning 1 is assumed.

Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-33	**Module Title:** Theory and Numerics for Constrained Optimisation Problems		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.		
Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity. he students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Theory and Numerics for Constrained Optimisation Problems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.

Literature

Possible References :

- Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.

Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further requirements.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-70-40**	**Module Title:** Game Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.		
Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Game Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Basic knowledge of analysis and numerics is assumed. The module is also suitable for students of related fields with basic mathematical knowledge.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System : g=graded, ng=not graded
Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-70-51**	**Module Title:** Financial Mathematics and Numerics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations. Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.		
Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Financial Mathematics and Numerics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.

Literature

Possible References :

- Steven Shreve: Stochastic Calculus for Finance. Springer 2005.

Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of calculus, linear algebra, basic programming, ordinary differential equations theory and introductory probability is recommended.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-01**	**Module Title:** Probability Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in contiuous time like Poisson processes and Brownian motion.		
Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Probability Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2010. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2002. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Martin Möhle, Martin Zerner

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-02**	**Module Title:** Combinatorics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Basic combinatorial objects. • Generating functions. • Partial orders, Möbius inversion. • Method of Polya and Redfield. • Symbolic combinatorics. • Transfer matrix method. • Euler-Maclaurin summation formula. • Asymptotic methods.		
Objectives	The students have learned the application of basic combinatoric methods. They can analyse discrete structures and counting problems. Furthermore they are familiar with applying common identities and handling counting coefficients. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Combinatorics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Martin Aigner: Combinatorial theory. Springer 1997. • Martin Aigner: A Course in Enumeration. Springer 2007. • Richard P. Stanley: Enumerative combinatorics. Volume 1. Cambridge University Press 2011. • Francois Bergeron, Gilbert Labelle, Pierre Leroux. Combinatorial species and tree-like structures. Cambridge University Press 1998. • Philippe Flajolet, Robert Sedgewick. Analytic Combinatorics. Cambridge University Press 2009.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of algebra (group actions), function theory (Cauchy's integral formula) and the foundations of discrete mathematics are expected.
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-03**	**Module Title:** Mathematical Statistics	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h / Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	regularly	
Term	1-3	
Language of Instruction	German	
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS	

Content

- Statistical models, exponential families, sufficient statistics.
- Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem.
- Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators.
- Hypothesis testing, confidence interval, Neyman-Pearson lemma.
- Testing methods, UMPU tests, 1- and 2-sample tests.
- Models with growing density quotients, non parametric models.
- Introduction in regression and variance analysis.

Objectives

Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Statistics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the probability theory module is helpful, but is not mandatory.
Responsible Persons	Martin Möhle, Martin Zerner

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-75-04</td><td colspan="2">Module Title:
Stochastic Processes</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Stochastic processes in continuous time, such as
• Markov processes;
• Martingale;
• Brownian motion, Poisson processes and general Levy processes;
• Gaussian processes.
Among other things, existence and convergence statements as well as path properties of these processes are analysed.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learnt the central concepts, results, methods and examples of the theory of stochastic processes in continuous time and can handle them mathematically. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Stochastic Processes</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>ü</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.</td></tr>
</table>

Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Joseph L. Doob: Stochastic Processes. Wiley 1990. • Samuel Karlin, Howard Taylor: A First Course in Stochastic Processes. Academic Press 1975. • Samuel Karlin, Howard Taylor: A Second Course in Stochastic Processes. Academic Press 1981. • Götz Kersting, Anton Wakolbinger: Stochastische Prozesse. Birkhäuser 2014. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013. • James R. Norris: Markov Chains. Cambridge University Press 1997.
Transfer	The module belongs to the *Study Specialisation ....* Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of the Probability Theory module is assumed.
Responsible Persons	Martin Möhle, Martin Zerner

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-05**	**Module Title:** Percolation Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Edge percolation on graphs, especially on multidimensional grids. • Phase transitions. • Number of clusters and cluster sizes. • Special features in two dimensions. • Alternative percolation models.		
Objectives	Students can interpret special spatially indexed families of random variables as random geometric structures and apply probability theory methods to analyse them. Using simple models, they learn how microscopic changes can result in macroscopic phase transitions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Percolation Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Literature	**Possible References :** • Béla Bollobás, Oliver Riordan. Percolation. Cambridge University Press 2006. • Geoffrey Grimmett: Percolation. Springer 1999.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Mathematical Physics* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge of the module Probability Theory is helpful, but not essential.
Responsible Persons	Elmar Teufl, Martin Zerner

