



Department of Mathematics

Module Handbook

Mathematical Physics

Master of Science*

Winter Semester 2026

Version: 13th July 2026

*The module handbook is valid for the 2026 study and examination regulations.

Contents

1	Program description	3
1.1	Study Concept	3
1.2	Qualification Goals	3
1.3	Program Structure	4
1.4	Mentoring	4
1.5	Information for students with a bachelor's degree in Physics at the University of Tübingen	4
2	Study Plans	6
2.1	Overview by Modules	6
2.2	Overview by the Course of Studies	7
2.3	Example Study Plans	8
2.4	Overview by Study Progress and Credit Requirements	12
3	Module Descriptions	14
	Section 1: Foundations	14
	Section 2: Knowledge Expansion	20
	Section 3: Elective Specialisation	23
	Section 4: Scientific Work	33
	Catalogue of Mathematics Modules	38

1 Program description

1.1 Study Concept

The Master of Science Mathematical Physics is an international research-oriented two year master's program offered jointly by the departments of Mathematics and Physics within the Faculty of Science of the University of Tübingen starting every year in the winter semester. It is geared towards students with a solid background in Mathematics as well as in Physics, and it requires a bachelor's degree in physics or mathematics or an equivalent degree. The scientific discipline *Mathematical Physics* is concerned with the mathematically rigorous formulation and analysis of physical theories and models. In this master's program students will thus deepen and broaden their knowledge of Mathematics and Physics in interdisciplinary courses in Mathematical Physics as well as in disciplinary courses in Mathematics and Theoretical Physics. At the end of the program they are particularly well prepared for jobs where the typical competences of mathematicians are needed in combination with applications of physics. The program is international and cannot be pursued without a solid knowledge of the English language. Language skill on the level of B2 according to the European Framework of Reference for Languages are therefore required. All mandatory modules and a large number of facultative modules are offered only in English. Some facultative modules may sometimes be offered only in German.

1.2 Qualification Goals

Students deepen and broaden their theoretical knowledge of different areas of mathematical physics, mathematics and theoretical physics. They become proficient in general and specific methods and principles in these areas. They can connect problems and questions from physics with their counterparts in mathematical models and are able to judge and critically question the relevance and adequacy of mathematical models and the derived consequences. They are able to report on and scrutinize the current state of research in the area of their specialisation. Graduates can apply their expanded knowledge in order to develop and successfully handle their own research projects. They are able to present, discuss, and defend the results of their research in writing and orally in front of a scientific audience. In the course of the Mathematical Physics Colloquium students practice scientific collaboration and discourse in interdisciplinary and internationally mixed groups.

Their education enables graduates in mathematical physics to successfully and professionally tackle complex mathematical modelling problems in physics and, after an appropriate familiarization with the subject, also in other areas of technology, finance or economics. They are moreover well prepared for interdisciplinary and international collaborations in mixed teams of different specialists from different cultural backgrounds, as are common nowadays in all areas of research and development.

1.3 Program Structure

The Master's Program is a two year (four terms) consecutive study program with a modular structure. Based on the foundational modules *Geometry in Physics*, *Functional Analysis in Geometry*, *Mathematical Quantum Theory*, and *Mathematical Relativity*, to be attended during the first year, students can specialise rather freely according to their personal preferences in one or more areas of Mathematical Physics, Mathematics and/or Theoretical Physics. The few restrictions are that every student must take at least one module from the Mathematics master's program and one module from the Theoretical Physics master's program, as well as a seminar. As a consequence, all graduates of the Master's Program have proven their ability to successfully conduct mathematical studies and theoretical physics studies at the master's level. A Scientific Project in the third term typically serves as a preparation for the Master Thesis (M.Sc. Thesis, 30 ECTS-points) written during the final term. During the second year students are also required to attend the Mathematical Physics Colloquium. This is a weekly colloquium where specialists lecture about recent developments in Mathematical Physics, and students have the opportunity to meet and discuss with international guest scientists and local researchers about current topics. The prescribed period of study is two years corresponding to a total of 120 ECTS points.

1.4 Mentoring

At the start of the program every student will be assigned to a mentor from the group of professors involved in the master's program for the whole duration of his/her studies. Students meet their mentor at the beginning and later at least once per term in order to plan and discuss the progress of their studies. In particular, at these meetings the study and examination plan in compliance with the examination regulations is discussed. The module selection is documented and passed on to the head of the examinations board for approval. During the first meeting possible gaps in the knowledge should be discussed in order to fill them by taking appropriate courses within the area of elective specialisation. The study and examination plan is then updated every semester during the meetings with the mentor. The mandatory mentoring program assures that students specialise in a purposeful way and select accordingly goal-oriented combinations of modules from mathematics and physics.

During the meetings with the mentor also possible time slots for a study period at a university abroad can be discussed. In principle, every semester is suitable, depending on the study progress of the student and the courses available at the other institution. It is also possible to write the master's thesis during a stay abroad under the cosupervision of a scientist there.

1.5 Information for students with a bachelor's degree in Physics at the University of Tübingen

Graduates of the 4-year degree program Bachelor of Science in Physics at the University of Tübingen can already gain up to 60 credit points for the degree program Master of Science in Mathematical Physics during their bachelor studies.

In particular,

- the module BMTPKFT Klassische Feldtheorie from the bachelor's program can be credited with

9 credit points for the module MAT-40-32 Advanced Topics in Theoretical Physics in the master's program, and

- up to 21 credit points in the section Vertiefungsfach in the bachelor's program can be credited in the section Elective Studies, provided the choice is suitable.

Moreover,

- up to 27 credit points in the section Ergänzungsmodule in the bachelor's program can be gained via the modules MAT-65-11 Geometry in Physics, MAT-65-12 Mathematical Quantum Theory, MAT-65-13 Mathematical Relativity or MAT-65-14 Mathematical Statistical Physics from the master's program, and
- the bachelor's thesis can be credited with 9 credit points in the module Scientific Project.

In order to finish the Master of Science in Mathematical Physics subsequently to the bachelor's degree in Physics at the University of Tübingen it is recommended to choose in the section Vertiefungsfach in the bachelor's program courses in theoretical physics, which can be credited in the section Elective Studies in the master's program in Mathematical Physics. Moreover, it is recommended to choose in the section Ergänzungsmodule in the bachelor's program at least two of the modules MAT-65-11, MAT-65-12, MAT-65-13 or MAT-65-14 from the master's program in Mathematical Physics. Good choices would be the combinations MAT-65-11 + MAT-65-13 and MAT-65-12 + MAT-65-14. Also the combination MAT-65-11 and MAT-65-12 would be suitable.

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Foundations							
1	MAT-65-11	Geometry in Physics	L+E	PM	EC	wr. o. or.	9
1	MAT-65-12	Mathematical Quantum Theory	L+E	PM	EC	wr. o. or.	9
2	MAT-65-13	Mathematical Relativity	L+E	PM	EC	wr. o. or.	9
Section 2: Knowledge Expansion							
1–3	MAT-40-31	Advanced Topics in Mathematics	L+E	PMW	EC	wr. o. or.	9
1–3	MAT-40-32	Advanced Topics in Theoretical Physics	L+E	PMW	EC	wr. o. or.	9
2–3	MAT-40-33	Seminar Knowledge Extension	S	PMW	s.M.	Pr	3
Section 3: Elective Specialisation							
1-3		Modules from the master's programmes of the Department of Mathematics or Physics according to Section 3.		WPM			30
Section 4: Scientific Work							
3	MAT-40-41	Scientific Project	P	PM	s.M.	-	9
3–4	MAT-40-42	Mathematical Physics Colloquium	C+C	PM	-	-	3
4	MAT-40-43	Master Thesis M.Sc. Mathematical Physics	MT	PM		MA+mP	30
Abbreviations: Type of Module : PM=compulsory module, PMW=compulsory module with choice, WPM=elective module Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Course Work : EC=exercise certificate Other : h=hours, o.=or, s.M.=see module description, ST=suggested term							

2.2 Overview by the Course of Studies

First we provide a general study plan showing the distribution of credit points over the different areas and the general time line. On the following pages example study plans for different types of specialisation are provided, where possible courses are assigned to the modules MAT-40-31 and MAT-40-32 as well as the modules from the area of Elective Specialisation.

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work
1.	27	27 CP	21 CP		
2.	30			30 CP	
3.	31				42 CP
4.	32				

Figure 2.1: General Study Plan

2.3 Example Study Plans

The example study plans shown below shall give an idea how the individual study in the different specialisations could look like. They are not meant as a recommendation, and it is neither guaranteed that the courses listed will be offered each year, nor that they all will be given in English.

Example Study Plan without Specialisation

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP) Mathematical Quantum Theory (9 CP)	Introduction to Partial Differential Equations (9 CP)			
2.	30	Mathematical Relativity (9 CP)	Seminar(3 CP)	Advanced Topics in Mathematical Quantum Theory (9 CP) Mathematical Statistical Physics (9 CP)		
3.	31		Quantum Field Theory and Particle Physics (9CP)	Advanced Topics in Mathematical Relativity (6 CP) Advanced Topics in Mathematical Statistical Physics (6 CP)	Mathematical Physics Colloquium (3 CP)	Scientific Project (9 CP)
4.	32					Master Thesis (30 CP)

Figure 2.2: The program Mathematical Physics can be completed to a large extent also without choosing a particular specialisation. In this case we recommend taking all four foundational modules and also all advanced courses offered. The modules from the area Knowledge Expansion should then be chosen in accordance with the planned specialisation in the Scientific Project and the Master Thesis, cf. e.g. the following study plans.

Example Study Plan Quantum Theory

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP)	Operator Theory (9 CP)			
		Mathematical Quantum Theory (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Quantum Field Theory and Particle Physics (9 CP)	Functional Analysis (9 CP)		
			Seminar(3 CP)			
3.	31			Advanced Topics in Mathematical Quantum Theory (9 CP)	Mathe- matical Physics Colloquium (3 CP)	Scientific Project (9 CP)
				Computational Methods in Physics / Astrophysics (6 CP)		
				Theoretical Condensed Matter Physics (6 CP)		
4.	32					Master Thesis (30 CP)

Figure 2.3: The mathematical foundations of quantum theory are predominantly allocated to areas of analysis. Thus we recommend that those specialising in one of the areas Mathematical Quantum Theory, Quantum Field Theory, Condensed Matter, Many-Body Quantum Systems, or Quantum Information attend mathematical courses from analysis, e.g. Operator Theory, Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Relativity

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP)	Astronomy and Astrophysics (9 CP)			
		Mathematical Quantum Theory (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Introduction to Partial Differential Equations (9 CP)	Riemannian Geometry (9 CP)		
			Seminar(3 CP)			
3.	31			Advanced Topics in Mathematical Relativity (9 CP)	Mathe- matical Physics Colloquium (3 CP)	Scientific Project (9 CP)
				Theoretical Astrophysics (6 CP)		
				Computational methods in Physics / Astrophysics (6 CP)		
4.	32					Master Thesis (30 CP)

Figure 2.4: The mathematical foundations of relativity are predominantly allocated to areas of geometry and analysis. Thus we recommend that those specialising in one of the areas Mathematical Relativity, Astronomy, Cosmology, or Astro Physics attend mathematical courses from geometry, e.g. Riemannian Geometry and Lorentz Geometry, and from analysis, e.g. Partial Differential Equations, Calculus of Variations, and Numerical Analysis.

Example Study Plan Statistical Physics

Term	CP	Foundations of Mathematical Physics	Knowledge Expansion	Elective Specialisation	Scientific Work	
1.	27	Geometry in Physics (9 CP)	Probability Theory (9 CP)			
		Mathematical Quantum Theory (9 CP)				
2.	30	Mathematical Relativity (9 CP)	Advanced Statistical Physics (9 CP)	Mathematical Statistical Physics (9 CP)		
				Density Functional Theory (6 CP)		
3.	31		Seminar (3CP)	Advanced Topics in Mathematical Statistical Physics (6 CP)	Mathematical Physics Colloquium (3 CP)	Scientific Project (9 CP)
				Mathematical Statistics (9 CP)		
4.	32					Master Thesis (30 CP)

Figure 2.5: The mathematical foundations of statistical physics are predominantly allocated to areas of probability. Thus we recommend that those specialising in one of the areas Mathematical Statistical Physics, Soft Matter, or Density Functional Theory attend mathematical courses from probability, e.g. Probability Theory and Mathematical Statistics.

2.4 Overview by Study Progress and Credit Requirements

Overview by Study Progress and Credit Requirements													
		Exam				Teaching				Term			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of exams / ECTS points to semesters is a recommendation only. Compulsory allocations are marked as such. The allocation of ECTS points to courses is for information only. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Foundations of Mathematical Physics:									27				
MAT-65-11 Geometry in Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-65-12 Mathematical Quantum Theory								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-65-13 Mathematical Relativity								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4			6		
2.	Exercises					E	o	2			3		
Knowledge Expansion:									21				
MAT-40-31 Advanced Topics in Mathematics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4		6			
2.	Exercises					E	o	2		3			
MAT-40-32 Advanced Topics in Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	o	4			6		
2.	Exercises					E	o	2			3		
MAT-40-33 Seminar								2	3				
1.	Seminar	Pres.	45–90	g	3	S	o	2				3	
Elective Specialisation:									30				
Here the modules MAT-65-15 and MAT-65-21 to MAT-65-26, as well as further suitable advanced modules from the Master's Programs in Mathematics, Physics, and Astro and Particle Physics, can be chosen. The choices need to be discussed and agreed upon with the Mentor. Modules from other areas need to be approved by the examinations board.													
MAT-65-14 Mathematical Statistical Physics								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	4			6		
2.	Exercises					E	f	2			3		
MAT-65-21 Advanced Topics in Mathematical Quantum Theory								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	4			6		
2.	Exercises					E	f	2			3		

Overview by Study Progress and Credit Requirements													
		Exam				Teaching				Term			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of exams / ECTS points to semesters is a recommendation only. Compulsory allocations are marked as such. The allocation of ECTS points to courses is for information only. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
MAT-65-22 Advanced Topics in Mathematical Quantum Theory (short version)								4	6				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	6	L	f	2			3		
2.	Exercises					E	f	2			3		
MAT-65-23 Advanced Topics in Mathematical Relativity								6	9				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	9	L	f	2				3	
2.	Exercises					E	f	2				3	
MAT-65-24 Advanced Topics in Mathematical Relativity (short version)								4	6				
1.	Lecture	Wr. or Or.	90–120 or 20–30	g	6	L	f	2				3	
2.	Exercises					E	f	2				3	
Scientific Work									42				
MAT-40-41 Scientific Project									9				
1.	Project	Proj.		ng	9		o					9	
MAT-40-42 Mathematical Physics Colloquium									3				
1.	Colloquium			ng			o					1	2
MAT-40-43 Master Thesis									30				
1.	Thesis	MA + Or.		g	30		o						30

Abbreviations:

Marking system : g=graded, ng=non graded

Form of examination : MA=Master Thesis, Or.=oral exam, Wr.=written exam, Pres.=presentation

Form of teaching : L=lecture, E=exercise class, S=seminar, Proj.=project work, Coll.=colloquium

Status : o=obligatory, f=fakultative

Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points

3 Module Descriptions

Section 1: Foundations

In the case that some of the mandatory modules in this section or modules, which are essentially identical as far as the contents and competences are concerned, have been part of the Bachelor studies, which are the prerequisite for this Master's Degree Program, according to the examination regulations these modules cannot be taken in the Master's Degree Program any more. They have to be replaced by other suitable modules in the framework of the studies and examination plan.

