EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-
Naturwissenschaftliche
Fakultät

Department of Mathematics

Module Handbook

Mathematics
Master of Education
Lehramt Gymnasium*

Summer Semester 2025

Status 20th January 2025

*This is a secondary school teaching degree with a major in mathematics.

Contents

1 Description of the Study Programme

1.1 Qualification Objectives

The M.Ed. Lehramt Gymnasium with Mathematics as a major is designed to equip future secondary-school teachers with the academic aptitude to teach Mathematics. The overarching and subject-specific qualification objectives, both with regard to the content of the degree programme and the skills to be acquired, are specified in the legal ordinance of the Ministry of Education and Cultural Affairs on framework requirements for the conversion of the general education teacher training degree programmes in Baden Württemberg.

As part of the M.Ed. Lehramt Gymnasium, graduates deepen their knowledge and skills in mathematics and mathematics didactics, which enable them to design targeted teaching, learning and educational processes in the subject of mathematics and to independently introduce new subject-related and interdisciplinary developments into teaching and school development.

Building on the fundamental issues in linear algebra, analysis, numerical mathematics, stochastics, geometry and algebra from the B.Ed. Lehramt Gymnasium, they expand their material and methodological skills in the areas of subject didactics, analysis and one further mathematical area from the specialisations offerd in Tübingen: Algebra and Geometry, Analysis and Differential Geometry, Mathematical Physics, Numerical Mathematics and Optimisation or Stochastics. Graduates are proficient in the theoretical explanatory approaches, principles and methods of mathematics, are familiar with its cognitive and working methods and can apply them in the central areas of mathematics. They can present mathematical facts adequately in oral and written form using suitable media and explain key issues in mathematical fields and their relationship to school mathematics. They are able to solve mathematical problems systematically, strategically and using suitable tools to solve mathematical problems and to comprehend and develop mathematical proofs. Based on their general knowledge, graduates are able to access and critically scrutinise fundamental current issues in mathematics. They can use their in-depth knowledge to develop and solve their own approaches and can derive, analyse, prove and interpret specific questions from general mathematical concepts. Graduates are able to present, explain and discuss the results of their work in depth in front of a scientific audience, both in writing and orally. They have learnt to acquire new specialist knowledge independently and are therefore able to access new mathematical theories in their later professional life, which are incorporated into school mathematics.

Students combine their scientific knowledge with didactic methods, use suitable media and are able to use theoretical concepts and empirical findings from mathematics-related teaching-learning research to analyse thought processes and ideas of pupils in exercises and to guide individual learning processes. They know and evaluate the concepts for learning and teaching mathematics at school on the basis of subject-didactic theories and empirical findings. They are able to analyse, plan and exemplarily implement mathematics lessons with heterogeneous learning groups on the basis of subject-didactic concepts. Students can justify the educational value of mathematical content, convey the societal significance of mathematics and place it in the context of the objectives and content of mathematics lessons.

1.2 Structure of the Study Programme

The standard period of study for the Master of Education Lehramt Gymnasium is four semesters; 28 credit points must be earned in the subject of mathematics. Depending on whether you start in the winter or summer semester, the first or second semester is mainly filled by the school internship component, which is accompanied by a subject didactics course at the university. In Mathematics, firstly, students will deepen their knowledge in the field of analysis with a module on the basic principles of complex analysis and ordinary differential equations. In addition, students have the opportunity to set their own focus in one of the specialisations offered by the department by attending a lecture with exercise classes and a suitable seminar. The study programme is completed with the Master's thesis (15 credit points) in one of the two chosen subject areas (including their didactics) or in the educational sciences. With the Master's degree, students (if they fulfil any further requirements) can enter the preparatory service traineeship, professional life, a doctorate in the field of didactics of mathematics or switch to a further degree course.

Integrating a study component at a foreign university into this degree programme in a meaningful way is a challenge, as it involves coordinating two subjects and the educational sciences; whether you try to complete parts of the programme in all areas during your stay at the other university, or whether you try to structure your studies at the University of Tübingen in such a way that parts of the programme are shifted to other semesters in order to create free time so that you do not have to complete all three areas at the other university. Adding to the difficulty is the fact that one semester is largely occupied by the practical component. Therefore, it is essential to plan a suitable timeframe for a study component at a foreign university through a personal consultation with the Faculty Course Advisor. Essentially, from the mathematics perspective, any academic semester is suitable for this purpose. The decision will depend on the student's previous achievements and the courses offered at the chosen foreign university.