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-06**	**Module Title:** Stochastic Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Martingales and stopping times in continuous time. • Doléans measure, compensator, Doob-Meyer decomposition. • Stochastic integral for square integrable martingales (in particular for non-continuous martingales). • Semimartingales, transformation of stochastic integrals. • Itô formula (in particular for processes with jumps). • Stochastic differential equations .		
Objectives	The students know the main notions, results, methods and examples from stochastic analysis and they know how to handle them. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Stochastic Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Fabrice Baudoin: Diffusion Processes and Stochastic Calculus. EMS 2014. • Kai Lai Chung and Ruth J. Williams: Introduction to Stochastic Integration. Birkhäuser 1990. • Richard Durrett: Stochastic Calculus. CRC Press 2006. • Albrecht Irle: Finanzmathematik. Teubner 2003. • Ioannis Karatzas, Steven Shreve: Brownian Motion and Stochastic Calculus. Springer 1991. • Michel Métivier: Semimartingales. De Gruyter 1982. • Bernt Oksendal: Stochastic Differential Equations. Springer 2007. • Nicolas Privault: Stochastic Analysis in Discrete and Continuous Settings. Springer 2009. • Daniel Revuz, Marc Yor: Continuous Martingales and Brownian Motion. Springer 1999. • Heinrich von Weizsäcker, Gerhard Winkler: Stochastic Integrals, Vieweg 1990.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	A solid knowledge on probability theory is a prerequisite.
Responsible Persons	Martin Möhle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-07	**Module Title:** Information Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Entropy and entropy rates in the discrete case. • Theorem of Shannon-McMillan-Breiman. • Entropy rates of Markov chains. • Kolmogorov complexity. • Data compression. • Chanel capacity. • Differential entropy.		
Objectives	Students learn to describe information content with entropy and entropy rate. They can apply the basic theory to concrete random experiments and stochastic processes. Students can also apply the theoretical concepts to specific problems in coding theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Information Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Robert B. Ash: Information Theory. Wiley. 1965. • Thomas M. Cover, Joy A. Thomas: Elements of Information Theory. Wiley 2006. • David J.C. MacKay: Information Theory, Inference and Learning Algorithms. Cambridge 2003. • Claude Shannon, Warren Weaver: The Mathematical Theory of Communication. University of Illinois Press 1949.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In terms of content, knowledge from the modules Stochastics and Probability Theory is assumed.
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-08**	**Module Title:** Mathematical Population Genetics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Exchangeable population models. • Probability of extinction. • Descendants and ancestors. • Duality of Markoff processes. • Coalescent processes and associated convergence rates. • Simple mutation models, Ewens sampling formula. • Statistical applications, e.g. estimating the mutation rate.		
Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Population Genetics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Jean Bertoin: Random Fragmentation and Coagulation Processes. Cambridge 2006. • Stewart N. Ethier, Thomas G. Kurtz: Markov Processes. Wiley 1986. • Warren J. Ewens: Mathematical Population Genetics. Springer 2004. • Jim Pitman: Combinatorial Stochastic Processes. LNM 1875. Springer 2006. • John Wakeley: Coalescent Theory. Roberts & Company Publishers 2008.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of probability theory is assumed.
Responsible Persons	Martin Möhle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-75-09	Point Processes		Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		

Content

- Random measures, point processes, Poisson processes.
- Factorial measure, Mecke equation.
- Transformation, labelling, thinning.
- Characterisation of point processes.
- Stationary Poisson processes.
- Poisson integrals.
- Cox processes.