Module Number: MAT-65-11	Module Title: Geometry in Physics		Type of Module: Compulsory Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.		
Objectives	Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. In particular, they develop a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometry in Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.										
Literature	Possible References : • John Lee: Introduction to smooth manifolds. Springer 2012. • John Lee: Riemannian manifolds: An introduction. Springer 1997. • Chris Isham: Modern differential geometry for physicists. World Scientific 1999. • Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.									
Transfer	Participation in the module is a prerequisite for participation in the module Mathematical Relativity. Successful completion of the module may be a prerequisite for participation in the module Seminar Knowledge Extension and is so for the participation in the module Scientific Project.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.									
Responsible Persons	Christoph Bohle, Carla Cederbaum, Stefan Teufel									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-65-12	Module Title: Mathematical Quantum Theory				Type of Module: Compulsory Module					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly in Winter Semester									
Term	1									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to mathematical quantum theory. Particular topics are the stationary and time-dependent Schrödinger equation, fundamental approximation methods as Rayleigh-Schrödinger perturbation theory and Hartree- resp. Hartree-Fock theory, the Fock space formalism, and elements of scattering theory. Optionally, other topics such as adiabatic theory and semiclassical approximations can be discussed.									
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle physics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle physics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Mathematical Quantum Theory	L E	o o	4 2	6 3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	Successful completion of module Mathematical Quantum Theory is a prerequisite for the participation in the module Advanced Topics in Mathematical Quantum Theory. Successful completion of one of the modules Mathematical Quantum Theory and Mathematical Relativity is a prerequisite for the participation in the module Scientific Project.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Stefan Teufel, Marius Lemm
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-13	Module Title: Mathematical Relativity					Type of Module: Compulsory Module				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	2									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to the mathematical theory of relativity. Particular topics are Newton's theory of gravity, special theory of relativity, relativistic effects, Einstein's equations, the Schwarzschild spacetime. Optionally, other topics such as cosmological models, matter models, black holes, Cauchy problem and ADM decomposition, singularity theorems or gravitational waves can be discussed.									
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from the theory of relativity. They are able to interrelate physical problems in cosmology and astrophysics and their mathematical models through methods from differential geometry and to question the relevance and adequacy of the mathematical model and the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular in module MAT-65-11. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Mathematical Relativity	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	Successful completion of module Mathematical Relativity is a prerequisite for the participation in the module Advanced Topics in Mathematical Relativity. Successful completion of one of the modules Mathematical Relativity or Mathematical Quantum Theory is a prerequisite for the participation in the module Scientific Project.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.									

Responsible Persons	Carla Cederbaum, Gerhard Huisken, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-40-32	Module Title: Advanced Topics in Theoretical Physics				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	every Semester									
Term	1–3									
Language of Instruction	English or German									
Forms of Teaching and Learning	Lectures 4 SWS + Exercise Classes 2 SWS, Homework Assignments									
Content	It is required to attend one or more advanced-level lectures from the field of theoretical physics as well as the respective exercise classes with the correspondent SWS-coverage from the Master's degree program in Physics or the Master's degree program Astro and Particls Physics. Recommended subjects are for instance Quantum field theory and Particle physics, Theoretical astrophysics, Relativistic astrophysics, Many-particle quantum systems, Advanced statistical physics, Yang-Mills theory, Condensed matter physics, Theoretical quantum optics, Quantum information theory, Cosmology, Numerical methods in physics and astrophysics, Current topics in theoretical physics. Further details can be found in the module handbook of the corresponding degree programs.									
Objectives	The students aquire deepend knowledge in one selected area of theoretical physics independently of rigorous mathematical formalism. They broaden the basis of their knowledge in theoretical physics and extend the methods at hand to tackle problems in physics. The further qualification goals, in particular the concrete content related qualifaction goals, will follow from the module description of the chosen course in the module handbook for the M.Sc. Pysics or the M.Sc. Astro and Particle Physics.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Advanced Topics in Theoretical Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	See prerequisites in the Module Handbook M.Sc. Physics or M.Sc. Astro and Particle Physics.									
Responsible Persons	Die Studiendekanin oder der Studiendekan des Fachbereichs Physik									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-40-33	Module Title: Seminar Knowledge Extension						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	every Semester									
Term	2–3									
Language of Instruction	English or German									
Forms of Teaching and Learning	Seminar: Presentation, Discussion, Teamwork, Handout									
Content	Various topics from various areas of Mathematical Physics, Mathematics or Theoretical Physics.									
Objectives	The students have learnt to develop independently or in team an acquaintance with an advanced topic in Mathematics or Physics by applying scientific methods and to present it in form of an oral presentation. They have improved their skills in the presentation of mathematical or physical results and are able to argue for these results in critical discussions.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Seminar	S	o	2	3	yes	Pr	45–90	g	100
	Erweiterungswissen									
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	Successful completion of one of the modules from the Section "Foundations of Mathematical Physics".									
Responsible Persons	Stefan Teufel									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Elective Specialisation

Within the study area Elective Specialisation students can choose modules from the Master Programs Mathematical Physics, Mathematics, Physics, and Astro and Particle Physics according to their individual interests. In particular, courses listed in the module descriptions MAT-40-31 and MAT-40-32 but not chosen there, the module MAT-65-13 respectively MAT-65-14 not yet chosen in the area Foundations, the modules MAT-65-15 and MAT-65-21 to MAT-65-24, as well as other appropriate advanced modules from the programs Mathematical Physics, Mathematics (see ??), Physics, and Astro and Particle Physics are available. Note that not all modules can be offered every year, but there is always a broad choice. Also note that some modules from other programs might be offered only in German, but also here a choice of English courses is ensured. The selection of modules within the area Elective Specialisation must be discussed and decided together with the mentor. Each module can be selected only once. In agreement with the mentor and upon request at the examinations board, 9 ECTS points within the area of Elective Specialisation can be allocated for modules that serve to close knowledge gaps either in mathematics or physics.

Within the area of Elective Specialisation students obtain relevant skills. They learn to independently judge which additional qualifications and competences are relevant to their studies and to select courses accordingly. They are able to acquire specific knowledge also beyond the mandatory parts of the study program. Within the area of their specialisation they can report on and scrutinize the current state of research. In the exercise classes students learn to work confidently, precisely and independently with the notions, statements and methods presented during the lectures. They also learn how to apply methods to new problems and to analyse and solve them alone or in groups.

- Advanced Topics in Mathematical Quantum Theory (MAT-65-21, 9 CP) 27
- Advanced Topics in Mathematical Quantum Theory (short version) (MAT-65-22, 6 CP) 29
- Advanced Topics in Mathematical Relativity (MAT-65-23, 9 CP) 31
- Foundations of Quantum Mechanics (MAT-65-15, 9 CP) 25
- Mathematical Statistical Physics (MAT-65-14, 9 CP) 23

Module Number: MAT-65-14	Module Title: Mathematical Statistical Physics		Type of Module: Elective Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly, in Summer Semester		
Term	2-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The module provides an introduction to mathematical statistical physics. Particular topics are concepts of probability theory, classical statistical mechanics of gases (equivalence of ensembles, thermal equilibrium, Boltzmann equation, entropy), Brownian motion (stochastic processes, Wiener process), lattice models (Ising model, Gibbs measure, thermodynamic limit, phase transitions), statistical quantum mechanics (quantum mechanical ensembles, transition to thermal equilibrium, Bose-Einstein condensate). Optionally, other topics such as open quantum systems, transport phenomena, renormalization group theory and the fluctuation-dissipation theorem can be discussed.		

[illegible]

Module Number: MAT-65-15	Module Title: Foundations of Quantum Mechanics					Type of Module: Elective Module				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly every two years									
Term	2-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to fundamental questions of quantum mechanics, including its mathematical and philosophical aspects. Various interpretations such as Copenhagen, Bohmian mechanics, many worlds and spontaneous collapse of the wave function are presented and analysed mathematically and physically. Other topics include Born's rule, Heisenberg's uncertainty principle, the quantum measurement problem, Bell's non-locality theorem, identical particles and no-hidden-variable theorems.									
Objectives	Students know and can apply the rules of quantum mechanics in different environments and understand several important theories of how the quantum world works. They acquire mathematical knowledge relevant to the application of these rules and theories and can connect the mathematical treatment with the physical meaning. They will familiarise themselves with the surprising phenomena and paradoxes of quantum mechanics. They will appreciate what is controversial about the orthodox interpretation and why, and will be able to follow the current debate on fundamental issues. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Foundations of Quantum Mechanics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	The type of examination is set by the instructor.									
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.									
Responsible Persons	Roderich Tumulka									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-21	Module Title: Advanced Topics in Mathematical Quantum Theory				Type of Module: Elective Module						
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h						
Duration	1 Semester										
Frequency	not regularly										
Term	2-3										
Language of Instruction	English										
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS										
Content	The module provides an introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent fermion systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.										
Objectives	Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.										
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	
		L	o	4	6						
	Advanced Topics in Mathematical Quantum Theory	E	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100	
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.										
Transfer	The module may be a prerequisite for the master thesis.										
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-12 Mathematical Quantum Theory.										
Responsible Persons	Stefan Teufel, Marius Lemm										

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-22	Module Title: Advanced Topics in Mathematical Quantum Theory (short version)					Type of Module: Elective Module				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly, in Summer Semester									
Term	2									
Language of Instruction	English									
Forms of Teaching and Learning	Lectures 2 SWS + Exercise Classes 2 SWS, Homework Assignments									
Content	The module provides a short introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent fermion systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.									
Objectives	Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and in parts also critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Advanced Topics in Mathematical Quantum Theory	L	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-12 Mathematical Quantum Theory.									
Responsible Persons	Stefan Teufel									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-23	Module Title: Advanced Topics in Mathematical Relativity					Type of Module: Elective Module				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly, in Winter Semester									
Term	3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to an advanced topic of mathematical theory of relativity. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.									
Objectives	Students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Advanced Topics in Mathematical Relativity	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.									
Transfer	The module may be a prerequisite for the master thesis.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-13 Mathematical Relativity.									
Responsible Persons	Carla Cederbaum, Gerhard Huiskens, Frank Loose									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-41	Module Title: Scientific Project						Type of Module: Compulsory Module			
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 15 h			Self-Study: 255 h			
Duration	1 Semester									
Frequency	every Semester									
Term	3									
Language of Instruction	English									
Forms of Teaching and Learning	Individual supervision by a mentor, study of scientific works.									
Content	• Definition of an advanced scientific project in coordination with the mentor. • Independent search and study of the relevant scientific literature. • Formulation of specific problems and methodical approach to their solution. • Written presentation of the project in the context of the current state of research on 5-10 pages. This module serves generally as a preparation for the Master Thesis.									
Objectives	Students • develop skills to systematically familiarize themselves with a new subject, • learn to work critically and to form a substantiated, professional and interdisciplinary judgement, • acquire qualifications in such areas as literature research, identification of relevant problems and appropriate methods, as well as in the written presentation of a research proposal.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Scientific Project	P	o	1	9	yes	-	-	ng	-
Transfer	Successful completion of this module is a prerequisite for participation in module Master Thesis.									
Prerequisites	Successful completion of module Geometry in Physics and of one of the modules Mathematical Quantum Theory or Mathematical Relativity.									
Responsible Persons	Stefan Teufel, Werner Vogelsang.									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-42	Module Title: Mathematical Physics Colloquium						Type of Module: Compulsory Module			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 60 h			Self-Study: 30 h			
Duration	2 Semester									
Frequency	every Semester									
Term	3–4									
Language of Instruction	English									
Forms of Teaching and Learning	Presentations, discussions. Specific form of study: during the final semester students present their Master thesis.									
Content	During each semester on 15 appointed dates (2 h each) there will take place presentations and discussions on current topics in mathematical physics. Speakers are the researchers of the involved departments, guest scientists and master's students, who present the results of their Master Thesis.									
Objectives	Students gain an insight into the current development of mathematical physics beyond the area of their own specialization. They develop the ability to follow scientific presentations and to discuss and challenge them within a larger group of scholars. They therefore also obtain interdisciplinary and intercultural competencies through regular cooperation and discussion in mixed groups.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Colloquium Winter Semester	C	o	2	1	no	-	-	ng	-
	Colloquium Summer Semester	C	o	2	2	no	-	-	ng	-
Transfer	-									
Prerequisites	-									
Responsible Persons	Carla Cederbaum, Stefan Teufel									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

[illegible]

Prerequisites	• 27 CP from the compulsory elective Section Foundations of Mathematical Physics, • a total of 18 CP from the Sections Knowledge Expansion and Elective Specialisation, • Successful completion of module Scientific Project.
Responsible Persons	Stefan Teufel, Werner Vogelsang
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Catalogue of Mathematics Modules

This section lists modules from the M.Sc. Mathematics programme that can be included in the area Elective Specialisation and which are not yet listed in Section 3.