2 Study Plans

2.1 Overview by Modules

We provide here an overview of the study plan as a table showing the modules to be taken.

Suggested Term	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Mathematics							
1-2	MAT-20-02	Introduction to Complex Analysis and Ordinary Differential Equations	L+E	PM	EC	wr. o. or.	9
3	MAT-40-51	Specialisation	L+E	PMW	EC	wr. o. or.	9
4	MAT-40-52	Seminar Mathematical Specialisation	S	PMW	s.M.	Pr	4
Section 2: Didactics of Mathematics							
1-2	MAT-80-03	Subject Didactics Mathematics 3	S+SV	PMW	-	K o. mP o. R o. H	6
Section 3: Master Thesis							
4	MAT-40-53	Master Thesis	MT	PM	s.M.	MT	15

Abbreviations:

Type of Module	: PM=compulsory module, PMW=compulsory module with choice, WPM=elective module
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Course Work	: EC=exercise certificate, PEC=practical exercise certificate, PC=practical certificate
Other	: h=hours, o.=or, s.M.=see module description

2.2 Overview by the Course of Studies

Firstly, we provide an overview of the possible course of study in the form of a table for both winter semester entry and summer semester entry.

Study Plan for Students Starting in the Winter Semester				
FS	**CP**	**Mathematics**	**Subject Didactics Mathematics**	**Master Thesis**
1	3		Subject Didactics 3 (6 CP)	
2	12	Introduction to Complex Analysis and Ordinary Differential Equations (9 CP)		
3	9	Consolidation of Special Areas of Mathematics (9 CP)		
4	4 + (15)	Seminar Mathematical Specialisation (4 CP)		Master Thesis (15 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)				

Figure 2.1: Study Plan for Students Starting in the Winter Semester

Study Plan for Students Starting in the Summer Semester				
FS	**CP**	**Mathematics**	**Subject Didactics Mathematics**	**Master Thesis**
1	12	Introduction to Complex Analysis and Ordinary Differential Equations (9 CP)	Subject Didactics 3 (6 CP)	
2	3			
3	9	Consolidation of Special Areas of Mathematics (9 CP)		
4	4 + (15)	Seminar Mathematical Specialisation (4 CP)		Master Thesis (15 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)				

Figure 2.2: Study Plan for Students Starting in the Summer Semester

2.3 Overview of Programme Structure with Semester Assignment

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester													
		Exam				**Teaching**				**Semester**			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Mathematics									**22**				
Introduction to Complex Analysis and Ordinary Differential Equations								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4			6		
2.	Exercise class					E	o	2			3		
Consolidation of Special Areas of Mathematics								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Seminar Mathematical Specialisation								2	4				
1.	Seminar	Pres		g	4	S	o	2					4
Section 2: Subject Didactics Mathematics									**6**				
Subject Didactics 3								4	6				
1.	Seminar	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	S	o	2		3			
2.	Seminar / Lecture	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2			3		
Section 3: Master Thesis									**15**				
Master Thesis									15				
1.	Master Thesis	MA		g		MA	o						15

Explanation of the Abbreviations:

Marking system	: g=graded, ng=non graded
Form of examination	: MA=master thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching	: L=lecture, SL=Seminar or lecture, E=exercise class, r=revision course, P=practical training, PS=proseminar, S=seminar
Status	: o=obligatory, f=facultative
Other	: o.=or, SWS=hours in class per week, CP=credit points=ECTS Points

Overview of Programme Structure with Semester Assignment for Students Starting in the Summer Semester													
		Exam				Teaching				Semester			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Mathematics									**22**				
Introduction to Complex Analysis and Ordinary Differential Equations								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4		6			
2.	Exercise class					E	o	2		3			
Consolidation of Special Areas of Mathematics								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Seminar Mathematical Specialisation								2	4				
1.	Seminar	Pres		g	4	S	o	2					4
Section 2: Subject Didactics Mathematics									**6**				
Subject Didactics 3								4	6				
1.	Seminar	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	S	o	2			3		
2.	Seminar / Lecture	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2		3			
Section 3: Master Thesis									**15**				
Master Thesis									15				
1.	Master Thesis	MA		g		MA	o						15