Objectives

The students have familiarised themselves with the central concepts, results, methods and examples of the theory of point processes and can handle them mathematically. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Point Processes	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	ü	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Daryl John Daley, David Vere-Jones: An Introduction to the Theory of Point Processes. Springer 2008. • Martin Jacobsen: Point Process Theory and Applications. Birkhäuser 2006. • Olav Kallenberg: Foundations of Modern Probability. Springer 2002. • John F. C. Kingman: Poisson Processes. Clarendon Press 1993. • Günter Last, Mathew D. Penrose: Lectures on the Poisson Process. Cambridge 2016.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	A sound knowledge of probability theory is assumed.
Responsible Persons	Martin Möhle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-75-10	Graph Theory		Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.		
Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Graph Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-11	**Module Title:** Markov Chains and Applications		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Fundamentals and advanced topics on Markov chains and related stochastic models are discussed. In particular, the long-term behaviour of Markov chains is examined. Furthermore, applications of Markov chains, such as Markov chain Monte Carlo simulation, randomised search algorithms, graphical models, entropy rates of Markov chains, are discussed.		
Objectives	The students have learnt the basic concepts of the theory of Markov chains and related models. They are also familiar with applications of the theory and have experienced the interaction of probability theory and algorithms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Markov Chains and Applications	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Pierre Bremaud: Discrete Probability Models and Methods. Springer 2017.
- Pierre Bremaud: Markov Chains. Springer 1999.
- Olle Häggström: Finite Markov Chains and Algorithmic Applications. Cambridge University Press 2002.
- Kevin Murphy: Machine Learning: A Probabilistic Perspective. MIT Press 2012.
- James Spall: Introduction to Stochastic Search and Optimization. Wiley 2003.

Transfer

The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Good knowledge of linear algebra and stochastics is required. Knowledge from the probability theory module is helpful, but is not required.
Responsible Persons	Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-12	**Module Title:** Foundations of Discrete Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Logic. • Sets, relations, functions. • Partial orders. • Combinatorics. • Number theory. • Graph theory. • Algorithms and formal languages. • Discrete optimization.		
Objectives	Students have learned how to use basic methods of discrete mathematics. They can analyze discrete structures and identify discrete structures in different contexts. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods on new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Foundations of Discrete Mathematics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Ronald Graham, Donald Knuth, Oren Patashnik: Concrete Mathematics. Addison-Wesley 1994. • Kenneth H. Rosen: Discrete Mathematics and Its Application. McGraw-Hill 2019. • Ralph P. Grimaldi: Discrete and Combinatorial Mathematics. Addison-Wesley 2004. • Norman L. Biggs: Discrete Mathematics. Oxford University Press 2002.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no further prerequisites.
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-75-20	Probability Distances for Data Science	Compulsory Module with Choice

ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	We study different concepts of distances between probability measures aimed at applications in data science. The classes of distances which are studied include optimal transport distances, f-divergences and integral probability metrics. The focus is on fundamental mathematical properties of these distances, like duality, famous inequalities, geometric aspects, and quantisation. Several applications in the area of data science and machine learning are illustrated throughout, for instance related to clustering, autoencoders, GANs, image processing, and compression.		
Objectives	Students are familiar with commonly used distances on the space of probability measures, particularly optimal transport distances, divergences, and integral probability metrics. They understand key mathematical results in this area, for instance related to duality, geometric aspects, and quantisation, as well as the interplay between different distances. They have further obtained an understanding of computational aspects and applicability in selected areas of data science. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed. In the exercises, they have developed a confident, precise and independent approach to the concepts, statements and methods from the lecture. They have learned to transfer the methods to new problems, to analyse them and to develop solution strategies alone or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Probability Distances for Data Science	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature	**Possible References :** • Gabriel Peyre, Marco Cuturi: Computational optimal transport: with ap- plications to data science. Foundations and Trends in Machine Learning 11.5-6 (2019): 355-607. • Alison L. Gibbs, Francis Edward Su: On choosing and bounding probability metrics. International Statistical Review 70.3 (2002): 419-435. • Cedric Villani: Topics in optimal transportation. American Mathematical Society, 2003. • Imre Csiszar, Paul C. Shields: Information theory and statistics: a tutorial. Foundations and Trends in Communications and Information Theory 1.4 (2004). 417-528. • Ily Tolstikhin et al.: Wasserstein auto-encoders. 6th International Conference on Learning Representations (ICLR 2018) • Siegfried Graf, Harald Luschgy: Foundations of quantization for probability distributions. Springer, 2007.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	The course is mostly self-contained, but students benefit from basic knowledge in analysis, probability theory, optimisation, and Python.
Responsible Persons	Stephan Eckstein

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-75-21**	**Module Title:** Bayesian Networks and Causality		**Type of Module:** Compulsory Module with Choice
ECTS-Points	5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as *Whenever the sprinkler is on in my greenhouse, the plants are wet* or *Whenever the plants are wet, the sprinkler is on*, to postulating a relationship of cause and effect: *If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler*. In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.		
Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Bayesian Networks and Causality	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	2					

Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.

Transfer

The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

The module Stochastics is assumed.

Responsible Persons

Stephan Eckstein

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week