• Algebraic Curves (MAT-45-14, 9 CP)	54
• Algebraic Curves and Riemann Surfaces (MAT-50-29, 9 CP)	128
• Algebraic Geometry (MAT-45-11, 9 CP)	48
• Algebraic Geometry and Toric Varieties (MAT-45-12, 9 CP)	50
• Algebraic Number Theory (MAT-45-21, 9 CP)	64
• Algebraic Topology 1 (MAT-50-21, 9 CP)	116
• Algebraic Topology 2 (MAT-50-22, 9 CP)	118
• Algebraic Topology 3 (MAT-50-23, 3 CP)	120
• Algebraic Transformation Groups (MAT-45-13, 9 CP)	52
• Algorithms of Numerical Mathematics (MAT-70-01, 9 CP)	243
• Area Minimising Flows (MAT-55-43, 4.5 CP)	176
• Automorphic Forms (MAT-55-53, 4.5 CP)	192
• Bayesian Networks and Causality (MAT-75-21, 4.5 CP)	298
• Calculus of Variations (MAT-55-49, 4.5 CP)	188
• Commutative Algebra (MAT-45-02, 9 CP)	44
• Computer Algebra (MAT-45-03, 9 CP)	46
• Consistency Proofs (MAT-55-62, 6 CP)	196
• Convex Geometry (MAT-50-02, 9 CP)	86
• Cox Rings (MAT-45-18, 9 CP)	60
• Cryptography (MAT-45-23, 4.5 CP)	68
• Elastic Curves (MAT-55-46, 3 CP)	182
• Elementary Number Theory (MAT-45-25, 6 CP)	72
• Elliptic Curves and Cryptography (MAT-45-27, 9 CP)	76
• Elliptic Curves and Taniyama-Shimura (MAT-45-28, 9 CP)	78
• Elliptic Functions and Elliptic Curves (MAT-45-24, 9 CP)	70
• Ergodic Theory (MAT-55-05, 9 CP)	144
• Explicit Mathematics (MAT-55-65, 6 CP)	201
• Financial Mathematics and Numerics (MAT-70-51, 6 CP)	280
• Fredholm Index Theory for Condensed Matter and Introduction to Spin Systems (MAT-65-41, 3 CP)	241
• Fully Non-Linear Elliptic Equations (MAT-55-27, 4.5 CP)	166
• Fully Non-Linear Elliptic and Parabolic Partial Differential Equations (MAT-60-36, 3 CP)	229
• Functional Analysis (MAT-55-01, 9 CP)	136
• Game Theory (MAT-70-40, 3 CP)	279
• Geometric Evolution Equations (MAT-60-01, 3 CP)	205
• Geometric Group Theory (MAT-50-30, 9 CP)	130

• Geometric Measure Theory (MAT-55-42, 9 CP)	174
• Geometric Measure Theory – Flows (MAT-55-48, 4.5 CP)	186
• Geometric Measure Theory – Varifolds (MAT-55-47, 4.5 CP)	184
• Geometric Variation Problems (MAT-60-02, 3 CP)	207
• Geometry of Manifolds 1 (MAT-50-10, 9 CP)	94
• Geometry of Manifolds 2 (MAT-50-11, 9 CP)	96
• Graph Theory (MAT-75-10, 9 CP)	294
• Gromov-Witten Theory (MAT-50-40, 6 CP)	132
• Groups and Representations (MAT-65-05, 9 CP)	231
• Hamiltonian Systems (MAT-65-38, 9 CP)	235
• Harmonic Analysis in Euclidean Space (MAT-55-12, 9 CP)	150
• Harmonic Analysis on Abelian Groups (MAT-55-13, 9 CP)	152
• Harmonic Analysis on General Groups (MAT-55-14, 9 CP)	154
• Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic (MAT-50-50, 9 CP)	134
• Information Geometry (MAT-50-12, 3 CP)	98
• Information Theory, Pattern Recognition, and Neural Networks (MAT-75-22, 3 CP)	300
• Integrable Systems (and Infinite Dimensional Lie Algebras) (MAT-50-18, 9 CP)	110
• Interacting Many-Body Quantum Systems (MAT-65-40, 9 CP)	239
• Introduction to Analytic Number Theory (MAT-45-26, 3 CP)	74
• Introduction to Berkovich Geometry (MAT-45-20, 3 CP)	62
• Introduction to Commutative Algebra and Algebraic Geometry (MAT-45-01, 9 CP)	41
• Introduction to Dynamical Systems (MAT-55-34, 3 CP)	170
• Introduction to Geometric Measure Theory (MAT-55-41, 9 CP)	172
• Introduction to Geometric Measure Theory – Measure Theoretic Methods (MAT-55-44, 4.5 CP)	178
• Introduction to Geometric Measure Theory – Varifolds (MAT-55-45, 4.5 CP)	180
• Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory) (MAT-50-17, 9 CP)	108
• Introduction to Mathematical Logic (MAT-55-60, 3 CP)	194
• Introduction to Optimisation (MAT-70-20, 6 CP)	261
• Introduction to Partial Differential Equations (MAT-55-21, 9 CP)	156
• Introduction to Partial Differential Equations – Part 1 (MAT-55-25, 4.5 CP)	162
• Introduction to Riemann Surfaces (MAT-50-15, 4.5 CP)	104
• Introduction to Schemes (MAT-45-16, 9 CP)	58
• Introduction to Stochastic Differential Equations - Part 1 (MAT-70-12, 4.5 CP)	257
• Introduction to set theory (MAT-55-63, 3 CP)	198
• Lie Groups (MAT-55-51, 9 CP)	190
• Linear Control Theory (MAT-55-07, 6 CP)	146
• Mathematical Aspects of 3d Computer Vision (MAT-50-13, 6 CP)	100

• Mathematical Aspects of Deep Learning 1 (MAT-50-14, 3 CP)	102
• Mathematical Aspects of Deep Learning 2 (MAT-50-19, 3 CP)	112
• Mathematical Aspects of Reinforcement Learning (MAT-50-24, 6 CP)	122
• Mathematical Introduction to Data Science (MAT-70-34, 4.5 CP)	275
• Mathematical Population Genetics (MAT-75-08, 6 CP)	290
• Mathematical Statistics (MAT-75-03, 9 CP)	284
• Morse Theory (MAT-55-28, 3 CP)	168
• Non-Linear Elliptic and Parabolic Partial Differential Equations (MAT-60-35, 6 CP)	227
• Non-Linear Functional Analysis (MAT-55-02, 9 CP)	138
• Non-Linear Optimisation (MAT-70-21, 9 CP)	263
• Nonlinear Elliptic Partial Differential Equations illustrated by the Example of Minimal Surfaces (MAT-55-24, 9 CP)	160
• Nonparametric Estimation of Distributions (MAT-75-23, 4.5 CP)	302
• Null Geometry in General Relativity (MAT-60-08, 4.5 CP)	217
• Number Theory and Cryptography (MAT-45-22, 9 CP)	66
• Numerics of Differential Equations of Surfaces (MAT-70-06, 6 CP)	253
• Numerics of Instationary Differential Equations (MAT-70-03, 9 CP)	247
• Numerics of Stationary Differential Equations (MAT-70-02, 9 CP)	245
• Numerics of Stochastic Differential Equations (MAT-70-15, 3 CP)	259
• Operator Algebras (MAT-55-04, 9 CP)	142
• Operator Algebras and their Applications to Statistical Mechanics (MAT-55-71, 6 CP)	203
• Operator Algebras in Mathematical Physics (MAT-65-31, 9 CP)	233
• Operator Theory (MAT-55-03, 9 CP)	140
• Optimal Control Theory with Ordinary Differential Equations (MAT-70-05, 4.5 CP)	251
• Optimal Transport (MAT-70-35, 9 CP)	277
• Optimisation with Differential Equations (MAT-70-22, 9 CP)	265
• Ordinary Differential Equations - Analysis and Numerics (MAT-70-04, 9 CP)	249
• Partial Differential Equations (MAT-55-22, 9 CP)	158
• Partial Differential Equations in Conformal Geometry: the Yamabe Problem (MAT-55-26, 3 CP)	164
• Point Processes (MAT-75-09, 6 CP)	292
• Probability Distances for Data Science (MAT-75-20, 6 CP)	296
• Probability Theory (MAT-75-01, 9 CP)	282
• Problem Solving Strategies (MAT-45-32, 4.5 CP)	84
• Propagation of Chaos (MAT-65-39, 9 CP)	237
• Representation Theory of Finite Groups (MAT-45-31, 6 CP)	82
• Riemannian Geometry (MAT-50-16, 6 CP)	106
• Software Obfuscation: Mixed Boolean-Arithmetic Expressions (MAT-45-30, 4.5 CP)	80
• Space-Like Hypersurfaces in Lorentzian Manifolds (MAT-60-04, 6 CP)	211
• Special Relativity (MAT-60-07, 3 CP)	215

• Special Topics in Evolution Equations for Submanifolds (with Exercise Class) (MAT-60-10, 3 CP)	221
• Special Topics in Evolution Equations for Submanifolds (without Exercise Classes) (MAT-60-11, 3 CP)	223
• Spectral Theory of Positive Operators (MAT-55-08, 3 CP)	148
• Statistical Learning 1 (MAT-70-31, 9 CP)	269
• Statistical Learning 2 (MAT-70-32, 9 CP)	271
• Stochastic Analysis (MAT-75-06, 9 CP)	288
• Stochastic Differential Equations (MAT-70-11, 9 CP)	255
• Stochastic Processes (MAT-75-04, 9 CP)	286
• The Einstein Constraint Equations (MAT-60-09, 6 CP)	219
• The Positive Mass Theorem (MAT-60-12, 6 CP)	225
• The Ricci Flow of Riemannian Metrics (MAT-60-06, 6 CP)	213
• Theoretical Aspects of Machine Learning (MAT-70-30, 6 CP)	267
• Theory and Numerics for Constrained Optimisation Problems (MAT-70-33, 9 CP)	273
• Theory of Mathematical Proofs (MAT-55-64, 6 CP)	199
• Topics in Mathematical Relativity (MAT-60-03, 6 CP)	209
• Topological Vector Spaces and Distributions (MAT-50-27, 6 CP)	124
• Topology (MAT-50-20, 3 CP)	114
• Toric Varieties and Mori Dream Spaces (MAT-45-15, 9 CP)	56
• Tropical Enumerative Geometry (MAT-50-04, 9 CP)	88
• Tropical Enumerative Geometry - Part 1 (MAT-50-05, 4.5 CP)	90
• Tropical Enumerative Geometry - Part 2 (MAT-50-06, 4.5 CP)	92
• Uniformisation of Riemann Surfaces (MAT-50-28, 4.5 CP)	126

Module Number: MAT-45-01	Module Title: Introduction to Commutative Algebra and Algebraic Geometry		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		

Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.																																		
Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.																																		
Requirements for obtaining Credits / Grading (Weighting if applicable)	<table><tr><td></td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr><tr><td>Title</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td rowspan="2">Introduction to Commutative Algebra and Algebraic Geometry</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr><tr><td>E</td><td>f</td><td>2</td><td>3</td></tr></table> In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	Title										Introduction to Commutative Algebra and Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	E	f	2	3
	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade																										
Title																																			
Introduction to Commutative Algebra and Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100																										
	E	f	2	3																															
Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.																																		
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Commutative Algebra' due to the large overlap in content.																																		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Jürgen Hausen
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-02	Module Title: Commutative Algebra			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly in Winter Semester									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.									
Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Commutative Algebra	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Commutative Algebra and Algebraic Geometry' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-03	Module Title: Computer Algebra				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Canonical forms and standard bases for ideals and modules. • Computation of important operations for ideals and modules. • Syzygies, free resolutions and the proof of the Buchberger criterion. • Calculation of the primary decomposition of ideals. • Hilbert functions.									
Objectives	Students are familiar with important problems in the interplay of commutative algebra and algebraic geometry as well as algorithmic approaches to solving them. In particular, they are familiar with the theory of standard bases and their diverse applications. They are also familiar with important software packages in the field of symbolic computing and have implemented algorithms in these. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse. They also learnt about important software packages in the field of symbolic computing and have implemented algorithms in them themselves.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Computer Algebra	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Gert-Martin Greuel, Gerhard Pfister: A SINGULAR Introduction to Commutative Algebra. Springer 2008. • Wolfram Decker, Christoph Lossen: Computing in algebraic geometry. A quick start using SINGULAR. Springer 2006. • Wolfram Decker, Gerhard Pfister: A first Course in computational algebraic geometry. Cambridge University Press 2013. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-02 Commutative Algebra. Knowledge from the following modules is helpful but not strictly required: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-45-11 Algebraic Geometry.
Responsible Persons	Daniele Agostini, Hannah Markwig, Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-11	Module Title: Algebraic Geometry				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Prevarieties and varieties. • Projektive varieties and homogeneous spectrum. • Finite and proper morphisms. • Blow-up and Grassmannians. • Rational maps. • Divisors and line bundles, class group and Picard group.									
Objectives	The students learn central terms, results and methods of modern Algebraic Geometry and they develop a deeper understanding of the interconnections between Geometry and Algebra. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry and Toric Varieties' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-02 Commutative Algebra.
Responsible Persons	Daniele Agostini, Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-12	Module Title: Algebraic Geometry and Toric Varieties				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Projective space. • Prevarieties, morphisms, tangent space and singularities. • Products and separation. • Projective varieties and Grassmannians. • Divisors and line bundles, class group and Picard group. • Toric varieties.									
Objectives	Students learn the central concepts, results and methods of modern Algebraic Geometry and they develop an advanced understanding of the relationships between Geometry and Algebra. Using the example of the class of toric varieties, they also learn how methods of Convex Geometry can facilitate the investigation of an important example class of Algebraic varieties, and expand the interplay of Algebra and Geometry by a further component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Geometry and Toric Varieties	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									

Literature	Possible References : • David A. Cox, John B. Little, Henry K. Schenck: Toric varieties. American Mathematical Society 2011: • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Jürgen Hausen, Daniele Agostini
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Armand Borel: Linear algebraic groups. Springer 1991. • Jean A. Dieudonne, James B. Carrell: Invariant theory. Academic Press 1971. • David Mumford: Geometric invariant theory. Springer 1965.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Groups' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-45-12 Algebraic Geometry and Toric Varieties.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Abbreviations:
 Grading System : g=graded, ng=not graded
 Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
 Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
 Status : o=obligatory, f=facultative
 Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-14	Module Title: Algebraic Curves				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Projective curves, divisors, Theorem of Riemann-Roch. • Ramified coverings, Theorem of Hurwitz. • Linear systems, embeddings, Castelnuovo inequality. • Singularities of plane curves, Puiseux expansions. • Classification and moduli spaces, Jacobi variety.									
Objectives	Students have familiarised themselves with the central concepts, results and methods in a selected sub-area of algebraic geometry. They have developed an in-depth understanding of algebraic curves and their classification. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Curves	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Robin Hartshorne: Algebraic geometry. Springer 2006. • Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994. • Rick Miranda: Algebraic Curves and Riemann Surfaces. AMS 1995.									