Explanation of the Abbreviations:

Marking system : g=graded, ng=non graded
Form of examination : MA=master thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching : L=lecture, SL=Seminar or lecture, E=exercise class, r=revision course, P=practical training, PS=proseminar, S=seminar
Status : o=obligatory, f=facultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS Points

3 Module Descriptions

Section 1: Mathematics

Module Number: **MAT-20-02**	**Module Title:** Introduction to Complex Analysis and Ordinary Differential Equations		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	jährlich im Sommersemester		
Term	1-2		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Complex Analysis: – Holomorphic functions, Cauchy-Riemann equations. – Antiderivatives, Cauchy's integral formula, Cauchy's integral theorem. – Compact convergence of families of functions, formal and convergent power series, complex-analytical functions, identity theorem. – Liouville's theorem, inverse function theorem for holomorphic functions, open mapping theorem, maximum principle. – Laurent series, holomorphic functions with isolated singularities, Casorati-Weierstrass theorem. – Residue theorem and applications. • Ordinary differential equations, an choice of the following: – Picard-Lindelöf existence and uniqueness theorem. – Linear ordinary differential equations, Gronwall's lemma. – Continous dependence on initial conditions, differential dependence on initial conditions. – Basics of dynamical systems, stability of equilibrium positions, characteristic exponents, first integrals, Liapunov-functions. – Ordinary differential equations over the complex numbers. – Regularity, the criterion of Fuchs. – The method of Frobenius.		

Objectives	The students know the foundations of the theory of complex analysis and ordinary differential equations. The are acquainted to essential calculation techniques and can calculate line integrals as well as explicitly solve simple differential equations. They know fundamental applications of the theory like e.g. the fundamental theorem of algebra and the Newtonian equations of motion. Also they have the ability to transfer abstract questions into concrete problems of complex analysis or respectively of ordinary differential equations and solve them this way. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students was trained by written assignments and the presentation of own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was aided by working in small groups.
Requirements for Obtaining Credit, Grading, Weight if applicable	(see table below)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Complex Analysis and ODEs.	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Literature

Possible References :

- Lars Valerian Ahlfors: Complex analysis. McGraw-Hill 1979.
- John B. Conway: Functions of one complex variable. Springer 1996.
- Wolfgang Fischer, Ingo Lieb: Einführung in die Komplexe Analysis. Springer 2010.
- Walter Rudin: Reelle und komplexe Analysis. Oldenbourg 2009.
- Earl A. Coddington, Norman Levinson: Theory of ordinary differential equations. McGraw-Hill 1955.
- William T. Reid: Ordinary differential equations. John Wiley & Sons 1971.
- Hille, Einar: Ordinary differential equations in the complex domain. Dover Publications 1997.
- Wasow, Wolfgang: Asymptotic expansions for ordinary differential equations. John Wiley 1965.

Transfer	If applicable the module is a prerequisite for the modules Seminar Specialisation Mathematics and Master Thesis.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-51	**Module Title:** Specialisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	Every semester		
Term	3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the chair of the examination board, upon written application by the student.		
Content	The content is determined by the choice of a course.		
Objectives	The students have acquired in-depth knowledge in a specific area of mathematics and gained further experience in presenting and communicating mathematical topics. They are capable of identifying the key statements of the lecture, reproducing the techniques used for their derivation and proof, and critically evaluating them. Additionally, they can integrate the methodological and theoretical foundations of the chosen mathematical subfield and place them within the broader mathematical context. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
see Comment	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the Board of Examiners.

Transfer	If applicable the module is a prerequisite for the modules Seminar Specialisation Mathematics and Master Thesis.
Prerequisites	-
Responsible Persons	The dean of studies at the department of mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-52	**Module Title:** Seminar Mathematical Specialisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	Every semester		
Term	4		
Language of Instruction	German		
Forms of Teaching and Learning	Seminar, talk, presentation, e-learning, blended learning		
Content	Various topics from the advanced fields of mathematics.		
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing fashion. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion. The work and the presentation may be the foundation or a deepened study in the scope of a master thesis.		

Requirements for Obtaining Credit, Grading, Weight if applicable

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	4	yes	Pr	60-90	g	100

The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. These further efforts constitute the coursework of the module.

Transfer	-
Prerequisites	The participation in the module requires the successful completion of at least one of the modules Introduction to Complex Analysis and Ordinary Differential Equations or Specialisation.
Responsible Persons	The dean of studies at the department of mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Didactics of Mathematics

Module Number: MAT-80-03	**Module Title:** Subject Didactics Mathematics 3		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	2 Semester		
Frequency	Every semester		
Term	1-2		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture, exercise, proseminar, talk, presentation, e-learning, blended learning, project work, case studies		
Content	In the first part varying topics are covered, who are especially related to the teaching profession and are also suited for a didactical refurbishment of the school internship semester. In the second part varying topics from didactics will be studied, which can lead up to recent research in didactics.		
Objectives	The students • know subject didactical principles and educational concepts and can evaluate and scrutinise them, • can compare and evaluate subject related approaches to central terms and theorems of the treated fields, • can plan, execute, analyse and evaluate competence oriented mathematical education on the basis of subject didactical concepts, • can motivate the general educating value of mathematical contents and methods and the social importance of mathematics and put it in context with the goals and contents of mathematical education, • can specifically use subject specific media, • can create a portfolio and document important experiences, findings and insights in a structured manner.		

Requirements for Obtaining Credit, Grading, Weight if applicable	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Subject didactics 3: professional knowledge	S	o	2	3	yes	K o. mP o. R o. H	90-180 o. 20-30	g	50
	Subject didactics 3: elective specialisation	SV	o	2	3	yes	K o. mP o. R o. H	90-180 o. 20-30	g	50
	The module consists of two parts (Professional Knowledge and Elective Specialisation), for which the teaching learning methods (lecture, exercise or seminar) as well as the type of examination (written or oral exam, presentation or paper) are usually different. This is taken into account by the fact that, the examination in this module consists of two equally weighted parts.									
Transfer	-									
Prerequisites	There are no prerequisites for participating in the module.									
Responsible Persons	Frank Loose, Walther Paravicini									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Master Thesis

<table>
<tr><td>Module Number:
MAT-40-53</td><td colspan="2">Module Title:
Master Thesis</td><td>Type of Module:
Compulsory Module</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">15</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
450 h</td><td>Time in Class:
0 h</td><td>Self-Study:
450 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">Every semester</td></tr>
<tr><td>Term</td><td colspan="3">4</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Master thesis</td></tr>
<tr><td>Content</td><td colspan="3">The master’s thesis represents the culmination of the master’s degree programme. Under the guidance of a supervisor, students are required to work on a well-defined problem in mathematics (including mathematics education), which may extend to the frontiers of current research. This involves using scientific methods and presenting the results in written form. Specifically, this includes:
formulating a scientific research question in consultation with the supervisor;
independently searching for and studying relevant scientific literature;
formulating appropriate research questions and methodological approaches for their solution;
independently carrying out the project, as well as presenting the project and its results in writing and, if applicable, orally, within the context of the current state of research.
The results should contribute to scientific knowledge.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students:
are capable of familiarising themselves with a problem, which may extend to the frontiers of current research, within a given timeframe and independently developing a solution approach,
can increasingly apply appropriate scientific methods independently and present the results in a scientifically appropriate manner,
are able to independently work on a scientific topic while applying their mathematical methodological knowledge,
deepen their problem-solving skills and can transfer their methodological knowledge,
are able to present the results of their project to a professional audience.</td></tr>
<tr><td>Requirements for Obtaining Credit, Grading, Weight if applicable</td><td colspan="3"><table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td>Master Thesis</td><td>MT</td><td>o</td><td>-</td><td>15</td><td>no</td><td>MT</td><td>-</td><td>g</td><td>100</td></tr></table></td></tr>
</table>

Transfer	-
Prerequisites	Academic admission requirements for the master's thesis module include, in addition to the general conditions specified in the general part of the study and examination regulations the successful completion of at least one of the modules Introduction to Complex Analysis and Ordinary Differential Equations or Specialisation.
Responsible Persons	The dean of studies at the department of mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, H=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=tutorial, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

4 Courses for the Module Specialisation

4.1 Catalogue of Courses

The following lists the courses that can be included in the module Specialisation in Section 3 Mathematics. Other courses may be approved upon written request to the chairperson of the examination board.