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required (for the latter, please also note the prerequisites specified there): • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Daniele Agostini, Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : - Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface. Cox rings. Cambridge University Press 2014. - Yi Hu, Sean Keel. Mori dream spaces and GIT. Michigan Math. J. 48: 331-348, 2000.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: - MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, - MAT-45-12 Algebraic Geometry and Toric Varieties.
Responsible Persons	Jürgen Hausen
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-16	Module Title: Introduction to Schemes			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Affine schemes. • General schemes: local and global properties. • Morphisms. • Projective schemes. • Coherent sheaves. • Cohomology.									
Objectives	The students understand the scheme-theoretic foundations of algebraic geometry. They can translate classical varieties into the language of schemes, work with sheaves and locally ringed spaces, and analyse morphisms of schemes and their key properties. They also gain a basic understanding of sheaf cohomology and its role in studying the geometric properties of schemes. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Schemes	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Andreas Gathmann: Algebraic Geometry. Lecture notes, RPTU Kaiserslautern. • Robin Hartshorne: Algebraic geometry. Springer 2006. • Ravi Vakil: The Rising Sea: Foundations of Algebraic Geometry. Princeton University Press 2025.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Daniele Agostini, Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-18	Module Title: Cox Rings		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Divisorial algebras. • Cox rings. • Charakteristic spaces. • Good quotients. • Geometric invariant theory. • Gale-duality. • Connections to toric geometry. • Defining data for varieties with finitely generated Cox ring. • Singularities. • Picard group. • Basis locus. • Ampleness. • Kanonical class. • Intrinsic quadrics. • k^*-surfaces. • Varieties with torus action.		
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra and combinatorics. They have familiarised themselves with the Cox ring as an algebraic object for investigating special classes of geometric spaces. In doing so, they expand the interplay between algebra and geometry with another important methodological component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Cox Rings	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, Antonio Laface: Cox Rings. CUP 2014.									
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: - MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, - MAT-45-12 Algebraic Geometry and Toric Varieties.									
Responsible Persons	Jürgen Hausen									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-20	Module Title: Introduction to Berkovich Geometry				Type of Module: Compulsory Module with Choice					
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h		Self-Study: 120 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.									
Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Berkovich Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.									

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-21	Module Title: Algebraic Number Theory					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Rings of integers. • Class numbers. • Dirichlet's unit theorem. • Extension of Dedekind rings. • Valuation theory. • Local fields. • Adeles and ideles.									
Objectives	The students have learned the central terms, results and methods of algebraic number theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Number Theory	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007. • Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007. • Andre Weil: Basic number theory. Springer 1995.									

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-22	Module Title: Number Theory and Cryptography			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h							
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.									
Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocols are working. Through studying many open problems of cryptography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Number Theory and Cryptography	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Elliptic Curves and Cryptography' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-23	Module Title: Cryptography				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h			Time in Class: 45 h			Self-Study: 105 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Brief review of key concepts and results from algebra and number theory. • Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes. • Diffie-Hellman protocol and fast exponentiation. • Discrete logarithms: Shanks' algorithm and Pollard's rho method. • RSA: correctness, security, and attacks. • Signature schemes.									
Objectives	Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Cryptography	L E	f f	2 1	3 1,5	yes	or.	20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired.										

Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Number Theory and Cryptography' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-24	Module Title: Elliptic Functions and Elliptic Curves					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Elliptic functions, Weierstrass-P-function, Riemann surfaces, complex tori. • Plane projective curves, Theorem of Bezout, elliptic curves. • Curves over finite fields, rational points. • Applications in cryptography. • Additionally a selection of the following: – Modular forms; – Classification of elliptic curves; – Moduli spaces.									
Objectives	The students have expanded their competence in mathematical-interdisciplinary studies. They have learnt about elliptic curves as a class of mathematical objects, which has comprehensive relevance in a wide spectrum of mathematical areas. The students have studied the notions, methods and results from the disciplines Complex Calculus, Algebraic Geometry, Number Theory, Topology and Cryptography, which are relevant in the given context, and they understand their mutual interrelations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Elliptic Functions and Elliptic Curves	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Wolfgang Fischer, Ingo Lieb: Funktionentheorie. Vieweg 2005. • Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Daniele Agostini
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Responsible Persons	Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Literature	Possible References : • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: <i>Moderne Verfahren in der Kryptographie</i>. Springer 2015. • Joseph H. Silverman: <i>The arithmetic of elliptic curves</i>. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: <i>Elliptic curves in cryptography</i>. CUP 1999.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the modules 'Number Theory and Cryptography' due to the large overlap in content
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required (for the latter, please also note the prerequisites specified there): • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Ivo Radloff
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Efficient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Artificial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.
Responsible Persons	Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. It can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Group Representations in Physics' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Jürgen Hausen, Thomas Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-45-32	Module Title: Problem Solving Strategies		Type of Module: Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Basic Techniques and Problem Reduction. • Proofs and Logic. • Sets and Functions. • Elementary Combinatorics. • Counting Techniques and Cardinality. • The Pigeonhole Principle (Box Principle). • Bounds, Estimation, and Inequalities. • The Extremal Principle. • The Invariance Principle: Invariants and Monovariants. • Proofs by Colouring. • Graph-Theoretic Modelling.		
Objectives	Students acquire a versatile toolkit of mathematical and heuristic problem-solving strategies that they can draw upon both in their subsequent studies and, where appropriate, in competition-oriented mathematical problem solving. This includes fundamental results and proof techniques, as well as the ability to recognise characteristic structural features of a problem that indicate suitable solution approaches or allow unsuitable strategies to be ruled out at an early stage. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Problem Solving Strategies	L	f	2	3	yes	or.	20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired.									
Literature	Possible References : • George Pólya: How to Solve It: A New Aspect of Mathematical Method. Princeton University Press 1973. • Arthur Engel: Problem-Solving Strategies. Springer 1998.									
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.									
Responsible Persons	Thomas Markwig									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-50-02	Module Title: Convex Geometry				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.									
Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develop a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Convex Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be counted alongside it.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

[illegible]

Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be included alongside it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-05 Tropical Enumerative Geometry - Part 1.
Responsible Persons	Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. Due to a significant overlap in contents the module cannot be taken together with either of the modules 'Geometry in Physics' and 'Introduction to Differential Geometry'.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Christoph Bohle, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-11	Module Title: Geometry of Manifolds 2					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Global aspects of Riemannian geometry • Cohomology of manifolds. • Analysis of differential operators on manifolds. • Applications on Riemann surfaces (and complex manifolds).									
Objectives	The students are familiar with the fundamental concepts of global real and complex differential geometry. They have deepened their understanding of methods in differential geometry, and they have exemplarily experienced how local and global aspects in geometry interact. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometry of Manifolds 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011. • John Milnor: Morse Theory. PUP 1963. • Donu Arapura: Algebraic Geometry over the Complex Numbers. Springer 2012. • Sundararaman Ramanan: Global Calculus. AMS 2005.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry and Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-10 Geometry of Manifolds 1.
Responsible Persons	Christoph Bohle, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-12	Module Title: Information Geometry					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). • Application to neural data processing (in particular supervised learning in artificial neural networks).									
Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Information Geometry	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. • Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. • Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. • Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.									

Responsible Persons	Christoph Bohle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14.
Responsible Persons	Christoph Bohle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-14	Module Title: Mathematical Aspects of Deep Learning 1						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	- Artificial neural networks and their training using backpropagation/stochastic gradient descent. Probabilistic interpretation. Mathematical problems arising in neural network science. - Classical architectures for computer vision (CNNs) and sequential data (RNNs). (Self-)supervised learning methods. - Transformer models, a modern architecture based on the attention mechanism, and its applications in natural language processing (GPT) and computer vision. - Simple neuroscience models for neural networks. Biological plausibility of backpropagation and transformers.									
Objectives	The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with contemporary architectures and mathematical questions arising in the framework. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Mathematical Aspects of Deep Learning 1	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : - Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. - Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. - Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> und <i>Stochastik</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Christoph Bohle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Riemann surfaces' due to the large overlap in content
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:
 Grading System : g=graded, ng=not graded
 Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
 Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
 Status : o=obligatory, f=facultative
 Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-16	Module Title: Riemannian Geometry				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• Riemannian manifolds. • Geodesics. • Curvature. • Geometry of submanifolds.									
Objectives	The students have learned and understood definitions and main examples of Riemannian manifolds from a classical point of view. In addition, topics related to geodesics were discussed. The students were exposed to important geometric results involving geodesics which are sufficient to study their role in different areas of differential geometry. Intuition for various notions of curvature was developed by the students and familiarity with computations in differential geometry was achieved in the exercise sessions. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
		Riemannian Geometry	L	f	2					
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • John M. Lee: Riemannian manifolds: An introduction to curvature. Springer 1997. • Barret O'Neill: Semi-Riemannian geometry. With applications to relativity. Academic Press 1983.									

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-17	Module Title: Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory)		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	Integrable systems are differential or difference equations with extraordinarily large symmetry group. The course will focus on equations related to the Korteweg de Vries (KdV) equation and discrete counterparts. Originally a mathematical model for the soliton phenomenon discovered during a famous horse ride along a canal, equations of KdV type have now many applications and the underlying theory involves various mathematical disciplines. A fundamental idea for understanding and solving KdV type equations is their interpretation as spectrum preserving deformations of underlying auxiliary linear operators - in the simplest case symmetric matrices. We study an important class of explicit solutions that includes solitons and finite gap (or algebro-geometric) solutions. This class of solutions can be described using a combination of Riemann surface theory and classical mechanics. The relevant parts of classical mechanics, Riemann surface theory, and spectral theory will be explained in the lecture. We will also briefly touch upon an integrable systems interpretation of the QR-algorithm of numerical linear algebra. The KdV equation is related to geometry in several different ways: for example, it can be interpreted as a dynamical system on the space of parametrized curves in the plane; it is deeply related to the geometry of Lie algebras and Lie groups; the special solutions discussed in the lecture are related to the geometry of Riemann surfaces... In a sequel to the lecture, it is planned to explain how infinite dimensional projective geometry allows to interpret generalizations of the KdV equation as quadratic equations and to finally linearize their dynamics.		
Objectives	The students have seen and understood relations between classical topics like Riemann surfaces, mechanics, and spectral theory – as well as other branches of mathematics – that were discovered mainly in the second half of the twentieth century during the emergence of a branch of mathematics sometimes called <i>soliton theory</i> or <i>integrable mathematics</i>. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Introduction to Integrable Systems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004. • Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003. • Alan C. Newell: Solitons in mathematics and physics. SIAM 1985. • Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-50-10 Geometry of Manifolds 1.									
Responsible Persons	Christoph Bohle, Frank Loose									

Abbreviations:
Grading System : g=graded, ng=not graded
Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

[illegible]

Literature	Possible References : • Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004. • Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003. • Alan C. Newell: Solitons in mathematics and physics. SIAM 1985. • Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus, Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-17 Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory).
Responsible Persons	Christoph Bohle, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-19	Module Title: Mathematical Aspects of Deep Learning 2					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Mechanisms behind (mathematical) reasoning in large language models (like GPT o1). Search and reinforcement learning. • Towards multimodal large language models: how to make large language models <i>see</i> and <i>draw</i> and allow comprehension and creation of images. • Uses of neural networks in applied mathematics (engineering) e.g. modeling with PDEs, simulation, inverse problems. For example physics informed neural networks, neural operators, 3d reconstruction from 2d images.									
Objectives	Students are familiar with some current problems in Deep Learning and with applications within or with relevance to Mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Mathematical Aspects of Deep Learning 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. • Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry, Analysis and Differential Geometry</i> und <i>Stochastik</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-14 Mathematical Aspects of Deep Learning 1.
Responsible Persons	Christoph Bohle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-20	Module Title: Topology			Type of Module: Compulsory Module with Choice						
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h		Time in Class: 30 h		Self-Study: 60 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.									
Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Topology	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry and Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Ulrich Groh, Rainer Nagel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-21	Module Title: Algebraic Topology 1				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.									
Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Topology	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Anton Deitmar, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-21 Algebraic topology 1.
Responsible Persons	Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-23	Module Title: Algebraic Topology 3					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	A selection of the following topics will be covered: • Basic concepts of homotopy theory; • Homotopy group of spheres; • Spectral sequences; • K-theory; • Characteristic classes.									
Objectives	With the in-depth knowledge of algebraic topology that they have acquired, students are now introduced to current areas of research and they tackle a small research project themselves, which can lead to a Master's thesis, for example. They will also lay the foundations for a possible doctorate in algebraic topology. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Algebraic Topology 3	L	f	2	3	no	P		g	100
	Specifics on the portfolio will be explained by the examiner at the beginning of the course.									
Literature	Possible References : • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Allen Hatcher: Vector bundles and K-theory. Manuskript 2009. • John W. Milnor, James D. Stasheff: Characteristic classes. Princeton University Press 1974. • John W. Milnor: Lectures on the h-cobordism theorem. Princeton University Press 1965.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-21 Algebraic topology 1, • MAT-50-22 Algebraic Topology 2.
Responsible Persons	Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14 and MAT-50-19.
Responsible Persons	Christoph Bohle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-27	Module Title: Topological Vector Spaces and Distributions				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	A selection of the following topics will be covered: • Locally convex topological vector spaces, Frechet spaces, LF spaces and LB spaces. • Duality: Hahn-Banach theorem, dual space, topologies on the dual space. • Generalised functions, Radon measures and distributions. • Properties of distributions and operations on the space of distributions. • Applications and examples.									
Objectives	Students master the basic principles and techniques of the theory of topological vector spaces and understand how to apply this to the theory of generalised functions according to L. Schwartz. Students are also able to name the main applications of the theory and show which classical questions of mathematical physics can be treated with it. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Topological Vector Spaces and Distributions	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									

Literature	Possible References : • Gerald Folland: Real Analysis. Wiley 1999. • Helmut H. Schäfer: Topological Vector Spaces. Springer 1999. • Laurant Schwartz: Theorie des Distributions. Hermann 1998. • Laurant Schwartz: Mathematics for the Physical Sciences. Dover 2008. • Francois Trèves: Topological Vector Spaces, Distributions and Kernel. Dover 1967.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry and Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Ulrich Groh, Rainer Nagel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-29	Module Title: Algebraic Curves and Riemann Surfaces		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Compact Riemann surfaces. • Normalisation of plane curves. • Topological genus. • Coverings. • Forms and integration. • Sheaves and cohomology. • Hodge theory. • Arithmetic and geometric genus. • Abel's theorem. • Riemann-Roch theorem. • Serre duality. • Jacobian and Abelian varieties. • Riemann bilinear relations. • Jacobi inverse problem. • Elliptic curves and functions. • j-Invariant. • Uniformisation. • Topology of non-compact Riemann surfaces.		
Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Curves and Riemann Surfaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.									
Literature	Possible References : • Frederice Mangolte: Real Algebraic Varieties. Springer 2020. • Robert Silhol: Real Algebraic Surfaces. Springer 1989. • Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Herrmann 1990. • Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.									
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module <i>Introduction to Riemann Surfaces</i> due to the large overlap in content.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.									
Responsible Persons	Daniele Agostini, Ivo Radloff									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-50-30	Module Title: Geometric Group Theory				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Group actions on graphs, free groups. • Quasi isometries. • Growth types. • Hyperbolic groups. • Ends.									
Objectives	Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometric Group Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Clara Löh: Geometric Group Theory - an Introduction. Springer 2017. • Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.									

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Joachim Kock, Israel Vainsencher: An invitation to quantum cohomology: Kontsevich's formula for rational plane curves. Birkhäuser 2007. • Ravi Vakil: The moduli space of curves and Gromov-Witten theory. Enumerative invariants in algebraic geometry and string theory. Lecture Notes in Mathematics, 1947. Springer 2008.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-50-50	Module Title: Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.									
Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra. The students are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : - Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. - Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. - Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.									

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures. Knowledge from the following modules is helpful but not strictly required: • MAT-20-06 Introduction to Geometry.
Responsible Persons	Hermann Hähl, Hannah Markwig
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus <i>Numerical Mathematics and Optimisation</i>.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Anton Deitmar, Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Melvyn Berger: Nonlinearity in Functional Analysis. Elsevier 1977. • Klaus Deimling: Nonlinear Functional Analysis. Springer 1985. • Eberhard Zeidler: Nonlinear Functional Analysis and its Applications I. Fixed-Point Theorems. Springer 1986.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-03	Module Title: Operator Theory					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Operator semigroups and abstract Cauchy problems. • Theorem of Hille-Yosida. • Applications of concrete evolution equations. • Spectral theory of semigroups and their generators. • Asymptotic of semigroups. • Applications: – Semigroups of ordinary and partial differential equations; – Semigroups of transport problems; – Semigroups of control theory.									
Objectives	Students have understood the concept of the operator-valued exponential function. They are able to deal with concrete evolution equations in this abstract form. They are able to prove well-posedness using the Hille-Yosida theorem and discuss the qualitative behaviour of solutions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Bruce Blackadar: Operator algebras. Springer 2006. • Klaus Jochen Engel, Rainer Nagel: One-parameter semigroups for linear evolution equations. Springer 2000. • Klaus Jochen Engel, Rainer Nagel: A short course on operator semigroups. Springer 2006. • Gert Pedersen: Analysis now. Springer 1995.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry and Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Rainer Nagel, Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-04	Module Title: Operator Algebras		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Geometry of Hilbert spaces. • Operators on Hilbert spaces and their spectral properties. • Spectral theory in Banach algebras. • Commutative Banach algebras and the representation theorem of Gelfand and Gelfand-Naimark. • The spectral theorem for normal operators of a Hilbert space. • Operator topologies and von Neumann's bicommutant theorem. • Kaplansky's density theorem. • Von Neumann algebras and their classification according to Murray-von Neumann, construction of examples. • The axiomatics of C^*- and W^*-algebras, the theorem of Gelfand-Naimark-Segal theorem for C^*-algebras and the representation theorem of Sakai for W^*-algebras. • Applications and outlook.		
Objectives	The students have familiarised themselves with the central concepts, results and methods of the theory of operator algebras. They have learnt the interplay between algebra and topology using the example of von Neumann algebras and their classification. The students also recognise how taking a higher point of view, i.e. the axiomatic nature of the problem, allows different questions to be dealt with and solved simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Algebras	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Bruce Blackadar: Operator algebras. Springer 2006. • Ola Bratelli, Derek Robinson: Operator Algebras and Quantum Physics. Springer 1997. • Richard Kadison, John Ringrose: Fundamentals of the Theory of Operator Algebras I - IV. AMS 1997. • Gert Pedersen: Analysis now. Springer 1995. • Shoichiro Sakai: C^*- and W^*-Algebras. Springer 1998. • Masamichi Takesaki: Theory of Operator Algebras I - II. Springer 2002.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.									
Responsible Persons	Ulrich Groh, Rainer Nagel									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Literature	Possible References : • Manfred Einsiedler, Thomas Ward: Ergodic Theory with a View Towards Number Theory. Springer 2011. • Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015. • Paul Halmos: Lectures on Ergodic Theory. Martino Fine Books 2013. • Marcelo Viana, Krenley Oliveira: Foundations of Ergodic Theory. CUP 2016.
Transfer	The module belongs to the <i>Study specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Rainer Nagel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-07	Module Title: Linear Control Theory			Type of Module: Compulsory Module with Choice						
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h							
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.									
Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Linear Control Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Rainer Nagel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-08	Module Title: Spectral Theory of Positive Operators					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	Starting from the classical theorems of Perron and Frobenius on the spectrum of positive matrices, positive linear mappings to C^* - and W^* -algebras and their spectral and algebraic properties are analysed. The ergodic properties of these operators, i.e. the convergence of powers and means, can then be derived from these. We then discuss the generalisation to operator semigroups. Applications of the theory can be found in mathematical physics, among others.									
Objectives	Students learn the basic spectral properties of positive operators on C^* - and W^* -algebras and the connections with non-commutative ergodic theory. In the seminar following the lecture, students can work on topics that lead to a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Spectral Theory of Positive Operators	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.									
Literature	Possible References : • Tanja Eisner, Markus Haase, Rainer Nagel : Operator Theoretic Aspects of Ergodic Theory. Springer 2015. • Ulrich Groh: Spectral Theory of Completely Positive Maps on C^*- and W^*-Algebras. Preprint.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Ulrich Groh, Rainer Nagel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-12	Module Title: Harmonic Analysis in Euclidean Space					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Fourier transformation. • Covering-, decomposition- and interpolation theorems. • Singular integrals, Poisson integrals. • Hardy- and BMO-spaces, multiplier theorems, Littlewood-Paley theory.									
Objectives	The students got to know the central terms, results and methods of harmonic analysis in euclidean space. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Harmonic Analysis in Euclidean Space	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Charles L. Feffermann, Elias M. Stein: H^p spaces of several variables. Acta Mathematica 129, pp. 137-193, 1972. • Christopher D. Sogge: Fourier integrals in classical analysis. Cambridge University Press 2017. • Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press 1970. • Elias M. Stein: Harmonic analysis. Princeton University Press 1993. • Elias M. Stein, Guido Weiss: Introduction to Fourier analysis on Euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-13	Module Title: Harmonic Analysis on Abelian Groups				Type of Module: Compulsory Module with Choice						
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h						
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German										
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS										
Content	• Locally compact groups, existence and uniqueness of Haar measures. • Convolution algebras, Banach algebras, the Gelfand-Neumark theorem. • LCA groups, Pontryagin duality, Plancherel theorem, structure theory of LCA groups.										
Objectives	The students have become familiar with the central concepts and methods of abstract harmonic analysis and know how to use them. They have understood the connection between topological/analytical/geometric concepts such as LCA groups and their expression in algebraic structures such as C^*-algebras and are able to apply this way of thinking to other theories. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Harmonic Analysis on Abelian Groups		L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
			E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.											
Literature	Possible References : • Anton Deitmar: A first course in Harmonic Analysis. Springer 2005. • Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008. • Edwin Hewitt, Kenneth Ross: Abstract harmonic analysis. Vol. I. Springer 1979. • Walter Rudin: Fourier analysis on groups. John Wiley 1990.										

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-14	Module Title: Harmonic Analysis on General Groups					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Representation theory of compact groups, Peter-Weyl theorem. • Representation theory of general groups. • trace formula and applications to the Heisenberg group and $SL_2(\mathbb{R})$.									
Objectives	The students have familiarised themselves with the deeper concepts and methods of abstract harmonic analysis and know how to use them. They have mastered the trace formula and understand its far-reaching implications, also for other areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Harmonic Analysis on General Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008. • Gerald B. Folland: A course in abstract harmonic analysis. Studies in Advanced Mathematics. Boca Raton 1995. • Michael E. Taylor: Noncommutative Harmonic Analysis. AMS 1986.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry and Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus <i>Numerical Mathematics and Optimisation</i>.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Gerhard Huisken, Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Luis Angel Caffarelli, Xavier Cabre: Fully nonlinear elliptic equations. American Mathematical Society 1995. • Michael G. Crandall, Hitoshi Ishii, Pierre-Louis Lions: User's Guide to Viscosity Solutions of second Order Partial Differential Equations. Bulletin of the American Mathematical Society 27, No. 1, pp. 1-67, 1992. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear elliptic equations.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry and Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Instationary Differential Equations, it can be included in the specialisation <i>Numerical Mathematics and Optimisation</i>.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken, Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Literature	Possible References : • Lawrence C. Evans: Partial Differential Equations. AMS 2010. • David Gilbarg, Neil Trudinger: Elliptic partial differential equations of second order. Springer 1083. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications. Siam 2000.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken

Abbreviations:
 Grading System : g=graded, ng=not graded
 Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
 Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
 Status : o=obligatory, f=facultative
 Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-25	Module Title: Introduction to Partial Differential Equations – Part 1			Type of Module: Compulsory Module with Choice						
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 105 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.									
Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Partial Differential Equations – Part 1	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry and Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. In combination with one of the modules 'Numerics of Stationary Differential Equations' or 'Numerics of Nonstationary Differential Equations', it can be included in the Specialisation <i>Numerical Mathematics and Optimisation</i>. The module is part of the module Introduction to Partial Differential Equations and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Gerhard Huisken, Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-26	Module Title: Partial Differential Equations in Conformal Geometry: the Yamabe Problem						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	The uniformisation theorem states that a closed two-dimensional Riemannian manifold is conformally equivalent to one with constant sectional curvature. The appropriate higher-dimensional analogue problem is finding a conformal metric with constant scalar curvature. This is the so-called Yamabe problem, introduced by Yamabe in 1960. The flaws in his proof were fixed only in 1984, by combining the works of Trudinger, Aubin, Schoen, and others. This course provides an overview of the complete proof of the Yamabe problem, which is extremely rich in techniques of calculus of variation, geometric analysis, and elliptic partial differential equations. It also employs crucial results of mathematical relativity, such as the Positive Mass Theorem, which is motivated by physics but possesses powerful analytical consequences. Moreover, basic results for non-compact manifolds or manifolds with boundary and the parabolic analogue, the Yamabe flow, will be discussed.									
Objectives	The students can state the Yamabe problem and explain the structure of the proof. The students are familiar with basic methods of the calculus of variation, particularly concerning the (sub-critical) Yamabe energy functionals and the associated elliptic semi-linear PDEs. They can study the conformal Green's function and relate it to the solvability of the Yamabe problem. The students are also familiar with the notion of mass in mathematical relativity and the renowned Positive Mass Theorem, still linking it to the solution of the Yamabe problem; they can provide the main ideas of the proof of the Positive Mass Theorem and, in particular, master elements of the minimal surface theory. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Partial Differential Equations in Conformal Geometry: the Yamabe Problem	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • John M. Lee, Thomas H. Parker: The Yamabe problem. Bull. Am. Math. Soc., New Ser. 17, 37–91, 1987. • Richard Schoen, Shing-Tung Yau: Lectures on differential geometry. International Press 1994. • Thierry Aubin: Some nonlinear problems in Riemannian geometry. Springer 1998. • Michael Struwe: Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems. Springer 2008.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Carla Cederbaum
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-27	Module Title: Fully Non-Linear Elliptic Equations				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h			Time in Class: 45 h			Self-Study: 105 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Solution of general fully non-linear equations with elliptic equations. • Solution of the Monge-Ampere equation.									
Objectives	The students learn the techniques to successively estimate the supremum, the gradient and the second derivatives of a given solution of a fully non-linear elliptic equation. The students learn how the modulus of continuity of the second derivatives is then estimated using the Evans-Krylov theorem and learn the continuity method, which leads to the existence of a solution. In particular, they are able to apply the methods to general uniformly elliptic equations and to the special, non-uniformly elliptic Monge-Ampere equation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Fully Non-Linear Elliptic Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.									
Literature	Possible References : • Luis A. Caffarelli, Joseph Kohn, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. I. Monge-Ampere equation. In: Communications on Pure and Applied Mathematics 37,3 pp. 369-402. • Luis A. Caffarelli, Joseph Kohn, Luis Nirenberg, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. II. Complex Monge-Ampere, and uniformly elliptic, equations. In: Communications on Pure and Applied Mathematics 38,2 pp. 209-252. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order. Springer 1998.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module MAT-60-36 'Fully nonlinear elliptic and parabolic partial differential equations' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-55-22 Partial Differential Equations.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Transfer	The module belongs to the <i>Study Specialisation Studienschwerpunkt Analysis and Differential Geometry</i> and <i>Algebra and Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-65-11 Geometry in Physics.
Responsible Persons	Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-34	Module Title: Introduction to Dynamical Systems					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.									
Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable? They are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introducation to Dynamical Systems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
Literature	Possible References : • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Dynamical Systems' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Responsible Persons	Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-41	Module Title: Introduction to Geometric Measure Theory			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.									
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Geometric Measure Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-42	Module Title: Geometric Measure Theory						Type of Module: Compulsory Module with Choice			
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• First and second variation for varifolds. • Monotonicity formula. • Allard's integral compactness theorem. • Lipschitz approximation. • tilt-excess descent. • Allard's regularity theorem. • General and rectifiable flows. • Deformation theorem. • Surface minimizing flows.									
Objectives	After having learned the basic terms and methods in introduction to geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Geometric Measure Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-43	Module Title: Area Minimising Flows					Type of Module: Compulsory Module with Choice				
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h			Time in Class: 45 h			Self-Study: 105 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Compactness theorem for integral flows. • Regularity of area minimising flows.									
Objectives	After having learned the essential notions and methods of geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Area Minimising Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory, • MAT-55-42 Geometric measure theory.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-44	Module Title: Introduction to Geometric Measure Theory – Measure Theoretic Methods				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 105 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.									
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Geometric Measure Theory – Measure Theoretic Methods	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

[illegible]

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-47	Module Title: Geometric Measure Theory – Varifolds				Type of Module: Compulsory Module with Choice						
ECTS-Points	4.5										
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 105 h						
Duration	1 Semester										
Frequency	not regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS										
Content	• First and second variation for varifolds. • Monotonicity formula. • Allard's integral compactness theorem. • Lipschitz approximation. • tilt-excess descent. • Allard's regularity theorem.										
Objectives	After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title		Type of Course	Status	SWS	ECTS	Coursework yes	Type of Exam wr. o. or.	Dur. of Exam (min) 90-180 o. 20-30	Grading g	Weight for Grade 100
	Geometric Measure Theory – Varifolds		L	f	2	3					
		E	f	1	1,5						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.											
Literature	Possible References : • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.										