Course Title:	Algebraic Topology 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Special Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented.		
Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Algorithms of Numerical Mathematics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calulation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.
Special Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics.
Literature	**Possible References :** • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.
Responsible Persons	Christian Lubich, Andreas Prohl

Course Title:	Introduction to K-theory		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Vector bundles. • Topological K-theory. • Künneth formula and Bott periodicity. • Characteristic classes. • Chern character. • Algebraic K-theory • Plus construction.		
Special Objectives	The students have learnt an important mathematical field that combines analysis, geometry, algebra and number theory. They have learnt to recognise and use the connections between different areas. They can understand and use terms such as vector or fibre bundles or categorical K-groups and apply them. They have learnt to think in large contexts.		

Literature	**Possible References :** • Michael Atiyah: K-theory. Addison-Wesley 1989. • Max Karoubi: K-theory. Springer 2008. • Emilio Lluis-Puebla, Jean-Louis Loday, Henri Gillet, Christophe Soule, Victor Snaith: Higher algebraic K-theory: an overview. Springer 1992.
Responsible Persons	Anton Deitmar

Course Title:	Introduction to Mathematical Logic		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Special Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content.		
Literature	**Possible References :** • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.		
Responsible Persons	Anton Deitmar		

Course Title:	Introduction to set theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	**Content:** •		
Special Objectives	-		
Literature	**Possible References :** •		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Optimisation		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise Class 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		
Special Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods.		

Literature	**Possible References :** • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.
Responsible Persons	Christian Lubich

Course Title:	Introduction to Dynamical Systems		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Special Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do exist mathematical solutions? Are there equilibrium states or periodic orbits?		
Literature	**Possible References :** • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Geometric Measure Theory
Specialisation	Analysis

Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and Cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Measure Theoretic Methods		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Measures, Covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.		

Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Responsible Persons	Reiner Schätzle

Course Title:	Introduction to Geometric Measure Theory – Varifolds		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Surface- and Cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Commutative Algebra and Algebraic Geometry
Specialisation	Algebra

Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		
Special Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.		
Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.		
Responsible Persons	Jürgen Hausen		

Course Title:	Introduction to Partial Differential Equations		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	English		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.
Special Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses.
Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Partial Differential Equations – Part 1		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Special Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses.		

Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Elementary Number Theory		
Specialisation	Algebra		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical Theorems. • Applications.		
Special Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds.		
Literature	**Possible References :** • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.		
Responsible Persons	Victor Batyrev, Thomas Markwig		

Course Title:	Functional Analysis		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Normed spaces, Banach spaces, Dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact Operators, normal operators, spectral theorems.		
Special Objectives	The students are aquainted with the basic principles and techniques of the theory of infinte dimensional spaces and can apply them to problems in analysis and geometry. They understand the complex of problems of spectral theory and can use its results for the solution of analytical problems.		
Literature	**Possible References :** • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.		
Responsible Persons	Carla Cederbaum, Anton Deitmar, Gerhard Huisken, Reiner Schätzle		

Course Title:	Geometry of Manifolds 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		

Language of Instruction	German or English
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.
Special Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. They have deepened their understanding especially of differential and integral calculus and have exemplarily experienced how mathematical concepts are naturally used in geometry.
Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Responsible Persons	Christoph Bohle, Frank Loose

Course Title:	Geometry in Physics		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.		
Special Objectives	Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories.		