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-48	Module Title: Geometric Measure Theory – Flows				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 105 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• General and rectifiable flows. • Deformation theorem. • Surface minimizing flows.									
Objectives	After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Geometric Measure Theory – Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-49	Module Title: Calculus of Variations					Type of Module: Compulsory Module with Choice				
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h			Time in Class: 45 h			Self-Study: 105 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.									
Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Calculus of Variations	L E	f f	2 1	3 1,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-01 Measure and Integration Theory, • MAT-20-04 Integration on Submanifolds of $\mathbb{R}^n$.
Responsible Persons	Anton Deitmar, Frank Loose
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-53	Module Title: Automorphic Forms				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 105 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.									
Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Automorphic Forms	L	f	2	3	yes	K o. mP o. H	90-180 o. 20-30	g	100
		E	f	1	1,5					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Deitmar, Anton: Automorphic Forms. Springer 2012. • Goldfeld, Dorian: Automorphic forms and L-functions for the group GL(n,R). Cambridge University Press 2015. • Serre, Jean-Pierre: A course in arithmetic. Springer 1973.									

Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. The module is part of the module Introduction to Geometric Measurement Theory and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Prerequisites	There are no prerequisites.
Responsible Persons	Anton Deitmar
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Transfer	The module belongs to the <i>Study Specialisation Algebra and Geometry and Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-63	Module Title: Introduction to set theory						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	Content: •									
Objectives	- The students are able to view mathematical theorems and theories in the context of mathematical logic. They understand the boundaries of the mathematically knowable, the difference between truth and provability, and are able to apply model-theoretic considerations to mathematical problems. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to set theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
Literature	Possible References : •									
Transfer	Mathematische Breitenbildung									
Prerequisites	There are no further prerequisites.									
Responsible Persons	Frank Loose									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-55-64	Module Title: Theory of Mathematical Proofs				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.									
Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
		Theory of Mathematical Proofs	L	f	2					
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Wolfram Pohlers. Proof Theory. Springer 2009.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-65	Module Title: Explicit Mathematics				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.									
Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and sub-systems of analysis and are familiar with their function in proof-theoretical investigations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Explicit Mathematics	L	f	2	3	yes	or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 6.									

Literature	Possible References : • Solomon Feferman: A language and axioms for explicit mathematics, in <i>Algebra and Logic</i>. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In <i>Logic Colloquium '78</i>, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, <i>Logic and Foundations of Mathematics</i>, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. <i>Textos de Matemática</i>, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. <i>Annals of Pure and Applied Logic</i>, 109(3), 141-162, 2001.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-55-71	Module Title: Operator Algebras and their Applications to Statistical Mechanics				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 3 SWS + Ex.cl. 1 SWS									
Content	- Basics on operator algebras (C*-algebras, algebraic states, inductive limits); - Introduction to algebraic deformation quantization (general set-up, coherent states, examples); - Applications to the classical limit of quantum mechanics and statistical mechanics including asymptotic emergence (phase transitions, large deviations (entropy), spontaneous symmetry breaking).									
Objectives	The students have obtained deepend knowledge in selections questions in algebraic quantum theory with an emphasis on algebraic deformation quantisation and their applications to the classical limit of quantum mechanics and statistical mechanics. They have learned algebraic techniques in order to develop abstract structures encoding the features of a physical theory. They are familiar with techniques to prove existence results of limits of sequences/nets encoded by algebraic states, examine these, and put them into a general perspective. Moreover, they understand the physical relevance of the results and are able to relate them to features of equilibrium thermodynamics, such as phase transitions and spontaneous symmetry breaking. They are able to describe the current state of research in the specific area. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Operator Algebras and their Applications to Statistical Mechanics	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : Klaas Landsman: Foundations of Quantum Theory, From Classical Concepts to Operator Algebras. Springer 2017.									

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry, Mathematical Physics</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-01	Module Title: Geometric Evolution Equations						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Examples of geometric evolution equations such as Mean curvature flow, Ricci flow, Inverse mean curvature flow. • Parabolic maximum principles. • Regularity theory for parabolic evolution equations. • Rescaling techniques and description of singularities. • Asymptotic behavior of solutions.									
Objectives	Students learn to combine their knowledge of differential geometry and partial differential equations and apply it to specific problems in selected geometric evolution equations. They learn techniques for checking solutions of non-linear evolution equations, which enables them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Geometric Evolution Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Nonstationary Differential Equations or Numerics of Differential Equations on Surfaces, it can be included in the subject specialisation <i>Numerical Mathematics and Optimisation</i> .									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.									

Responsible Persons	Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-02	Module Title: Geometric Variation Problems			Type of Module: Compulsory Module with Choice						
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h		Self-Study: 60 h						
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Examples of geometric variation problems such as minimal surfaces, capillary surfaces, harmonic mappings and associated boundary value problems. • Direct methods of calculating variations. • Regularity theory for solutions of variational problems. • Relationship between variational problems and partial differential equations. • Stability properties of solutions.									
Objectives	Students learn to combine their knowledge of differential geometry and analysis and apply it to specific problems in selected geometric variational problems. They learn techniques for proving solutions to various variational problems and for analysing the properties of solutions, which provide a basis for independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Geometric Variation Problems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.									
Responsible Persons	Carla Cederbaum, Gerhard Huisken									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-03	Module Title: Topics in Mathematical Relativity				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• A selection of concrete models of Mathematical Relativity, such as black holes, static metrics, physical invariants of isolated systems, positivity estimates for energy and mass. • Geometric and analytical structure of the models, existence and properties of concrete models as solutions to Einstein's equations.									
Objectives	Students acquire in-depth knowledge of selected issues in the mathematical theory of relativity. They learn analytical and geometric techniques for proving and investigating solutions to Einstein's equations and are able to categorise the physical relevance of the mathematical results. The lecture introduces students to their first independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Topics in Mathematical Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 6.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-13 Mathematical Relativity.									
Responsible Persons	Carla Cederbaum, Gerhard Huisken									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-04	Module Title: Space-Like Hypersurfaces in Lorentzian Manifolds			Type of Module: Compulsory Module with Choice						
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h							
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	Space-like hypersurfaces of a Lorentzian manifold play a crucial role in the study of solutions to Einstein’s equations modelling some phenomenon in General Relativity. The course explores how geometric choices of space-like hypersurfaces such as maximal surfaces, constant mean curvature surfaces or solutions of mean curvature flow and inverse mean curvature flow can be used to achieve a splitting of space and time that is suitable for the study of both isolated gravitating systems and cosmological spacetimes.									
Objectives	The students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Space-Like Hypersurfaces in Lorentzian Manifolds	L E	f f	2 2	3 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Barrett O'Neill: Semi-Riemannian Geometry - With applications to Relativity. Academic Press 1983. • Andrejs E. Treibergs: Entire space-like hypersurfaces of constant mean curvature in Minkowski space. <i>Inventiones Math.</i> 66, (1982) 39–56. • Klaus Ecker, Gerhard Huisken: Parabolic methods for the construction of spacelike slices of prescribed mean curvature in cosmological spacetimes. <i>Comm. Math. Phys.</i> 135 (1991), 595–613. • Helmut Friedrich, Alan Rendall: The Cauchy Problem for the Einstein Equations. In: Schmidt B.G. (eds) <i>Einstein's Field Equations and Their Physical Implications</i>. Lecture Notes in Physics, vol 540. Springer 1999. • Hans Ringström: The Cauchy Problem in General Relativity. European Math. Society 2009.
Transfer	The module belongs to the <i>Study Specialisation Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-13 Mathematical Relativity.
Responsible Persons	Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-06	Module Title: The Ricci Flow of Riemannian Metrics					Type of Module: Compulsory Module with Choice				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	The lecture introduces the basic properties of the Ricci flow and develops the necessary techniques, e.g. tensor maximum principles and regularity estimation. The long-term existence of solutions and resulting classifications for metrics of positive curvature are presented. Finally, the monotonicity of functionals according to Perelman is derived and used for the classification of possible singularities, with an outlook on the surgery methods of Hamilton and Perelman, which have led to the proof of the Poincaré and geometrisation conjectures.									
Objectives	The students have learnt basic methods for the treatment of geometric evolution equations in Riemannian geometry. At the same time, they have experienced the interplay of local geometric assumptions on the curvature properties of a metric with analytic techniques for the study of parabolic equations and have learnt and understood how the local assumptions have global consequences for the geometry and topology of the underlying manifolds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	The Ricci Flow of Riemannian Metrics	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 6.									
Literature	Possible References : • Simon Brendle: Ricci-flow and the sphere theorem. AMS 2010. • Peter Topping: Lectures on the Ricci-Flow. Lecture Notes 2006. • Richard Hamilton: Riemannian 3-manifolds with positive Ricci curvature. J. Diff. Geom. 17, 1982.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-07	Module Title: Special Relativity						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Derivation of the Minkowski metric from basic physical assumptions. • Physical consequences of relativity such as length contraction, time dilation and some popular paradoxes.									
Objectives	Students have learnt and understood the derivation of the special theory of relativity and important concepts such as length contraction and time dilation. They are familiar with important paradoxes that arise. Students have developed an intuition for various aspects of the theory of relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Special Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Albert Einstein: Relativity: the special and general theory. Public domain 1920. • Thomas A. Moore: Six ideas that shaped physics: unit R. McGraw-Hill 2003. • Robert Resnick: Introduction to Special Relativity. Wiley 2007. • Bernard Schutz: A First Course in General Relativity. Cambridge University Press 2009.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-08	Module Title: Null Geometry in General Relativity			Type of Module: Compulsory Module with Choice						
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h							
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	This module provides an introduction to null geometry. Topics include the properties of light-like vector fields and curves, as well as the geometry of light-like hypersurfaces that carry a degenerate induced metric. Another major topic is the extrinsic curvature of space-like surfaces in higher codimension, which are considered in particular along light-like hypersurfaces. Optionally, geometric flows along light-like hypersurfaces can also be treated.									
Objectives	Students know and understand the concepts and methods mentioned and can use them to analyse known and new questions from null geometry. Furthermore, they link physical problems in cosmology and astrophysics and their mathematical modelling using differential geometric methods and are able to question the relevance and adequacy of mathematical modelling and the mathematical results derived from it. In particular, they expand on the methods learnt in the MAT-65-11 module and connect their methodological and specialist knowledge. They are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Null Geometry in General Relativity	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.									
Literature	Possible References : • Barrett O'Neill: Semi-Riemannian Geometry. Academic Press 1983. • Johannes Sauter: Foliations of Null hypersurfaces and the Penrose Inequality. Dissertation (ETH Zürich), url: https://www.research-collection.ethz.ch/handle/20.500.11850/150826.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-09	Module Title: The Einstein Constraint Equations		Type of Module: Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Introduction to GR and the Einstein equations: – The Einstein equations, special solutions and geometric constructions; – The Cauchy problem for the Einstein equations. • The constraint equations and the conformal method: – The conformal method; – Overview of the elliptic theory on closed manifolds; – Classification of constant mean curvature on closed manifolds. • Asymptotically Euclidean (AE) initial data: – AE manifolds and elliptic operators; – Constructions of AE initial data sets.		
Objectives	Students can use conformal methods to transform Einstein's constraints into a system of elliptic partial differential equations and thus describe parts of the solution spaces of Einstein's equations and analyse properties of the associated solutions. They have learnt about connections between the theory and questions of geometric analysis such as the scalar curvature problem and the Yambe problem and are familiar with the interplay of methods of Riemannian geometry, geometric analysis and physics for answering questions of general relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	The Einstein Constraint Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.									
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics, • MAT-50-16 Riemannian Geometry. Knowledge from the following modules is helpful but not strictly required: • MAT-55-21 Introduction to Partial Differential Equations.									
Responsible Persons	Carla Cederbaum									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-10	Module Title: Special Topics in Evolution Equations for Submanifolds (with Exercise Class)							Type of Module: Compulsory Module with Choice		
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.									
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Special Topics in Evolution Equations for Submanifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> und <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-11	Module Title: Special Topics in Evolution Equations for Submanifolds (without Exercise Classes)						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.									
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Special Topics in Evolution Equations for Submanifolds	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: - MAT-55-21 Introduction to Partial Differential Equations, - MAT-65-11 Geometry in Physics.									
Responsible Persons	Gerhard Huisken									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-12	Module Title: The Positive Mass Theorem					Type of Module: Compulsory Module with Choice				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	This course revolves around fundamental theorems in Geometric Analysis, which have applications in Mathematical Relativity. The center of gravity is the so called Positive Mass Theorem, which basically says that non-negative matter density implies non-negative total mass. We discuss the (by now) classical proofs using minimal surfaces and more recent proofs based on level sets of carefully chosen functions, which arise as solutions to an elliptic partial differential equation. Interesting follow-up questions such as rigidity and stability of the case of zero or near zero total mass will be a substantial part of the lecture.									
Objectives	The students acquire advanced knowledge of selected topics in mathematical relativity. They learn analytical and geometrical techniques that are used in the Positive Mass Theorem and related results. Through the lecture, students are guided towards their first independent academic research, for example in the context of a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	The Positive Mass Theorem	L E	f f	2 2	3 3	no	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.									
Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.									
Responsible Persons	Carla Cederbaum									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-35	Module Title: Non-Linear Elliptic and Parabolic Partial Differential Equations			Type of Module: Compulsory Module with Choice						
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h		Self-Study: 120 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• Semilinear and quasilinear elliptic and parabolic partial differential equations; • Minimum surface operator and surfaces of prescribed mean curvature; • Parabolic geometric equations, e.g. flow along the mean curvature; • Hölder continuity according to De Giorgi and Nash; • Inner regularity and boundary regularity of solutions.									
Objectives	Students have learnt analytical methods that are central to the treatment of non-linear partial differential equations of second order of the elliptic and parabolic type. Using concrete examples of partial differential equations from mathematical physics and differential geometry, techniques were learnt to prove the existence and regularity of solutions to such equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes, students have acquired confidence in the technical handling of the methods they have learnt and can apply them independently to other problems. They are able to present their problem solutions and participate in discourses on problems in this field of research.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Non-Linear Elliptic and Parabolic Partial Differential Equations	L E	f f	2 2	3 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptic PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Fritz John: Introduction to Partial Differential Equations. Springer 1982. • Jürgen Jost: Partielle Differentialgleichungen. Springer 1998. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications, Pure and Applied Mathematics, Vol. 88. Academic Press 1980.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry and Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-60-36	Module Title: Fully Non-Linear Elliptic and Parabolic Partial Differential Equations					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	The lecture investigates fully non-linear elliptic and parabolic partial differential equations of second order. Classical examples are the Monge-Ampère equation, the equation of prescribed Gaussian curvature or, more general, equations of prescribed other scalar invariants of curvature together with their parabolic analogues. They also arise in problems of stochastic control and optimal transport. The course establishes basic techniques for solving Dirichlet- and Neumann boundary value problems for such equations, in particular techniques for deriving the necessary a priori estimates for solutions.									
Objectives	The students have learnt analytical methods that are central for the treatment of strongly non-linear partial differential equations of the elliptic and parabolic type. Using concrete examples of such differential equations, techniques were learnt to prove the existence and regularity of solutions of such equations and the associated boundary value problems. Students can apply the methods they have learnt to other problems and related equations independently. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Fully Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptic PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Ilya J. Bakelman: Convex functions and nonlinear geometric elliptic equations. Springer 1994. • Luis Caffarelli, Xavier Cabré: Fully nonlinear elliptic equations. AMS 1995.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'MAT-55-27 Fully Nonlinear Elliptic Equations' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Gerhard Huisken
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-05	Module Title: Groups and Representations		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. • Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. • Applications: symmetries and degeneracies in quantum mechanics, selection rules. • Representations of finite groups: group algebra, regular representation, ideals, idempotents. • Symmetric groups: Young tableaux, Young operators, dimensions and characters. • Applications: identical particles in quantum theories. • Lie groups: Haar measure, representations, Lie algebras. • Tensor representations of classical groups: symmetry classes, Young tableaux. • Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour) • Moreover a selection of the following: – Irreducible representations of the Lorentz and Poincaré groups. – Applications: notion of particles in quantum theories. – Roots and weights; Killing-Cartan classification of semi-simple Lie algebras		
Objectives	The students know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deep understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Groups and Representations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : - Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. - Barry Simon: Representations of Finite and Compact Groups. AMS 1996. - Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.									
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Mathematical Physics</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Representation Theory of Finite Groups' due to the large overlap in content.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: MAT-10-12 / MAT-20-06 Algebraic Structures.									
Responsible Persons	Stefan Keppeler									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-65-31	Module Title: Operator Algebras in Mathematical Physics				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to operator algebras and their role in mathematical physics. Particular topics are C^* -algebras and the GNS construction, von Neumann algebras and the double commutant theorem, and operator algebras and quantum theory. The mathematical theory is developed in parallel with applications in mathematical physics, most importantly the dynamics and statistical physics of extended many-body quantum systems.									
Objectives	The students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from operator algebras. They are able to interrelate fundamental physical concepts with their mathematical modelling via operator algebra methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the earlier semesters, in particular on functional analysis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Algebras in Mathematical Physics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Kenneth R. Davidson: Functional Analysis and Operator Algebras. Springer 2025. • Ola Bratelli, Derek W. Robinson: Operator Algebras and Quantum Statistical Mechanics 1. Springer 1987. • Ola Bratelli, Derek Robinson: Operator Algebras and Quantum Statistical Mechanics 2: Springer 1997. • Masamichi Takesaki: Operator Algebras I. Springer 1979.									
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-55-01 Functional Analysis or MAT-65-12 Mathematical Quantum Theory.
Responsible Persons	Stefan Teufel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-38	Module Title: Hamiltonian Systems			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	The module provides an introduction to the theory of Hamiltonian systems as used in classical mechanics. This builds a bridge between the fields of differential geometry, symplectic geometry and dynamical systems as well as theoretical physics. The main points of the lecture are: • Symplectic manifolds and the canonical 1-form of the cotangent bundle. • Darboux-Moser theorem. • Lagrangian and Hamiltonian systems. • Integrable systems and Arnold-Liouville theorem. • Moment mappings. • Symplectic reduction. • Symplectic manifolds and toric effects.									
Objectives	The students are familiar with the theory of Hamiltonian systems and their investigation using methods of symplectic geometry. They are familiar with the interplay of methods and questions of different areas of mathematics (differential geometry, geometry, dynamical systems) and theoretical physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Hamiltonian Systems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 1989. • Ana Cannas da Silva: Lectures on symplectic geometry. Springer 2001.
Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-39	Module Title: Propagation of Chaos					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS									
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.									
Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Propagation of Chaos	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		?	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. • Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.									
Transfer	The module belongs to the <i>Study Specialisations Mathematical Physics</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Peter Pickl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-40	Module Title: Interacting Many-Body Quantum Systems				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Bosons and fermions. • Bose-Einstein condensation. • Mean-field theory. • Nonlinear Schrödinger equation. • Hartree- and Hartree-Fock theory. • Bogoliubov approximations.									
Objectives	Students have learned about the difference between Fermions and Boson and its impact on many body systems. They understand that, at cold temperatures, the fluctuations of the particle number in a certain area is significantly suppressed for Fermions. They understand how the Bosonic nature leads to the famous effect of Bose Einstein condensation and that, in certain scaling regimes, interacting many body systems can in good approximation be described by nonlinear evolution equations. Finally they are able to give higher order corrections and to calculate the leading order correlations between the particles. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Propagation of Chaos	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, Jakob Yngvason: The Mathematics of the Bose Gas and its Condensation. Birkhäuser 2000. • Niels Benedikter , Marcello Porta , Benjamin Schlein: Effective Evolution Equations from Quantum Dynamics. Springer 2016.									