Literature	**Possible References :** • John Lee: Introduction to smooth manifolds. Springer 2012. • John Lee: Riemannian manifolds: An introduction. Springer 1997. • Chris Isham: Modern differential geometry for physicists. World Scientific 1999. • Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.
Responsible Persons	Christoph Bohle, Carla Cederbaum, Stefan Teufel

Course Title:	Foundations of Discrete Mathematics		
Specialisation	Stochastics		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Logic. • Sets, relations, functions. • Partial orders. • Combinatorics. • Number theory. • Graph theory. • Algorithms and formal languages. • Discrete optimization.		
Special Objectives	Students have learned how to use basic methods of discrete mathematics. They can analyze discrete structures and identify discrete structures in different contexts.		
Literature	**Possible References :** • Ronald Graham, Donald Knuth, Oren Patashnik: Concrete Mathematics. Addison-Wesley 1994. • Kenneth H. Rosen: Discrete Mathematics and Its Application. McGraw-Hill 2019. • Ralph P. Grimaldi: Discrete and Combinatorial Mathematics. Addison-Wesley 2004. • Norman L. Biggs: Discrete Mathematics. Oxford University Press 2002.		
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl		

Course Title:	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Special Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge of the interweaving of geometry and algebra.		
Literature	**Possible References :** • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.		
Responsible Persons	Hermann Hähl, Hannah Markwig		

Course Title:	Measure and Integration Theory		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	jährlich im Wintersemester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content	• Measures and integrals. • Lebesgue integral, Fubini's theorem, transformation formula. • Convergence theorems. • L^p-spaces, Radon-Nikodym theorem and Riesz representation theorem. • Submanifolds in the $\mathbb{R}^n$, differential forms, Stokes' theorem.
Special Objectives	The students know the basic terms, constructions, results and proving methods of integration theory in several real variables and in general measure spaces. Furthermore they are able to calculate the surface contents and the volumes even of complex bodies as well as multidimensional integrals. They have learned to transfer abstract questions of the subject into concrete problems and know important applications, e.g. in probability theory and physics.
Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 1978. • Anton Deitmar: Analysis. Springer Spektrum 2017. • Jürgen Elstrodt: Maß- und Integrationstheorie. Springer 2011. • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Otto Forster: Analysis 3. Friedr. Vieweg+Teubner 2011. • Edwin Hewitt, Karl Robert Stromberg: Real and Abstract Analysis. Springer 1975. • Georg Nöbeling: Integralsätze der Analysis. De Gruyter 1979. • Walter Rudin: Reelle und komplexe Analysis. Oldenbourg 2009.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Commutative Algebra		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		

Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.
Special Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Victor Batyrev, Thomas Markwig

Course Title:	Convex Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.		

Special Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Besides they enhance their geometric view and their spatial sense.
Literature	**Possible References :** • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Responsible Persons	Hannah Markwig

Course Title:	Lie Groups		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		
Special Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can disciplines of mathematics can work together extremely successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena.		
Literature	**Possible References :** • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Linear Control Theory
Specialisation	Analysis

Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Special Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications.		
Literature	**Possible References :** • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.		
Responsible Persons	Rainer Nagel		

Course Title:	Non-Linear Optimisation		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method. • Restricted optimisation, Farkas' lemma, tangent cone. • Abadie CQ, KKT conditions, Slater conditions. • Linear programme, duality, simplex method. • Penalty and barrier methods, interior point method. • Nonlinear programs, SQP methods, non-smooth optimisation.		

Special Objectives	Students master the basic principles and techniques of analysis and numerics of constrained optimisation problems.
Literature	**Possible References :** • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.
Responsible Persons	Andreas Prohl

Course Title:	Topology		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 2 SWS		
Content	• Review of metric spaces: Closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Special Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics.		
Literature	**Possible References :** • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.		
Responsible Persons	Rainer Nagel		

Course Title:	Calculus of Variations
Specialisation	Analysis

Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise Class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Special Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations.		
Literature	**Possible References :** • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.		
Responsible Persons	Reiner Schätzle		

Course Title:	Probability Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in contiuous time like Poisson processes and Brownian motion.		

Special Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2010. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2002. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Responsible Persons	Martin Möhle, Martin Zerner

Course Title:	Number Theory and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise Class 2 SWS		
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.		
Special Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically.		

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Responsible Persons	Elena Klimenko, Thomas Markwig