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry and Mathematical Physics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Peter Pickl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-65-41	Module Title: Fredholm Index Theory for Condensed Matter and Introduction to Spin Systems		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Reminder on compact and trace class operators in infinite dimensional Hilbert spaces. • Fredholm index theory in Hilbert spaces: definitions and proofs of stability of the index and homotopy invariance. • The Fredholm index as a difference of traces. • Introduction to the physics of the Integer Quantum Hall Effect and application of Fredholm index theory. • Definition and properties of the Bott index of two unitary matrices and application to the index theory of the Integer quantum Hall effect on a two-dimensional torus. • The bulk-boundary correspondence for chiral topological system in one dimension. • Tensor products of finitely many finite dimensional Hilbert spaces and introduction to the main models of spin systems. • Lieb-Robinson bounds.		
Objectives	Students have learned about operators in infinite dimensional Hilbert spaces with finite dimensional kernel and finite codimension (Fredholm operators), their index and its stability (topological invariance). They know the meaning of an index theorem in the context of condensed matter physics, that relates a physically measurable quantity to an integer (in certain units), and are able to prove it for the Integer Quantum Hall Effect in two dimensions. The students have learned about the equality of the topological indexes defined at the boundary and in the bulk of a one-dimensional systems of non-interacting particles with chiral. They recognize the main models of spin 1/2 system: Ising model, XY and XXZ models and can sketch their phase diagrams. The students can discuss the meaning of the dynamical spread of information in a spin system and prove the associated Lieb-Robinson bounds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Module Number: MAT-70-01	Module Title: Algorithms of Numerical Mathematics				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calculation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.									
Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
		Algorithms of Numerical Mathematics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Christian Lubich, Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-02	Module Title: Numerics of Stationary Differential Equations					Type of Module: Compulsory Module with Choice					
ECTS-Points	9										
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h				
Duration	1 Semester										
Frequency	regularly										
Term	1-3										
Language of Instruction	German or English										
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS										
Content	Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.										
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.										
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
			L	f	4	6					
	Numerics of Stationary Differential Equations		E	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.											
Literature	Possible References : • Dietrich Braess: Finite Elemente. Springer Spektrum 2013. • Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.										
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.										

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Christian Lubich
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Responsible Persons	Christian Lubich
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

[illegible]

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-06	Module Title: Numerics of Differential Equations of Surfaces				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h			Self-Study: 120 h				
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	• Numerical treatment of differential equations on moving (or stationary) surfaces. • Semi- and fully discretization of elliptic and parabolic equations on surfaces using surface finite elements and efficient time integrators. • Implementation of the algorithms.									
Objectives	Students have learned the basic methods and techniques of numerics for problems on (moving) surfaces. In particular, they are familiar with the discussed energy techniques, which are very strong, general and rich in application, even in surface-independent areas of numerics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Numerics of Differential Equations of Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Gerhard Dziuk: Finite elements for the Beltrami operator on arbitrary surfaces. 1988. • Gerhard Dziuk, Charles M. Elliott: Finite elements on evolving surfaces. 2007.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Christian Lubich
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-11	Module Title: Stochastic Differential Equations				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Stochastic processes, filtrations, martingales. • Wiener process, random walk, Donsker's theorem. • Diffusion semigroup, Ito's integral. • Solution of a stochastic differential equation. • Markov property, Malliavin calculus, rough path theory.									
Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Stochastic Differential Equations	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Bernt Oksendal: Stochastic differtial equations. Springer 2000.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-12	Module Title: Introduction to Stochastic Differential Equations - Part 1				Type of Module: Compulsory Module with Choice					
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	• Introduction to Brownian motion and stochastic integration. • Solution concepts for stochastic differential equations. • Stability of stochastic differential equations. • Numerical approximation of stochastic differential equations.									
Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Stochastic Differential Equations - Part 1	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Bernt Oksendal: Stochastic differential equations. Springer 2000.									
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation and Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-15	Module Title: Numerics of Stochastic Differential Equations					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.									
Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Numerics of Stochastic Differential Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.									
Responsible Persons	Andreas Prohl									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-20	Module Title: Introduction to Optimisation			Type of Module: Compulsory Module with Choice						
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h		Self-Study: 120 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 3 SWS + Ex.cl. 1 SWS									
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.									
Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Optimisation	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.									

Transfer	The module belongs to the <i>Study Specialisation Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Christian Lubich
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-21	Module Title: Non-Linear Optimisation					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method. • Restricted optimisation, Farkas' lemma, tangent cone. • Abadie CQ, KKT conditions, Slater conditions. • Linear programme, duality, simplex method. • Penalty and barrier methods, interior point method. • Nonlinear programs, SQP methods, non-smooth optimisation.									
Objectives	Students master the basic principles and techniques of analysis and numerics of constrained optimisation problems. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Non-Linear Optimisation	L E	f f	4 2	6 3	yes	or.	20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i>. Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i>, <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i>, in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-22	Module Title: Optimisation with Differential Equations				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Direct method in the calculus of variations, Euler-Lagrange equation. • Brouwer-Minty theorem, non-linear evolution equations. • Gateaux and Frechet differentiability. • Proof of existence of optimal controls, necessary optimality conditions. • Adjoint, convergent optimisation methods in Banach spaces. • Variational discretisation concepts.									
Objectives	Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Optimisation with Differential Equations	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.									

Transfer	The module belongs to the <i>Study Specialisations Analysis and Differential Geometry</i> and <i>Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-30	Module Title: Theoretical Aspects of Machine Learning				Type of Module: Compulsory Module with Choice					
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h		Self-Study: 120 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	The lecture covers some recent aspects of theoretical machine learning such as: • The theory of Reproducing Kernel Hilbert Spaces (RKHS). • Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings. • Approximation capabilities of neural networks. • Dynamics of neural networks and the neural tangent kernel. • Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.									
Objectives	The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Theoretical Aspects of Machine Learning	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
E		f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.										

Literature	Possible References : • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation and Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-31	Module Title: Statistical Learning 1			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.									
Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Statistical Learning 1	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.									

Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-32	Module Title: Statistical Learning 2						Type of Module: Compulsory Module with Choice																										
ECTS-Points	9																																
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h																										
Duration	1 Semester																																
Frequency	regularly																																
Term	1-3																																
Language of Instruction	German																																
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS																																
Content	- The uniform law of large numbers on function classes (Vapnik-Chervonenkis theory). - Abstract (strong) consistency theory for <i>least-squares</i> regression estimators on (approximating) function classes. - Examples, in particular the <i>data dependent partitioning</i> estimator and the <i>least squares neural networks</i> estimator. - Rate convergence for <i>least-squares</i> estimators.																																
Objectives	Students are familiar with in-depth methods of stochastic learning and their analysis, as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.																																
Requirements for obtaining Credits / Grading (Weighting if applicable)	<table border="1"> <thead> <tr> <th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr> </thead> <tbody> <tr> <td rowspan="2">Statistical Learning 2</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr> <tr> <td>E</td><td>f</td><td>2</td><td>3</td></tr> </tbody> </table> In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade	Statistical Learning 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100	E	f	2	3
Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade																								
Statistical Learning 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100																								
	E	f	2	3																													
Literature	Possible References : Laslo Györfi, Michael Kohler, Adam Krzyżak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.																																
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation and Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.																																

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-70-31 Statistical Learning 1.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-33	Module Title: Theory and Numerics for Constrained Optimisation Problems				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.									
Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity. he students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Theory and Numerics for Constrained Optimisation Problems	L E	f f	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.									
Literature	Possible References : • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.									

Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-34	Module Title: Mathematical Introduction to Data Science			Type of Module: Compulsory Module with Choice						
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h		Self-Study: 105 h						
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	The lecture provides a mathematically rigorous survey of core data-science techniques from regression and k-nearest neighbours to high-dimensional phenomena, dimensionality reduction, and modern classifiers (Perceptron, SVMs, Kernel Methods, Neural Networks). Emphasis is put on both theoretical foundations and hands-on Python implementation with real datasets.									
Objectives	The students can formulate and solve linear, polynomial and logistic regression models. They are able to implement k-nearest neighbours classification and build and evaluate classifiers (Perceptron, SVM (kernel methods), Neural Networks). Moreover, they know how to optimise convex objectives using gradient-descent techniques. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Mathematical Introduction to Data Science	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.									
Literature	Possible References : • Sven A. Wegner: Mathematical Introduction to Data Science. Springer 2024.									
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-35	Module Title: Optimal Transport			Type of Module: Compulsory Module with Choice						
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• The course introduces the theory of optimal transport at the interface of analysis, probability, and optimisation. Starting from the classical problems of Monge and Kantorovich, the fundamental measure-theoretic and analytical tools are developed. • Building on this, central results are presented, including the existence and duality of optimal transport plans, c-cyclical monotonicity, Brenier's theorem, and the geometric structure of Wasserstein spaces. • In the second part of the course, the algorithmic perspective of optimal transport is explored. Topics include entropic regularisation, the Sinkhorn algorithm, and selected applications, for example in image processing, statistics, and machine learning.									
Objectives	The students are familiar with the fundamental concepts, methods, and key results of the theory of optimal transport and are able to situate these within a broader mathematical context spanning analysis, probability, and optimisation. They can understand the classical formulations due to Monge and Kantorovich, as well as essential structural concepts such as duality, c-cyclical monotonicity, and the geometry of Wasserstein spaces, and apply them in simple settings. Furthermore, they are acquainted with basic algorithmic approaches to optimal transport and can assess their relevance for selected applications. In the exercise classes, they deepen their understanding of the lecture material, further develop their ability to construct precise mathematical arguments, and practise the written and oral presentation of their own solution methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Optimal Transport	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3						
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Alessio Figalli, Federico Glaudo: An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. EMS 2023. • Gabriel Peyré, Marco Cuturi: Computational Optimal Transport. Now Publishers 2019.
Transfer	The module belongs to the <i>Study Specialisation Analysis, Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 Submodules Measure and Integration Theory, and Integration on Submanifolds of $\mathbb{R}^n$. • MAT-20-12 Stochastics.
Responsible Persons	Nicolai Jork, Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-70-40	Module Title: Game Theory						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 2 SWS									
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.									
Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Game Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.									
Transfer	The module belongs to the <i>Study Specialisation Numerical Mathematics and Optimisation</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.									
Responsible Persons	Andreas Prohl									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Module Number: MAT-70-51	Module Title: Financial Mathematics and Numerics						Type of Module: Compulsory Module with Choice			
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations. Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.									
Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Financial Mathematics and Numerics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.									
Literature	Possible References : Steven Shreve: Stochastic Calculus for Finance. Springer 2005.									

Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-11 / MAT-20-13 Numerical Mathematics, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-01	Module Title: Probability Theory				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly in Winter Semester									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in continuous time like Poisson processes and Brownian motion.									
Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interpret stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Probability Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2019. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2021. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Martin Möhle, Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-03	Module Title: Mathematical Statistics				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Statistical models, exponential families, sufficient statistics. • Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. • Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. • Hypothesis testing, confidence interval, Neyman-Pearson lemma. • Testing methods, UMPU tests, 1- and 2-sample tests. • Models with growing density quotients, non parametric models. • Introduction in regression and variance analysis.									
Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									

Literature	Possible References : • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics. Knowledge from the following modules is helpful but not strictly required: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle, Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-04	Module Title: Stochastic Processes				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h		Time in Class: 90 h		Self-Study: 180 h					
Duration	1 Semester									
Frequency	regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	Stochastic processes in continuous time, such as • Markov processes; • Martingale; • Brownian motion, Poisson processes and general Levy processes; • Gaussian processes. Among other things, existence and convergence statements as well as path properties of these processes are analysed.									
Objectives	The students have learnt the central concepts, results, methods and examples of the theory of stochastic processes in continuous time and can handle them mathematically. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Stochastic Processes	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Joseph L. Doob: Stochastic Processes. Wiley 1990. • Olav Kallenberg: Foundations of Modern Probability. Springer 2021. • Samuel Karlin, Howard Taylor: A First Course in Stochastic Processes. Academic Press 1975. • Samuel Karlin, Howard Taylor: A Second Course in Stochastic Processes. Academic Press 1981. • Götz Kersting, Anton Wakolbinger: Stochastische Prozesse. Birkhäuser 2014. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • James R. Norris: Markov Chains. Cambridge University Press 1997.
Transfer	The module belongs to the <i>Study Specialisation</i> Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle, Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-06	Module Title: Stochastic Analysis				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Martingales and stopping times in continuous time. • Doléans measure, compensator, Doob-Meyer decomposition. • Stochastic integral for square integrable martingales (in particular for non-continuous martingales). • Semimartingales, transformation of stochastic integrals. • Itô formula (in particular for processes with jumps). • Stochastic differential equations .									
Objectives	The students know the main notions, results, methods and examples from stochastic analysis and they know how to handle them. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Stochastic Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Fabrice Baudoin: Diffusion Processes and Stochastic Calculus. EMS 2014. • Kai Lai Chung and Ruth J. Williams: Introduction to Stochastic Integration. Birkhäuser 1990. • Richard Durrett: Stochastic Calculus. CRC Press 2006. • Albrecht Irle: Finanzmathematik. Teubner 2003. • Ioannis Karatzas, Steven Shreve: Brownian Motion and Stochastic Calculus. Springer 1991. • Michel Métivier: Semimartingales. De Gruyter 1982. • Bernt Oksendal: Stochastic Differential Equations. Springer 2007. • Nicolas Privault: Stochastic Analysis in Discrete and Continuous Settings. Springer 2009. • Daniel Revuz, Marc Yor: Continuous Martingales and Brownian Motion. Springer 1999. • Heinrich von Weizsäcker, Gerhard Winkler: Stochastic Integrals, Vieweg 1990.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Jean Bertoin: Random Fragmentation and Coagulation Processes. Cambridge 2006. • Stewart N. Ethier, Thomas G. Kurtz: Markov Processes. Wiley 1986. • Warren J. Ewens: Mathematical Population Genetics. Springer 2004. • Jim Pitman: Combinatorial Stochastic Processes. LNM 1875. Springer 2006. • John Wakeley: Coalescent Theory. Roberts & Company Publishers 2008.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Literature	Possible References : • Daryl John Daley, David Vere-Jones: An Introduction to the Theory of Point Processes. Springer 2008. • Martin Jacobsen: Point Process Theory and Applications. Birkhäuser 2006. • Olav Kallenberg: Foundations of Modern Probability. Springer 2002. • John F. C. Kingman: Poisson Processes. Clarendon Press 1993. • Günter Last, Mathew D. Penrose: Lectures on the Poisson Process. Cambridge 2016.
Transfer	The module belongs to the <i>Study Specialisation Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-10	Module Title: Graph Theory					Type of Module: Compulsory Module with Choice				
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.									
Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Graph Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Transfer	The module belongs to the <i>Study Specialisations Algebra and Geometry</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-20	Module Title: Probability Distances for Data Science			Type of Module: Compulsory Module with Choice						
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h		Time in Class: 60 h		Self-Study: 120 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS									
Content	We study different concepts of distances between probability measures aimed at applications in data science. The classes of distances which are studied include optimal transport distances, f-divergences and integral probability metrics. The focus is on fundamental mathematical properties of these distances, like duality, famous inequalities, geometric aspects, and quantisation. Several applications in the area of data science and machine learning are illustrated throughout, for instance related to clustering, autoencoders, GANs, image processing, and compression.									
Objectives	Students are familiar with commonly used distances on the space of probability measures, particularly optimal transport distances, divergences, and integral probability metrics. They understand key mathematical results in this area, for instance related to duality, geometric aspects, and quantisation, as well as the interplay between different distances. They have further obtained an understanding of computational aspects and applicability in selected areas of data science. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed. In the exercises, they have developed a confident, precise and independent approach to the concepts, statements and methods from the lecture. They have learned to transfer the methods to new problems, to analyse them and to develop solution strategies alone or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Probability Distances for Data Science	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									

Literature	Possible References : • Gabriel Peyre, Marco Cuturi: Computational optimal transport: with applications to data science. Foundations and Trends in Machine Learning 11.5-6 (2019): 355-607. • Alison L. Gibbs, Francis Edward Su: On choosing and bounding probability metrics. International Statistical Review 70.3 (2002): 419-435. • Cedric Villani: Topics in optimal transportation. American Mathematical Society, 2003. • Imre Csiszar, Paul C. Shields: Information theory and statistics: a tutorial. Foundations and Trends in Communications and Information Theory 1.4 (2004). 417-528. • Ily Tolstikhin et al.: Wasserstein auto-encoders. 6th International Conference on Learning Representations (ICLR 2018) • Siegfried Graf, Harald Luschgy: Foundations of quantization for probability distributions. Springer, 2007.
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Stephan Eckstein
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-75-21	Module Title: Bayesian Networks and Causality		Type of Module: Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as <i>Whenever the sprinkler is on in my greenhouse, the plants are wet</i> or <i>Whenever the plants are wet, the sprinkler is on</i>, to postulating a relationship of cause and effect: <i>If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler.</i> In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.		
Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Bayesian Networks and Causality	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	1	1,5					
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.									
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: - MAT-20-01 Measure and Integration Theory, - MAT-20-12 Stochastics.									
Responsible Persons	Stephan Eckstein									

Abbreviations:
Grading System : g=graded, ng=not graded
Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-22	Module Title: Information Theory, Pattern Recognition, and Neural Networks		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Introduction to information theory: – The possibility of reliable communication over unreliable channels. The Hamming code and repetition codes. • Entropy and data compression: – Entropy, conditional entropy, mutual information, Shannon information content. The idea of typicality and the use of typical sets for source coding. Shannon's source coding theorem. Codes for data compression. Uniquely decodeable codes and the Kraft-MacMillan inequality. Completeness of a symbol code. Prefix codes. Huffman codes. Arithmetic coding. • Communication over noisy channels: – Definition of channel capacity. Capacity of binary symmetric channel; of binary erasure channel; of Z channel. Joint typicality, random codes, and Shannon's noisy channel coding theorem. Real channels and practical error-correcting codes. Hash codes. • Statistical inference, data modelling and pattern recognition: – The likelihood function and Bayes' theorem. Clustering as an example. • Approximation of probability distributions: – Laplace's method. (Approximation of probability distributions by Gaussian distributions.) Monte Carlo methods: Importance sampling, rejection sampling, Gibbs sampling, Metropolis method. (Slice sampling, Hybrid Monte Carlo, Overrelaxation, exact sampling.) Variational methods and mean field theory. Ising models. • Neural networks and content-addressable memories: – The Hopfield network.		
Objectives	Students are familiar with fundamental concepts of information theory – such as entropy and noisy channel coding – and how these are applied in computing and approximating probabilities. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Information Theory, Pattern Recognition, and Neural Networks	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	Possible References : David J.C. MacKay: Information Theory, Inference, and Learning Algorithms. CUP 2003.									
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation</i> and <i>Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: - MAT-20-01 Measure and Integration Theory, - MAT-20-12 Stochastics.									
Responsible Persons	Stephan Eckstein									

Abbreviations:
 Grading System : g=graded, ng=not graded
 Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
 Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
 Status : o=obligatory, f=facultative
 Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-23	Module Title: Nonparametric Estimation of Distributions			Type of Module: Compulsory Module with Choice						
ECTS-Points	4.5									
Workload - Time in Class - Self-Study	Workload: 150 h		Time in Class: 45 h		Self-Study: 115 h					
Duration	1 Semester									
Frequency	not regularly									
Term	1-3									
Language of Instruction	English									
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS									
Content	This course explores a fundamental question in statistics: Given n data points from an unknown random vector, how well can we infer the true characteristics of the random vector, i.e., its distribution? In particular, we will focus on the role of the dimension of the vector, leading to a treatment of the curse of dimensionality. Further, we will explore how prior knowledge (e.g., smoothness or structural understanding of the random vector) can help overcome the curse of dimensionality. In contrast to parametric statistics, where the distribution is assumed to belong to a finite-dimensional family (e.g., estimating mean and covariance matrix of a normal distribution), the focus is purely on nonparametric settings, where no such parametric forms are given a priori. These settings require suitably chosen evaluation criteria based on distances between probability distributions, and we will treat both density estimation with L^p -type losses and estimation in Wasserstein distance.									
Objectives	The statistical learning problem for probability distributions can be formulated precisely. The importance of the chosen metric and of the dimension of the distribution is well understood. Students are familiar with the fundamental proof techniques for establishing both upper bounds, using explicit estimators, and lower bounds, using information-theoretic methods. They develop a structural understanding of why $(G^{\frac{1}{2}}(Bandwhich(G^{\frac{1}{2}}(Bassumptions are necessary to achieve convergence rates that are better than those dictated by the curse of dimensionality. Through the exercise classes, students demonstrate their ability to work independently on topics covered in the lectures. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.$									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Nonparametric Estimation of Distributions	L E	f f	2 1	3 1,5	yes	wr. o. or.	90-180 o. 20-30	g	100
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										

Literature	Possible References : • Alexandre B. Tsybakov: Introduction to Nonparametric Estimation. Springer 2009. • Jonathan Niles-Weed, Quentin Berthet: Minimax estimation of smooth densities in Wasserstein distance. The Annals of Statistics 50.3 (2022): 1519-1540. • Nicolas Fournier, Arnaud Guillin: On the rate of convergence in Wasserstein distance of the empirical measure. Probability Theory and Related Fields 162.3 (2015): 707-738. • Daniel Bartl, Stephan Eckstein: Fast Wasserstein rates for estimating probability distributions of probabilistic graphical models. arXiv:2510.09270 (2025).
Transfer	The module belongs to the <i>Study Specialisations Numerical Mathematics and Optimisation and Stochastics</i> . Taking into account the chosen personal Study Specialisation, it can be included in the Sections <i>Study Focus</i> , <i>Advanced Knowledge in Mathematics</i> or <i>Elective Specialisation</i> , in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Stephan Eckstein
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	